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Erratum

Erratum to "A genetic algorithm approach for solving a closed loop supply chain model: A case of battery recycling" [Appl. Math. Modell. 34 (2010) 655–670]

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ABSTRACT

This study analyzes Mixed Integer Linear Program (MILP) proposed by G. Kannan, P. Sasikumar M. Devika, (2010) in their paper titled 'A genetic algorithm approach for solving a closed loop supply chain model: A case of battery recycling', *Applied Mathematical Modelling*, (34, 655–670). The model in Kannan et al. (2010) is found to be inadequate for the problem described. It is erroneous/infeasible in terms of constraints, objective and variables. In this work, we list down the flaws in the published work and propose modifications to rectify the flaws. The revised model is presented and illustrated using hypothetical problems.

1. Problem description

The CLSC network, considered by Kannan et al. [1], is detailed below.

A network has J manufacturers catering to the needs of M retailers through a set of K distributors and L wholesalers. Manufacturers produce a set of products $P \in \{p_1, p_2, \ldots, p_p\}$. Raw materials $I \in \{i_1, i_2, \ldots, i_i\}$, for producing product set P are purchased from S suppliers. Retailers' demand for products P in each time period $t \in \{1, 2, \ldots, T\}$ is known. Products returned by customers are aggregated at X initial collection points, i.e., Return quantity is known for each time period. The products are subsequently transferred to a centralized return center (CRC), where the products are inspected for quality and sorted for potential repair or recycling. The products fit for recycling are transferred to any of Z recycling plants while products that cannot be recycled are pretreated and disposed off at any of Y disposal sites. Disposal rate gives the portion of non-recyclable products from the returns received. The recycled products are dismantled to raw material stage. Only some raw materials are sent to the manufacturing plant for usage in the new product. In this case, Lead is reclaimed and used in manufacturing new batteries. The remaining raw materials are sold off to external agencies. There is a cost associated with third party selling from each recycling plant. The task is to analyze a multi-product, multi-period allocation problem for closed loop supply chain network (see assumption of the model in Kannan et al. [1]). Fig. 1 shows the general structure of the CLSC network.

The remainder of this note is organized as follows: Section 2 discusses the errors/infeasibilities found in Kannan et al. [1] model and our revisions. The revised mathematical model for the scenario addressed in the paper is given in Section 3. A hypothetical problem is discussed in Section 4. Section 5 presents the summary.

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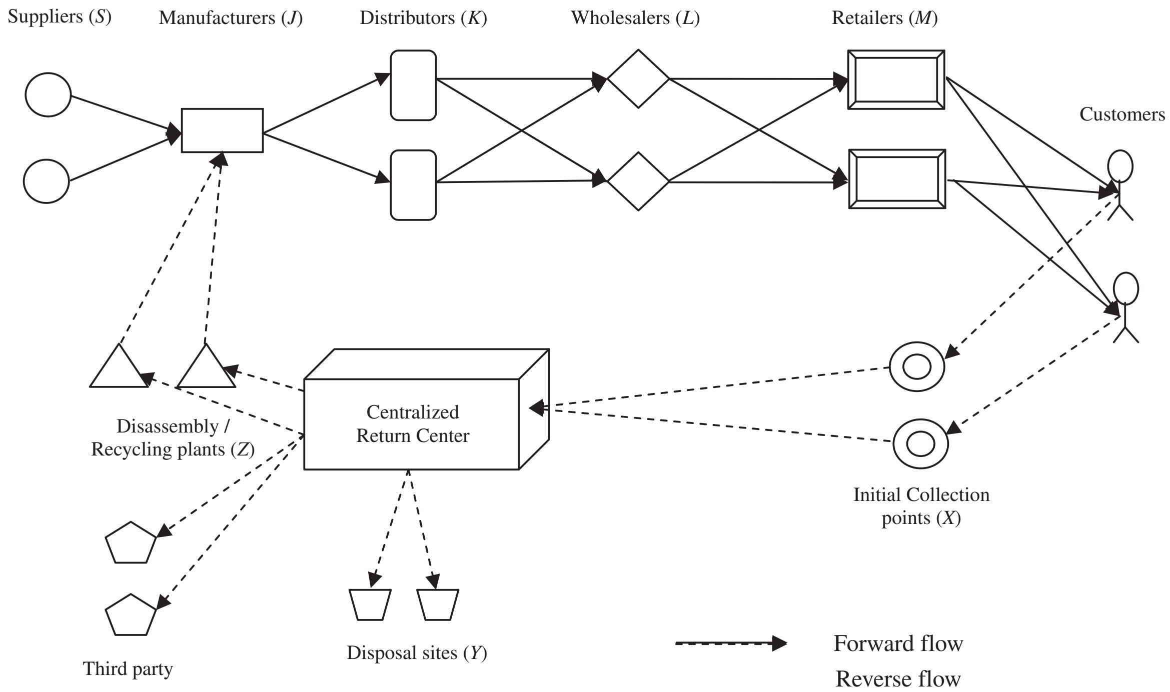

Fig. 1. General structure of the CLSC network.

2. Limitations and revisions

The errors/infeasibilities in the constraints and the objective function of the model and the corresponding revisions are discussed in this section.

2.1. Constraints

(1) Consider the constraints to determine the disposal and return quantities [1, pp. 661, 662].
The constraints of Kannan et al. [1] will make the model infeasible since the RHS of the constraint has a fractional element, the constraint is an equation and the LHS has to be integer. So, the model has to terminate with an infeasible solution. It is surprising that the authors report a feasible solution and compare it with their proposed GA. Such a case is simply solved by using a Gauss's symbol in the LHS value. The present constraints and the revised constraints are given below:

Present constraints	Revised constraints*
$\sum_y QTCD_{ypt} = QRC_{pt}.DR_{pt},\ \forall t, \forall p$	$\sum_y QTCD_{ypt} = \lfloor QRC_{pt}.DR_p \rfloor,\ \forall t, \forall p$
$\sum_z QTCR_{zpt} = QRC_{pt}.(1 - DR_{pt}),\ \forall t, \forall p$	$\sum_z QTCR_{zpt} = \lfloor QRC_{pt}.(1 - DR_p) \rfloor,\ \forall t, \forall p$

* Disposal rate of the product does not vary across time periods (i.e.) DR_p is used.

In the revised constraints, since the values inside the gauss's transformation are input variables and known beforehand, there is no need for an integer transformation However, in case where any of the variables inside the Gauss's symbol is a decision variable, an integer transformation would be needed (Rangwani et al. [2]; Subramanian et al. [3]; Wang and Hsu, [4]).

Dealing with return materials (in the raw material stage), the decision variables related to them have to be defined as either continuous or integers. In our formulation, the return quantities (in the raw material stage), is considered as integers.

(2) The basic difference in a CLSC network design problem is considering the returns in the forward logistics decisions. Here, the recycled Lead gives a savings in the purchase of Virgin Lead for producing new products (Batteries). However, the recycled/reclaimed component is not considered in the purchasing constraint. The assumption that logically and economically, a manufacturer chooses recycled raw material first and then moves onto buying new products is clear. But the model does not incorporate that (See assumption in page 659 of Kannan et al. [1]). The model in the paper could have been two separate formulations – one for forward logistics and another for reverse logistics (with corrected constraints) rather than a CLSC problem. The present constraint and the revised constraint are given below.

Present constraints	Revised constraints
$RMI_{ijt} = RMI_{ij(t-1)} + \sum_s RMP_{isjt} - \sum_p X_{ip} * QP_{jpt}$, $\forall i, \forall j, \forall t$	$RMI_{ijt} + \sum_p X_{ip} * QP_{jpt} = RMI_{ij(t-1)} + \sum_s RMP_{isjt} + \sum_z RMRP_{izjt}$, $\forall i, \forall j, \forall t$

The above revised constraints are valid only if the X_{ip} are integer values. The authors have considered X_{ip} could have continuous values. To incorporate this option into the model, the constraint is further revised as follows:

$$RMI_{ijt} + \left\lfloor \sum_p X_{ip} * QP_{jpt} \right\rfloor = RMI_{ij(t-1)} + \sum_s RMP_{isjt} + \sum_z RMRP_{izjt}, \quad \forall i,\ \forall j,\ \forall t.$$

As a decision variable is present inside the Gauss's symbol, an integer transformation is done as follows:

2.1.1. Transformation of Gaussian variables

To transform the above constraint with Gauss's symbol, three additional inequalities are needed as defined below and the transformed constraints are given.

$$\sum_p X_{ip} * QP_{jpt} \geqslant TQP_{ijt}, \quad \forall i,\ \forall j,\ \forall t,$$

$$\left(\sum_p X_{ip} * QP_{jpt} \right) - \varepsilon \leqslant TQP_{ijt}, \quad \forall i,\ \forall j,\ \forall t,$$

TQP_{ijt} is an integer and $\geqslant 0$; $\varepsilon \to 1$;

$$RMI_{ijt} + TQP_{ijt} = RMI_{ij(t-1)} + \sum_s RMP_{isjt} + \sum_z RMRP_{izjt}, \quad \forall i,\ \forall j,\ \forall t.$$

Now, the constraint will take an appropriate value according to the variable definition.

(3) Consider the following constraint. [1, p. 662].

$$QPDR_{zpt} = \sum_p \sum_t QTCR_{zpt}, \quad \forall t,\ \forall p.$$

The constraint should not exist at all. The notations defined by Kannan et al. [1] paper are as follows:

$QTCR_{zpt}$-quantity of useful (recyclable) returned products of p transported from centralized return center to the disassembly/recycling center z at time period t
$QPDR_{zpt}$-quantity of returned products of p processed (recycling) at the disassembly/recycling center z at time period t

Note that there is no inventory holding at the disassembly/recycling center z. This implies that the quantity of products coming into a disassembly/recycling center z is processed. However, Kannan et al. [1] have defined 2 different variables and have used an inappropriate constraint to relate them. We remove $QPDR_{zpt}$ from the formulation and use $QTCR_{zpt}$ appropriately.

(4) Consider the following [1, p. 662].

RMS_{izt}-amount of recycled raw material i from the disassembly/recycling plant z sold to third party during the time period t
Y_{ip}-percentage contribution of raw material i for the returned product p

The constraint below defines the recycled amount for raw material $i \in I$, which also takes into account the raw materials which is reclaimed for production (i.e.) Lead is also assumed to be sold to a third party. The equation given by Kannan et al. [1] is

$$RMS_{izt} = \sum_p (QTCR_{zpt} * W_p * Y_{ip}), \quad \forall i,\ \forall z,\ \forall t,$$

Y_{ip} could be defined as zero for raw materials which are reclaimed. But, there is no way to capture the percentage contribution of reclaimed raw materials such as Lead, in this case! Kannan et al. [1] did not face this issue because they did not consider the reclaiming equation and presented an infeasible/erroneous MILP model. In the revised formulation, to model the given scenario, we define a binary input variable TPL_i. It is a user defined variable and can be defined for each raw material (i.e.) $TPL_{(i=lead)}$ can be defined as 0.

$$TPL_i = \left\{ \begin{matrix} 1 - \text{if raw material } i \text{ is sold to third party} \\ 0 - \text{otherwise} \end{matrix} \right\}.$$

The present and the revised constraint are as follows:

Present constraints	Revised constraints
$RMS_{izt} = \sum_p (QTCR_{zpt} * W_p * Y_{ip}),\ \forall i, \forall z, \forall t$	$RMS_{izt} = \left\lfloor \sum_p (QTCR_{zpt} * W_p * Y_{ip} * TPL_i) \right\rfloor,\ \forall i, \forall z, \forall t$

As a decision variable is present inside the Gauss's symbol, an integer transformation is done as follows:

2.1.2. Transformation of Gaussian variables

To transform the above constraint with Gauss's symbol, three additional inequalities are needed as defined below and the transformed constraints are given.

$$\sum_p (QTCR_{zpt} * W_p * Y_{ip} * TPL_i) \geqslant TRANS_{izt}, \quad \forall i,\ \forall z,\ \forall t,$$

$$\sum_p (QTCR_{zpt} * W_p * Y_{ip} * TPL_i) - \varepsilon \leqslant TRANS_{izt}, \quad \forall i,\ \forall z,\ \forall t,$$

$TRANS_{izt}$ is an integer and $\geqslant 0$; $\varepsilon \to 1$;

$$TRANS_{izt} = RMS_{izt}.$$

The decision variable RMS_{izt} would now take an appropriate value according to the problem definition.

(5) Consider the following [1, p. 662].

(i) $\sum_j RMRP_{izjt} = \alpha_{izt}(1 - RMS_{izt}), \forall i, \forall z, \forall t$
RMS_{izt} is the amount of recycled raw material i from the disassembly/recycling plant z sold to third party during the time period t. When this is subtracted from 1, RHS becomes a negative value. Now the LHS, which is a decision variable that is constrained to be non-negative, takes a negative value! This problem should terminate with an infeasible solution. The solutions given by the authors need to be scrutinized.

(ii) α_{izt}-Recycling rate of the required raw material i to be reclaimed for production of new batteries at the disassembly/recycling plant z in time period t

Note:

- Does the recycling rate per product vary from time to time in a recycling plant?
- Does the recycling rate per product vary from recycling plant to plant? Variable recycling rate can at least be accepted to some extent.

In the revised formulation, we define, α_{iz}-Recycling rate of the required raw material i to be reclaimed for new battery production from a product p at the disassembly/recycling plant z.

The revised constraint is for the total reclaimed raw material i in disassembly/recycling plant z at time period t.

The present and the revised constraint are as follows:

Present constraints	Revised constraints
$\sum_j RMRP_{izjt} = \alpha_{izt}(1 - RMS_{izt}),\ \forall i, \forall z, \forall t$	$\sum_j RMRP_{izjt} = \left\lfloor \sum_p (QTCR_{zpt} * W_p * Y_{ip} * (1 - TPL_i) * \alpha_{iz}) \right\rfloor,\ \forall i, \forall z, \forall t$

Similar to the case explained earlier, the integer transformation for the constraint is done as follows:

2.1.3. Transformation of Gaussian variables

Three additional inequalities are needed. These are given below along the transformed constraints.

$$\sum_p (QTCR_{zpt} * W_p * Y_{ip} * (1 - TPL_i) * \alpha_{iz}) \geqslant TRMRP_{izt}, \quad \forall i,\ \forall z,\ \forall t,$$

$$\left(\sum_p (QTCR_{zpt} * W_p * Y_{ip} * (1 - TPL_i) * \alpha_{iz}) - \varepsilon \right) \leqslant TRMRP_{izt}, \quad \forall i,\ \forall z,\ \forall t,$$

$TRMRP_{izt}$ is an integer and $\geqslant 0$; $\varepsilon \to 1$;

$$TRMRP_{izt} = \sum_{j} RMRP_{izjt}.$$

Thus, the decision variable $RMRP_{izjt}$ would now take an appropriate value according to the problem definition.

(6) Consider the following demand constraint [1, p. 663].

$$\sum_{j}\sum_{p}\sum_{t} QP_{jpt} \geqslant \sum_{m}\sum_{p}\sum_{t} DD_{mpt}.$$

This constraint requires the demand summed up across all retailers, all products and all time periods to be less than the production rate summed up over all manufacturing plants, all products and all time periods.

The following two cases will prove that the constraint in Kannan et al. [1] is erroneous.

(1) Let the demand for product 1 in retailer 1 and retailer 2 be 100 and 10. There are only 2 retailers in the network. Suppose that the cost of supplying to retailer 1 is 20 and retailer 2 is 10. Now, based on the Kannan et al. [1] constraint, the whole shipping will be delivered to retailer 2 (minimizing the cost). The solution is feasible to their constraint, but, it is erroneous and infeasible to the scenario.
(2) Let the demand for product 1 be 100 and product 2 be 20 at retailer 1. A simple case, in which there is only one retailer. Suppose transportation cost and inventory carrying cost for product 2 is lesser than product 1. Based on Kannan et al. [1] formulation, 120 of product 2 will be shipped to retailer 1. The solution is infeasible / erroneous. Similarly, the constraint can cause infeasible quantity allocation in line with time periods and manufacturing plants.

The present and revised constraints are as follows:

Present constraints	Revised constraints
$\sum_j\sum_p\sum_t QP_{jpt} \geqslant \sum_m\sum_p\sum_t DD_{mpt}$	$\sum_l QTWR_{lmpt} \geqslant DD_{mpt}, \ \forall m, \forall p, \forall t$

(7) Consider the following capacity constraints: [1, p. 663]

$$\sum_{x}\sum_{p}\sum_{t} QC_{xpt} \leqslant \sum_{z}\sum_{p}\sum_{t} CDS_{zpt}.$$

The constraint requires that the total quantity collected at the initial collection point summed over all products and all time periods to be less than the capacity of all recycling points summer over all products and all time periods.

A constraint is written to bind the feasible region for a decision variable or for a set of decision variables. In this constraint of Kannan et al. [1], both the LHS and the RHS are input variables. This is not a constraint and hence removed.

(8) Let us consider the following constraint [1, p. 663].

$$\sum_{y}\sum_{p}\sum_{t} QTCD_{ypt} \leqslant \sum_{y}\sum_{p}\sum_{t} CDS_{ypt}.$$

Does the disposal site have product-wise capacity?

This constraint requires that the quantity disposed from CRC summed up across all disposal sites, all products and all time periods to be less than the capacities summed up over all disposal sites, all products and all time periods. The entire disposal quantities will follow the path of lowest cost.

The present and revised constraints are as follows:

Present constraints	Revised constraints
$\sum_y\sum_p\sum_t QTCD_{ypt} \leqslant \sum_y\sum_p\sum_t CDS_{ypt}$	$\sum_p QTCD_{ypt} \leqslant CDS_y, \ \forall y, \forall t$

Similarly, the present and revised constraints for the disassembly/recycling plant are as follows:

Present constraints	Revised constraints
$\sum_x\sum_p\sum_t QC_{xpt} \leqslant \sum_z\sum_p\sum_t CDS_{zpt}$	$QTCR_{zpt} \leqslant CD_{zp}, \forall z, \forall p, \forall t$

2.2. Objective function

(1) Consider the inventory carrying costs at the CRC [1, p. 662]. The following proof shows that calculating inventory at the CRC is not meaningful.
Given in the paper [1, pp. 661, 662]

$$TCICC = \sum_p \sum_t CI_{pt}.ICC_{pt},$$

where

$$CI_{pt} = CI_{p(t-1)} + QRC_{pt} - \sum_y QTCD_{ypt} - \sum_z QTCR_{zpt}, \tag{I}$$

$$\sum_y QTCD_{ypt} = QRC_{pt} * DR_{pt}, \tag{a}$$

$$\sum_z QTCR_{zpt} = QRC_{pt} * (1 - DR_{pt}), \tag{b}$$

DR_{pt} is the disposal rate of product p in time period t (Note that the disposal rate of a product does not change in each time period. But to illustrate the errors in the case, we use the terms as such. We will revise them in our formulation).

Proof. Substitute (a) and (b) in the equation (I),

$$\begin{aligned} CI_{pt} &= CI_{p(t-1)} + QRC_{pt} - (QRC_{pt} * DR_{pt}) - (QRC_{pt} * (1 - DR_{pt})) \\ &= CI_{p(t-1)} + QRC_{pt} - (QRC_{pt} * (DR_{pt} + 1 - DR_{pt})) \\ &= CI_{p(t-1)} + QRC_{pt} - QRC_{pt}, \\ CI_{pt} &= CI_{p(t-1)}. \end{aligned}$$

So CI_{pt} always remains the same in all the time periods. If the beginning inventory at the CRC (i.e.) $CI_{p,0}$, is given as Zero, then the $CI_{p,1}, CI_{p,2}, CI_{p,3}, \ldots, CI_{p,T}$ will all be Zero.

(2) Consider the following objective function term for the total collection cost of returned products [1, p. 661].

$$TCC = \sum_x \sum_p \sum_t QC_{xpt}.TCIC_{xp},$$

QC_{xpt} is the quantity of returned items of product p collected at the initial collection point x during time period t. Similarly, $TCIC_{xp}$ is the cost of collection at collection center x. Both QC_{xpt} and $TCIC_{xp}$ are deterministic input variables. A constant in the objective function has no effect on the solution. So, we remove TCC from the objective function. For the given function in the objective function, calculating a savings matrix is inappropriate. When the task is to collect and deliver the returned items at the initial collection points to the CRC, using a set of vehicles, the vehicle routing problem emerges. However, the problem of collecting returned materials to the CRC could be seen as a separate problem in this case and can be solved separately for each time period. There is no need to jointly solve the VRP and the CLSC allocation problem. The authors have merged the collection points according to the savings (Clark and Wright, 1964 not referred!).

(3) While the stated assumption is that the transportation costs and inventory costs do not vary according to time and remains constant throughout the period of study (See assumptions 3 and 5), all the input variables (transportation costs, inventory costs, disposal costs, procurement costs) are shown with subscript t! This is in conflict with the model assumptions. In the revised formulation, we remove all the subscripts t from the input variables which are assumed to remain constant across time periods. □

Based on the discussions in Sections 2.1 and 2.2, we now propose a revised mathematical model as an ILP accounting all the forward and reverse logistics constraints in Section 3.

3. Revised mathematical model

We propose an Integer Linear Program (ILP) for the problem considered by Kannan et al. [1]. For the sake of clarity of the readers, we have used the same notations, indices and equation numbers wherever possible.

Notations

Indices		
s	Suppliers	$(s = 1,2,\ldots,S)$
i	Raw materials	$(i = 1,2,\ldots,I)$
j	Manufacturing plants	$(j = 1,2,\ldots,J)$
k	Distributors	$(k = 1,2,\ldots,K)$
l	Wholesalers	$(l = 1,2,\ldots,L)$
m	Retailers	$(m = 1,2,\ldots,M)$
x	Initial collection points	$(x = 1,2,\ldots,X)$
y	Disposal sites	$(y = 1,2,\ldots,Y)$
z	Disassembly/recycling plants	$(z = 1,2,\ldots,Z)$
p	Products	$(p = 1,2,\ldots,P)$
t	Time periods	$(t = 1,2,\ldots,T)$

Costs in the objective function

TC	Total closed loop supply chain cost
TCRTC	Total transportation cost from CRC to disassembly/recycling plant
TDC	Total disposal costs
TDIC	Inventory carrying cost at Distributor
TDWTC	Total transportation cost from distributors to wholesalers
TFGIC	Finished goods inventory carrying costs at the manufacturing plant
TPC	Total processing cost
TPDTC	Total transportation cost from manufacturing plant to distributors
TPUC	Total purchasing cost
TRC	Total recycling cost from the disassembly/recycling plant to the third party
TRMIC	Raw material inventory carrying costs at the manufacturing plant
TRPC	Total disassembly/reclaiming cost at the disassembly/recycling plant
TRPTC	Total transportation cost from disassembly/recycling plant to manufacturing plant
TWIC	Inventory carrying cost at Wholesaler
TWRTC	Total transportation cost from wholesalers to retailers

Input parameters

PUC_{is}	Purchasing cost/ unit of raw material i from supplier s
PC_{jp}	Processing cost per product p at manufacturing plant j
$TCPD_{jkp}$	Transportation cost/unit of product p from manufacturing plant j to distributor k
$TCDW_{klp}$	Transportation cost/unit of product p from distributor k to wholesaler l
$TCWR_{lmp}$	Transportation cost/unit of product p from wholesaler l to retailer m
RIC_{ij}	Inventory carrying cost/unit of raw material i at manufacturing plant j
FIC_{jp}	Inventory carrying cost/unit of product p at manufacturing plant j
ICD_{kp}	Inventory carrying cost/unit of product p at distributor k
ICW_{lp}	Inventory carrying cost/unit of product p at wholesaler l
QC_{xpt}	Quantity of retuned products p at initial collection point x in time period t
QRC_{pt}	Quantity of product p collected at the centralized return center=$\sum_x QC_{xpt}$
X_{ip}	Amount of raw material i required to produce one unit of product p
DC_{yp}	Disposal cost/unit of useless returned product p at disposal site y
$TCCR_{zp}$	Transportation cost/unit of product p from CRC to disassembly/recycling plant z
DR_p	Disposal rate of product p
DRC_{zp}	Disassembly/recycling cost per unit for the returned product p at the disassembly/recycling center z
RC_{iz}	Recycling cost / unit of raw material i sold to third party from disassembly/recycling center z
W_p	Weight of the product p
Y_{ip}	Percentage contribution of raw material i for the returned product p; [0,1]
$TCRP_{izj}$	Transportation cost per tonne for the reclaimed raw material i from disassembly/recycling center z to manufacturing plant j
α_{iz}	Recycling rate of the raw material i reclaimed for new battery production at the disassembly/recycling center z; [0,1]

(*continued on next page*)

PRS_j	Raw material storage capacity at manufacturing plant j
PFS_j	Finished goods storage capacity at manufacturing plant j
SC_{ps}	Supply capacity of supplier s for product p
PT_{jp}	Available processing time for product p in manufactur j
DSC_k	Storage capacity of distributor k
WSC_l	Storage capacity of wholesaler l
DD_{mpt}	Demand of product p in retailer m at time period t
CD_{zp}	Recycling capacity of disassembly/recycling center z for product p
CDS_y	Capacity of disposal site y

$$TPL_i = \left\{ \begin{matrix} 1 - \text{if raw material } i \text{ is sold to third party} \\ 0 - \text{otherwise} \end{matrix} \right\}.$$

Decision variables

RMP_{isjt}	Amount of raw material i Purchased from supplier s by manufacturing plant j in time period t
QP_{jpt}	Quantity of product p processed at manufacturing plant j in time period t
$QTPD_{jkpt}$	Quantity of product p transported from manufacturing plant j to distributor k in time period t
$QTDW_{klpt}$	Quantity of product p transported from distributor k to wholesaler l in time period t
$QTWR_{lmpt}$	Quantity of product p transported from wholesaler l to retailer m in time period t
$QTCD_{ypt}$	Quantity of useless product p transported from CRC to disposal site y in time period t
$QTCR_{zpt}$	Quantity of returned product p transported from CRC to disassembly/recycling center z in time period t
RMI_{ijt}	Inventory of raw material i at manufacturer j in time period t
FGI_{jpt}	Inventory of finished product p at manufacturer j in time period t
DI_{kpt}	Inventory of product p at distributor k in time period t
WI_{lpt}	Inventory of product p at wholesaler l in time period t
RMS_{izt}	Amount of recycled raw material i from the disassembly/recycling center z to the third party in time period t
$RMRP_{izjt}$	Amount of raw material i reclaimed for new battery production and transported from disassembly/recycling center z to manufacturing plant j in time period t

$$\begin{aligned}\text{Minimize } TC = {} & TPUC + TPC + TPDTC + TDWTC + TWRTC + TRMIC + TFGIC + TDIC \\ & + TWIC + TDC + TCRTC + TRPC + TRC + TRPTC\end{aligned}$$

$$TPUC = \sum_i \sum_s \sum_j \sum_t RMP_{isjt}.PUC_{is}$$

$$TPC = \sum_j \sum_p \sum_t QP_{jpt}.PC_{jp}$$

$$TPDTC = \sum_j \sum_k \sum_p \sum_t QTPD_{jkpt}.TCPD_{jkp}$$

$$TDWTC = \sum_k \sum_l \sum_p \sum_t QTDW_{klpt}.TCDW_{klp}$$

$$TWRTC = \sum_l \sum_m \sum_p \sum_t QTWR_{lmpt}.TCWR_{lmp}$$

$$TRMIC = \sum_i \sum_j \sum_t RMI_{ijt}.RIC_{ij}$$

$$TFGIC = \sum_j \sum_p \sum_t FIG_{jpt}.FIJ_{jp}$$

$$TWIC = \sum_l \sum_p \sum_t WI_{lpt}.ICW_{lp}$$

$$TDIC = \sum_k \sum_p \sum_t DI_{kpt}.ICD_{kp}$$

$$TDC = \sum_y \sum_p \sum_t QTCD_{ypt}.DC_{yp}$$

$$TCRTC = \sum_z \sum_p \sum_t QTCR_{zpt}.TCCR_{zp}$$

$$TRPC = \sum_z \sum_p \sum_t QTCR_{zpt}.DRC_{zp}$$

$$TRC = \sum_i \sum_z \sum_t RMS_{izt}.RC_{iz}$$

$$TRPTC = \sum_i \sum_z \sum_j \sum_t RMRP_{izjt}.TCRP_{izj}.$$

Subject to,

Supplier

Supply capacity constraints

$$\sum_j RMP_{isjt} \leqslant SC_{is} \quad \forall i,\ \forall s,\ \forall t.$$

Manufacturer

Raw material constraint

$$RMI_{ijt} = RMI_{ij(t-1)} + \sum_s RMP_{isjt} + \sum_z RMRP_{izjt} - \left\lfloor \sum_p X_{ip}.QP_{jpt} \right\rfloor \quad \forall i,\ \forall j,\ \forall t.$$

Transformation of Gaussian variables

$$\left(\sum_p X_{ip} * QP_{jpt}\right) \geqslant TQP_{ijt}, \quad \forall i,\ \forall j,\ \forall t,$$

$$\left(\sum_p X_{ip} * QP_{jpt}\right) - \varepsilon \leqslant TQP_{ijt}, \quad \forall i,\ \forall j,\ \forall t,$$

TQP_{ijt} is an integer and $\geqslant 0$; $\varepsilon \to 1$;

$$RMI_{ijt} + TQP_{ijt} = RMI_{ij(t-1)} + \sum_s RMP_{isjt} + \sum_z RMRP_{izjt}, \quad \forall i,\ \forall j,\ \forall t.$$

Finished goods constraints

$$FGI_{jpt} = FGI_{jp(t-1)} + QP_{jpt} - \sum_k QTPD_{jkpt} \quad \forall j,\ \forall p,\ \forall t.$$

Storage capacity constraints-Raw materials

$$\sum_i \sum_s RMP_{isjt} \leqslant PRS_j \quad \forall j,\ \forall t.$$

Storage capacity constraints- Finished goods

$$\sum_p QP_{jpt} \leqslant PFS_j \quad \forall j,\ \forall t.$$

Processing time constraints

$$QP_{jpt} \leqslant PT_{jp} \quad \forall p,\ \forall j,\ \forall t.$$

Distributor

Storage capacity constraints

$$\sum_j \sum_p QTPD_{jkpt} \leqslant DSC_k \quad \forall k,\ \forall t.$$

Flow constraint forDistributor

$$DI_{kpt} = DI_{kp(t-1)} + \sum_j QTPD_{jkpt} - \sum_l QTDW_{klpt} \quad \forall k,\ \forall p,\ \forall t.$$

Wholesaler

Storage capacity constraints

$$\sum_k \sum_p QTDW_{klpt} \leqslant WSC_l \quad \forall l,\ \forall t$$

Flow constraint for Wholesaler

$$WI_{lpt} = WI_{lp(t-1)} + \sum_k QTDW_{klpt} - \sum_m QTWR_{lmpt} \quad \forall l,\ \forall t,\ \forall p.$$

Retailer

Demand satisfying/Flow constraints

$$\sum_i QTWR_{lmpt} \geqslant DD_{mpt} \quad \forall m,\ \forall p,\ \forall t.$$

Disassembly/Recycling center

Storage capacity constraints

$$QTCR_{zpt} \leqslant CD_{zp} \quad \forall z,\ \forall p,\ \forall t.$$

Recycling quantity

$$\sum_z QTCR_{zpt} = \lfloor QRC_{pt}.(1 - DR_p) \rfloor \quad \forall t,\ \forall p.$$

Reclaimed raw materials – to manufacturing plants

$$\sum_j RMRP_{izjt} = \left\lfloor \sum_p \left(QTCR_{zpt} * W_p * Y_{ip} * (1 - TPL_i) * \alpha_{iz}\right) \right\rfloor \quad \forall i,\ \forall z,\ \forall t.$$

Transformation of Gaussian variables

$$\sum_p (QTCR_{zpt} * W_p * Y_{ip} * (1 - TPL_i) * \alpha_{iz}) \geqslant) TRMRP_{izt} \quad \forall i,\ \forall z,\ \forall t,$$

$$\left(\sum_p \left(QTCR_{zpt} * W_p * Y_{ip} * (1 - TPL_i) * \alpha_{iz}\right) - \varepsilon \right) \leqslant TRMRP_{izt} \quad \forall i,\ \forall z,\ \forall t,$$

$TRMRP_{izt}$ is an integer and $\geqslant 0$; $\varepsilon \to 1$;

$$TRMRP_{izt} = \sum_j RMRP_{izjt}.$$

Recycled raw materials – to Third party

$$RMS_{izt} = \left\lfloor \sum_p (QTCR_{zpt} * W_p * Y_{ip} * TPL_i) \right\rfloor, \quad \forall i,\ \forall z,\ \forall t.$$

Transformation of Gaussian variables

$$\sum_p (QTCR_{zpt} * W_p * Y_{ip} * TPL_i) \geqslant TRANS_{izt} \quad \forall i,\ \forall z,\ \forall t,$$

$$\sum_p (QTCR_{zpt} * W_p * Y_{ip} * TPL_i) - \varepsilon \leqslant TRANS_{izt} \quad \forall i,\ \forall z,\ \forall t,$$

$TRANS_{izt}$ is an integer and $\geqslant 0$; $\varepsilon \to 1$;

$$TRANS_{izt} = RMS_{izt}.$$

Disposal site

Storage capacity constraints

$$\sum_p QTCD_{ypt} \leqslant CDS_y \quad \forall y,\ \forall t.$$

Disposal quantity

$$\sum_y QTCD_{ypt} = \lfloor QRC_{pt}.DR_p \rfloor \quad \forall t,\ \forall p.$$

Initial inventory levels

Initial inventory levels at manufacturers, distributors, wholesalers are set to zero. It is a user defined value and can be given a value as per the inventory levels at the beginning of the planning horizon.

Integral constraints

$$RMP_{isjt}, QP_{jpt}, QTPD_{jkpt}, QTDW_{klpt}, QTWR_{lmpt}, QTCD_{ypt},$$
$$QTCR_{zpt}, RMI_{ijt}, FGI_{jpt}, DI_{kpt}, WI_{lpt}, RMS_{izt}, RMRP_{izjt} \quad \geqslant 0 \text{ and Integers}.$$

4. Model testing and validation

The proposed ILP is tested using two hypothetical datasets. Dataset 1 is similar to the illustration given by Kannan et al. [1] (An optimization problem for a 2 retailer problem is unrealistic / unacceptable. The maximum size of the problems considered by Kannan et al. [1] has 2 retailers and 1 product). To demonstrate the utility of the proposed model, we also consider a larger network in Dataset 2. The datasets are available with the authors and can be provided for interested readers.

The network sizes considered in the datasets are given in Table 1. The hypothetical datasets design parameters are shown in Table 2.

Table 1
Network structure for the hypothetical datasets.

Elements	Dataset 1	Dataset 2
Suppliers (*S*)	2	10
Raw materials (*I*)	2	2
Manufacturers (*J*)	1	3
Distributors (*K*)	2	5
Wholesalers (*L*)	2	7
Retailers (*M*)	2	10
Initial collection points (*X*)	5	5
Disposal sites (*Y*)	1	3
Disassembly/Recycling centers (*Z*)	1	2
Products (*P*)	2	2
Time periods (*T*)	2	5

Table 2
Hypothetical dataset parameters.

Description	Parameter	Values (Uniform distribution)
Storage capacities		
Supply capacity of supplier *s*	SC_{is}	25000
Manufacturing plant – raw material	PRS_j	30000
Manufacturing plant – finished goods	PFS_j	5000
Distributor	DSC_k	~Unif (2000, 2500)
Wholesaler	WSC_l	~Unif (1500, 2000)
Recycling plant	CD_{zp}	~Unif (400, 600)
Disposal site	CDS_y	~Unif (300, 400)
Demand	D_{mpt}	Dataset 1: ~Unif (400, 600) Dataset 2: ~Unif (100, 150)
Manufacturing plant – processing times	PT_{jp}	Product 1-4000 ; Product 2-3500;
Amount of raw material *i* in product *p*	X_{ip}	P1-RM1: 3.069 P2-RM1: 3.001 P1-RM2: 2.559 P2-RM2: 3.301
Cost parameters		
Cost of Raw material	PUC_{is}	~Unif (100, 120)
Cost of processing at manufacturing plants	PC_{jp}	~Unif (250, 300)
Raw material carrying cost Manufacturer	RIC_{ij}	~Unif (1, 2)
Finished goods carrying cost (Manufacturer, Distributor, Wholesaler)	FIC_{jp}, ICD_{kp}, ICW_{lp}	~Unif (4, 5)
Disposal cost	DC_{yp}	~Unif (8, 11)
Recycling cost	DRC_{zp}	~Unif (10, 20)
Third party selling rate cost	RC_{iz}	~Unif (4, 6)
Transportation cost between *i* to *j*		~Unif (5, 10)
Return parameters		
Returns at collection center *x*	QC_{xpt}	~Unif (100, 160)
Recycling rate	α_{iz}	~Unif (0.6, 0.8)
Percentage contribution of raw material in the returned product	Y_{ip}	Set depending on the number of raw materials and their contribution in the returned product
Disposal rate	DR_p	~Unif (0.2, 0.4)

Table 3
Result for the hypothetical datasets.

Integer variables	CPU time (seconds)	Optimal CLSC cost
104	0.01	34,39,104.95
1910	0.14	1,02,71,968.26

The ILP model is solved using ILOG CPLEX 9.0 on a PC with INTEL(R) Core (TM) 2 Duo, 3 GHz processor with 2 GB of RAM. Table 3 presents the results.

5. Summary

We have analyzed the MILP proposed by Kannan et al. [1], listed the errors and the infeasibilities in their model. A revised and comprehensive ILP is proposed, to suit the CLSC application. Two hypothetical test data sets are solved using the model.

References

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