International Journal of Solids and Structures 47 (2010) 1325–1326

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Erratum

Erratum to "Interaction integrals for fracture analysis of functionally graded piezoelectric materials", International Journal of Solids and Structures 45 (20) (2008) 5237–5257

B.N. Rao [a,*], M. Kuna [b]

[a] Structural Engineering Division, Department of Civil Engineering, Indian Institute of Technology, Madras, Chennai 600036, India
[b] TU Bergakademie Freiberg, Institut für Mechanik und Fluiddynamik, Lampadiusstraße 4, 09596 Freiberg, Germany

The authors regret that unfortunately errors cropped up in Section 4.2.4 of in our paper "Interaction integrals for fracture analysis of functionally graded piezoelectric materials", International Journal of Solids and Structures 45 (20) (2008) 5237–5257. The errors should be corrected as follows.

4.2.4. Proof of existence of interaction integral for FGPMs

Eqs. (46), (50) and (54) contains the second integral involving extra terms due to material nonhomogeneity. The existence of the second integral in Eqs. (46), (50) and (54), as the limit $r \to 0$ is proved below. Since, spatial derivatives of the material properties, C_{ijks}, e_{sij}, κ_{is}, are bounded at the crack tip, i.e., $C_{ijks,1}$, $e_{sij,1}$, and $\kappa_{is,1}$ are $O(r^{\alpha})$ with $\alpha \geqslant 0$, the second integral in Eqs. (46), (50) and (54) exists as the limit $r \to 0$. In the limit $r \to 0$, $C_{ijks}(r,\theta) = C^{*}_{ijks}$, $e_{sij}(r,\theta) = e^{*}_{sij}$, $\kappa_{is}(r,\theta) = \kappa^{*}_{is}$, in Eqs. (55)–(57). Hence in the limit $r \to 0$, in constant constitutive tensor formulation $\sigma^{(1)}_{ij}\frac{\partial \varepsilon^{(2)}_{ij}}{\partial x_1} = \frac{\partial \sigma^{(1)}_{ij}}{\partial x_1}\varepsilon^{(2)}_{ij}, \sigma^{(2)}_{ij}\frac{\partial \varepsilon^{(1)}_{ij}}{\partial x_1} = \frac{\partial \sigma^{(2)}_{ij}}{\partial x_1}\varepsilon^{(1)}_{ij}, D^{(1)}_{j}\frac{\partial E^{(2)}_{j}}{\partial x_1} = \frac{\partial D^{(1)}_{j}}{\partial x_1}\ E^{(2)}_{j}, D^{(2)}_{j}\frac{\partial E^{(1)}_{j}}{\partial x_1} = \frac{\partial D^{(2)}_{j}}{\partial x_1}E^{(1)}_{j}$, and in incompatibility formulation $\frac{\partial^2 u^{(2)}_{i}}{\partial x_j \partial x_1} = \frac{\partial \varepsilon^{(2)}_{ij}}{\partial x_1}, \frac{\partial^2 \phi^{(2)}}{\partial x_j \partial x_1} = -\frac{\partial E^{(2)}_{j}}{\partial x_1}$. Therefore the integrand of the second integral in Eq. (46) and the incompatibility terms in the integrand of the second integral in Eq. (50) naturally vanish. Note that in incompatibility formulation the auxiliary fields are compatible very near the crack tip (asymptotically).

(i) In addition, since in constant constitutive tensor formulation the auxiliary stress, and electrical displacement are related to the auxiliary strain, and electrical fields through a constant constitutive tensor comprising of the elastic, piezoelectric and dielectric material constants, evaluated at the crack tip, substituting $\sigma^{(2)}_{ij} = \left(C^{*}_{ijks} - C_{ijks}(r,\theta)\right)\varepsilon^{(2)}_{ks} - \left(e^{*}_{sij} - e_{sij}(r,\theta)\right) E^{(2)}_{s} + C_{ijks}(r,\theta)\varepsilon^{(2)}_{ks} - e_{sij}(r,\theta)E^{(2)}_{s}$, and $D^{(2)}_{i} = \left(e^{*}_{iks} - e_{iks}(r,\theta)\right) \varepsilon^{(2)}_{ks} + \left(\kappa^{*}_{is} - \kappa_{is}(r,\theta)\right)E^{(2)}_{s} + e_{iks}(r,\theta)\varepsilon^{(2)}_{ks} + \kappa_{is}(r,\theta)E^{(2)}_{s}$, the terms in the second integral of Eq. (46) in the limit $r \to 0$ becomes,

$$
\begin{aligned}
&\lim_{r\to 0}\int_A \frac{1}{2}\begin{pmatrix} \sigma^{(1)}_{ij}\frac{\partial \varepsilon^{(2)}_{ij}}{\partial x_1} - \frac{\partial \sigma^{(1)}_{ij}}{\partial x_1}\varepsilon^{(2)}_{ij} + \sigma^{(2)}_{ij}\frac{\partial \varepsilon^{(1)}_{ij}}{\partial x_1} - \frac{\partial \sigma^{(2)}_{ij}}{\partial x_1}\varepsilon^{(1)}_{ij} \\ -D^{(1)}_{j}\frac{\partial E^{(2)}_{j}}{\partial x_1} + \frac{\partial D^{(1)}_{j}}{\partial x_1}E^{(2)}_{j} - D^{(2)}_{j}\frac{\partial E^{(1)}_{j}}{\partial x_1} + \frac{\partial D^{(2)}_{j}}{\partial x_1}E^{(1)}_{j} \end{pmatrix} q\ dA \\
&= \lim_{r\to 0}\int_A \frac{1}{2}\begin{pmatrix} \sigma^{(1)}_{ij}\frac{\partial \varepsilon^{(2)}_{ij}}{\partial x_1} - \frac{\partial \sigma^{(1)}_{ij}}{\partial x_1}\varepsilon^{(2)}_{ij} + \sigma^{(2)}_{ij}\frac{\partial \varepsilon^{(1)}_{ij}}{\partial x_1} - \frac{\partial \sigma^{(2)}_{ij}}{\partial x_1}\varepsilon^{(1)}_{ij} \\ -D^{(1)}_{j}\frac{\partial E^{(2)}_{j}}{\partial x_1} + \frac{\partial D^{(1)}_{j}}{\partial x_1}E^{(2)}_{j} - D^{(2)}_{j}\frac{\partial E^{(1)}_{j}}{\partial x_1} + \frac{\partial D^{(2)}_{j}}{\partial x_1}E^{(1)}_{j} \end{pmatrix} q\ rdrd\theta \\
&= \lim_{r\to 0}\int_\theta\int_r \left(O(r)O\left(r^{-\frac{1}{2}}\right)O\left(r^{-\frac{3}{2}}\right) - O(r^{\alpha})O\left(r^{-\frac{1}{2}}\right)O\left(r^{-\frac{1}{2}}\right)\right) q\ rdrd\theta \\
&= \lim_{r\to 0} O(r) = 0
\end{aligned}
$$

(ii) Similarly the incompatibility terms in the second integral of Eq. (50) in the limit $r \to 0$ becomes,

$$
\begin{aligned}
&\lim_{r\to 0}\int_A \left(\sigma^{(1)}_{ij}\left(\frac{\partial^2 u^{(2)}_{i}}{\partial x_j \partial x_1} - \frac{\partial \varepsilon^{(2)}_{ij}}{\partial x_1}\right) + D^{(1)}_{j}\left(\frac{\partial^2 \phi^{(2)}}{\partial x_j \partial x_1} + \frac{\partial E^{(2)}_{j}}{\partial x_1}\right)\right) q\ dA \\
&= \lim_{r\to 0}\int_A \left(\sigma^{(1)}_{ij}\left(\frac{\partial^2 u^{(2)}_{i}}{\partial x_j \partial x_1} - \frac{\partial \varepsilon^{(2)}_{ij}}{\partial x_1}\right) + D^{(1)}_{j}\left(\frac{\partial^2 \phi^{(2)}}{\partial x_j \partial x_1} + \frac{\partial E^{(2)}_{j}}{\partial x_1}\right)\right) q\ rdrd\theta \\
&= \lim_{r\to 0}\int_\theta\int_r O(r)O\left(r^{-\frac{1}{2}}\right)O\left(r^{-\frac{3}{2}}\right) q\ rdrd\theta \\
&= \lim_{r\to 0} O(r) = 0
\end{aligned}
$$

(iii) Rest of the terms in the second integral of Eq. (50) in the limit $r \to 0$ becomes,

$$
\begin{aligned}
&\lim_{r\to 0}\int_A \left(-\varepsilon^{(1)}_{ij}\frac{\partial C_{ijkl}}{\partial x_1}\varepsilon^{(2)}_{kl} + \left(E^{(1)}_{n}\frac{\partial e_{nkl}}{\partial x_1}\varepsilon^{(2)}_{kl} + E^{(2)}_{n}\frac{\partial e_{nkl}}{\partial x_1}\varepsilon^{(1)}_{kl}\right) + E^{(1)}_{n}\frac{\partial \kappa_{nm}}{\partial x_1}E^{(2)}_{m}\right) q\, dA \\
&= \lim_{r\to 0}\int_A \left(-\varepsilon^{(1)}_{ij}\frac{\partial C_{ijkl}}{\partial x_1}\varepsilon^{(2)}_{kl} + \left(E^{(1)}_{n}\frac{\partial e_{nkl}}{\partial x_1}\varepsilon^{(2)}_{kl} + E^{(2)}_{n}\frac{\partial e_{nkl}}{\partial x_1}\varepsilon^{(1)}_{kl}\right) + E^{(1)}_{n}\frac{\partial \kappa_{nm}}{\partial x_1}E^{(2)}_{m}\right) q\, rdrd\theta \\
&= \lim_{r\to 0}\int_\theta\int_r O\left(r^{-\frac{1}{2}}\right)O(r^{\alpha})O\left(r^{-\frac{1}{2}}\right) q\, rdrd\theta = \lim_{r\to 0} O(r^{\alpha+1}) = 0
\end{aligned}
$$

(iv) Following similar procedure as given Paulino and Kim (2003), in the limit $r \to 0$ the nonequilibrium terms in the integrand of the second integral in Eq. (54) becomes,

DOI of original article: 10.1016/j.ijsolstr.2008.05.020.
* Corresponding author. Tel.: +91 44 2257 4285; fax: +91 44 2257 5286.
E-mail address: bnrao@iitm.ac.in (B.N. Rao).

0020-7683/$ - see front matter
doi:10.1016/j.ijsolstr.2010.01.001

$$\lim_{r\to 0}\int_A \left(\frac{\partial\sigma_{ij}^{(2)}}{\partial x_j}\frac{\partial u_i^{(1)}}{\partial x_1}+\frac{\partial D_j^{(2)}}{\partial x_j}\frac{\partial \phi^{(1)}}{\partial x_1}\right)q\ dA$$
$$=\lim_{r\to 0}\int_A \left(\frac{\partial\sigma_{ij}^{(2)}}{\partial x_j}\frac{\partial u_i^{(1)}}{\partial x_1}+\frac{\partial D_j^{(2)}}{\partial x_j}\frac{\partial \phi^{(1)}}{\partial x_1}\right)q\ rdrd\theta$$
$$=\lim_{r\to 0}\int_\theta\int_r \mathrm{O}(r)\mathrm{O}\left(r^{-\frac{3}{2}}\right)\mathrm{O}\left(r^{-\frac{1}{2}}\right)q\ rdrd\theta$$
$$=\lim_{r\to 0}\mathrm{O}(r)=0$$

In the limit $r \to 0$ rest of the terms in the second integral of Eq. (54) becomes as given in (iii). Thus all the proposed interaction integrals for non-homogeneous piezoelectric materials in Eqs. (46), (50) and (54) are well posed as the limit $r \to 0$ exists.

It should be pointed out that the above changes in Section 4.2.4 do not practically affect the results presented in the paper, because the numerical results are based on Eqs. (46), (50) and (54) and not on their proof of existence.