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Economic back-off selection based on optimal multivariable controller

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Abstract: This paper discusses the minimum backed off operating point selection problem based on process economics. In this work, we consider the case where the nominal operating point is not completely constrained, i.e., there are some unconstrained degrees of freedom or manipulations available. In this regard, we propose a stochastic formulation that ensures feasible dynamic operating region within the prescribed confidence limit. Furthermore, the formulation also finds a suitable multivariable controller to achieve economic benefits. The problem is nonlinear and non-convex and hence an iterative solution procedure is proposed such that at each step in the iteration, a convex problem is solved. Finally, the approach is illustrated using an evaporation process.

Keywords: Process control, multivariable control, convex optimization

1. INTRODUCTION

With increase in global competition, it has become increasingly important and relevant to operate a chemical plant at the profitable operating point within the operating window. Often, the optimal operation is found to occur at the boundary of the operating window. These boundaries define the design constraints, environmental and safety limits, etc. However, presence of uncertainties in the form of measurement noise, modeling error, parametric uncertainties and control errors caused by disturbances cause violation of these constraints. In this paper, we consider the effect of dynamic control errors caused by the normally distributed random disturbances. Thus, back off from the constraints is required to remain feasible. Back-off is defined as the amount by which the actual operating point is departed from the optimal operating point to ensure feasibility. On the other hand, departure from optimality will result in loss of profit. Hence, the rational solution is to formulate and solve an optimization problem that accounts for the trade-off between feasibility and profitability and this is the focus of this contribution.

The notion of back off is illustrated in Fig. 1 where the rectangular region represents the feasible operating window. Contours of the objective function are shown in dashed line. Generally, the Optimal Operating Point (OOP) is determined by solving a non-linear steady state optimization problem. Often, the OOP is constrained and is marked by star. To determine the economic back-off, the knowledge of dynamic operating region is required. Dynamic region indeed depends on the selected controller. Under the assumption of Gaussian uncertainty, the Expected Dynamic Operating Region (EDOR) can be represented as ellipses. The size of the region is characterized by the confidence limit and variance of the disturbance considered while the orientation of the ellipse depends the controller. Hence, the selection of Minimum Back-off Operating Point (MBOP) depends on the structural decisions in the control layer. Figueroa et al. (1996) solved the dynamic optimization problem to compute the necessary back off using open loop indicators. However, the formulation does not include controller as decision variable rather only alternative controller performance are studied. Later, Peng et al. (2005) addressed the simultaneous controller and MBOP selection in the stochastic framework for a fully constrained case (i.e., number of manipulated inputs equals the number of active constraints). As their formulation contains a set of convex and reverse convex constraints, a branch and bound type procedure for obtaining globally optimal solution has been proposed. Several other authors have also addressed the issue of how process operations are limited by process design (Narraway et al. (1991); Pistikopoulos (1995)) and structural decisions on online optimizer and controller (Loeblein and Perkins (1999); Kookos and Perkins (2002)).

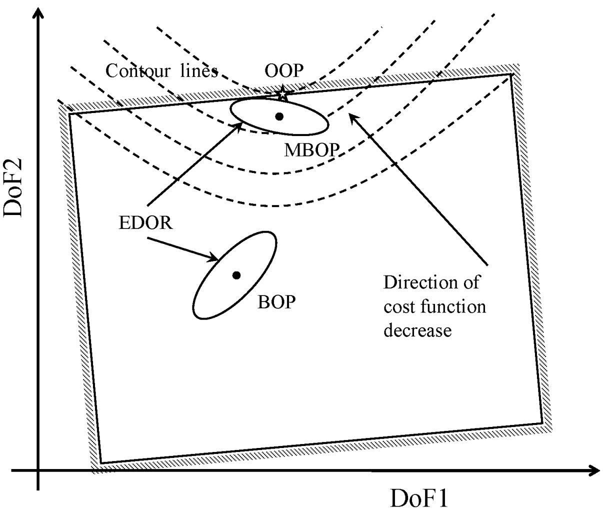

Fig. 1. Idea of back off operating point selection

Loeblein and Perkins (1999) and Peng et al. (2005) have computed the amount of back off required using first order sensitivity of the cost function which applies to fully constrained optimal solution. It is well known that the solution of the Linear Program is always constrained and hence linear approximation yields a solution at the corner points of the operating envelope. As a result, the back off solution will be near the corner points. To consider a more general case where some unconstrained degrees of freedom are available, it is important to include quadratic information in the cost function. In this work, we have extended the formulation for a partially constrained case (i.e., number of manipulated inputs is greater than the number of active constraints) to determine MBOP and the controller. Furthermore, convex relaxation of the constraints are presented and a solution methodology is developed. Finally, the proposed formulation is exemplified using an evaporator system.

2. PROBLEM FORMULATION

In this section, we develop a stochastic formulation that ensures feasible operation modulo, a prescribed confidence limit i.e., the probability that the constraints are satisfied is greater than or equal to the confidence limit. In this formulation, we have assumed full state feedback and disturbance as the only source of uncertainty and is characterized by zero mean Gaussian white noise. Following Peng et al. (2005), the dynamic operating region is defined for the given disturbances which follow from the closed loop covariance analysis of the state space model of the process. Therefore, the current objective is to formulate the optimization problem that aims at determining the MBOP and optimal controller such that the dynamic operating region remains feasible for the given confidence limit while minimizing the loss in profit.

We start by determining the optimal operating point at steady state operation that minimizes the economic cost (the negative of the operating profit) $J(x_0, u_0, \overline{d}_0)$ where x_0,u_0 and $\overline{d}_0$ denote the states, manipulated inputs and expected value of disturbances. Thus, the steady state optimizer solves the nonlinear steady state optimization problem of the form,

$$\min_{x_0,u_0} \; J(x_0, u_0, \overline{d}_0) \tag{1a}$$

$$s.t. \; g(x_0, u_0, \overline{d}_0) = 0 \tag{1b}$$

$$h(x_0, u_0, \overline{d}_0) \leq 0 \tag{1c}$$

At OOP, the states, manipulated inputs are denoted as x_0^*, u_0^* respectively. It is common that at OOP, some of the inequality constraints are active. As mentioned previously, operating at OOP is usually not possible because of random disturbances, therefore it is necessary to back off from OOP. The back off is defined as,

$$\begin{aligned} Back-off \; &= |Actual \; steady \; state \; operating \; point \\ &\quad -Nominal \; steady \; state \; operating \; point| \end{aligned} \tag{2}$$

Linearizing the process model (1b) around the nominally optimal operating point $(x_0^*, u_0^*, \overline{d}_0)$, we have: $\dot{\tilde{x}} = A\tilde{x} + B\tilde{u} + G\tilde{d}$ by defining deviation variables $(\tilde{x}, \tilde{u}, \tilde{d})$ with respect to the optimal point where A, B and G are the partial derivative of g evaluated at $(x_0^*, u_0^*, \overline{d}_0)$. For the assumed zero mean disturbances, the linearized model in terms of the steady state backed-off variables $(\tilde{x}_{ss}, \tilde{u}_{ss})$ is given by

$$0 = A\tilde{x}_{ss} + B\tilde{u}_{ss} \tag{3}$$

The above equation defines the set of feasible backed-off operating points. The inequality performance limits (1c) is linearized around $(x_0^*, u_0^*, \overline{d}_0)$ and writing in bounded form by defining a new variable z_0 as:

$$z_0 = Z_x x_0 + Z_u u_0 + Z_d \overline{d}_0 \tag{4a}$$

$$z_{min} \leq z_0 \leq z_{max} \tag{4b}$$

where Z_x, Z_u and Z_d are the partial derivative of h evaluated at $(x_0^*, u_0^*, \overline{d}_0)$. Now, rewriting the inequalities in terms of backed-off variables $(\tilde{x}_{ss}, \tilde{u}_{ss})$ yield,

$$\tilde{z}_{ss} = Z_x \tilde{x}_{ss} + Z_u \tilde{u}_{ss} \tag{5a}$$

$$\tilde{z}_{min} \leq \tilde{z}_{ss} \leq \tilde{z}_{max} \tag{5b}$$

where $\tilde{z}_{min} = z_{min} - Z_x x_0^* - Z_u u_0^* - Z_d \overline{d}$ and $\tilde{z}_{max} = z_{max} - Z_x x_0^* - Z_u u_0^* - Z_d \overline{d}$. To determine the Backed-off Operating Point (BOP) that yield a maximum profit, it is necessary to account for the dynamic effect of disturbances which might cause constraint violation. As random disturbances are assumed, we follow stochastic framework to define the EDOR around the given BOP. The dynamic model is rewritten in terms of the new deviation variables around the BOP ($\tilde{x}_{ss}$,$\tilde{u}_{ss}$,$\overline{d}$) and is given by

$$\dot{x} = Ax + Bu + Gd \tag{6}$$

$$z = Z_x x + Z_u u + Z_d d \tag{7}$$

$$\tilde{z}_{min} - \tilde{z}_{ss} \leq z \leq \tilde{z}_{max} - \tilde{z}_{ss} \tag{8}$$

where $x = \tilde{x} - \tilde{x}_{ss}$,$u = \tilde{u} - \tilde{u}_{ss}$ and $d = d_0 - \overline{d}_0$. In this framework, the EDOR is a region such that the probability that the system is confined to the EDOR is greater than the prescribed confidence limits. When the model is linear and uncertainties are Gaussian, the EDOR is usually described as an ellipsoid and can be computed given the covariance and the prescribed confidence limits. This covariance matrix depends on the process dynamics, controller and measurement. Assuming full state information and linear feedback $u = Lx$, the closed loop steady state covariance matrix of the state vector ($\Sigma_x := \lim_{t\to\infty} \mathbf{E}[x(t)^T x(t)]$) is given by the symmetric positive semi-definite solution to the Lyapunov equation

$$(A + BL)\Sigma_x + \Sigma_x(A + BL)^T + G\Sigma_d G^T = 0 \tag{9}$$

Respectively, the covariance of the signal z is given by

$$\Sigma_z = (Z_x + Z_u L)\Sigma_x(Z_x + Z_u L)^T + Z_d \Sigma_d {Z_d}^T \tag{10}$$

Convex optimization tools are highly useful in transforming the "difficult-to-solve" non linear constraints into solvable Linear Matrix Inequality (LMI) forms (Boyd and Vandenberghe (2004)). In this regard, it has been shown that the above non linear matrix inequalities (9)-(10) can be converted to LMIs in terms of relaxation variables (X, Y) as:

$$(AX + BY) + (AX + BY)^T + G\Sigma_d G^T \prec 0 \tag{11}$$

$$\begin{bmatrix} \Sigma_z - Z_d \Sigma_d {Z_d}^T & Z_x X + Z_u Y \\ (Z_x X + Z_u Y)^T & X \end{bmatrix} \succeq 0 \tag{12}$$

where $Y = LX$ and $X = \Sigma_x \succ 0$ ($\succeq 0$) denotes that X is positive definite (respectively positive semi-definite). The last LMI (12) is the consequence of the following trick

$$\Sigma_z = (Z_x + Z_u L)XX^{-1}X(Z_x + Z_u L)^T + Z_d \Sigma_d {Z_d}^T$$

$$\Sigma_z = (Z_x X + Z_u LX)X^{-1}(Z_x X + Z_u LX)^T + Z_d \Sigma_d {Z_d}^T$$

and applying Schur complement (see Boyd and Vandenberghe (2004)) yields the LMI (12). This holds true when there exists a stabilizing feedback gain L (For more details, please refer theorem 2.1 of Chmielewski and Manthanwar (2004)).

2.1 Fully constrained case

In this subsection, we present two formulations of the minimum backed off operating point selection problem for fully constrained case, which differ in the way, the feasibility constraint of dynamic operating region is expressed.

MBOP formulation 1 The MBOP formulation is formulated as Peng et al. (2005):

$$min \ {J_x}^T \tilde{x}_{ss} + {J_u}^T \tilde{u}_{ss} \tag{13a}$$

$$s.t. \ 0 = A\tilde{x}_{ss} + B\tilde{u}_{ss} \tag{13b}$$

$$\tilde{z}_{ss} = Z_x \tilde{x}_{ss} + Z_u \tilde{u}_{ss} \tag{13c}$$

$$(AX + BY) + (AX + BY)^T + G\Sigma_d G^T \prec 0 \tag{13d}$$

$$\begin{bmatrix} \Sigma_z - Z_d \Sigma_d {Z_d}^T & Z_x X + Z_u Y \\ (Z_x X + Z_u Y)^T & X \end{bmatrix} \succeq 0 \tag{13e}$$

$$\Sigma_z(i,i) < (\tilde{z}_{ss,i} - \tilde{z}_{min,i})^2; \ i = 1 \cdots n_z \tag{13f}$$

$$\Sigma_z(i,i) < (\tilde{z}_{max,i} - \tilde{z}_{ss,i})^2; \ i = 1 \cdots n_z \tag{13g}$$

$$\tilde{z}_{min} \leq \tilde{z}_{ss} \leq \tilde{z}_{max} \tag{13h}$$

where $\tilde{x}_{ss}$, $\tilde{u}_{ss}$, $\tilde{z}_{ss}$, Y, $X \succeq 0$ and $\Sigma_z \succeq 0$ are the decision variables. The above problem has a linear objective function with a set of convex (linear, LMIs) and reverse-convex (13f)-(13g) constraints which requires a branch and bound scheme for solution. It is also important to note that the above reverse convex constraints ensure the feasibility of dynamic operating region.

MBOP formulation 2 As mentioned previously, the EDOR can be expressed as ellipsoids which can be mathematically represented by defining $P = \Sigma_z^{1/2}$ as:

$$\mathcal{E}_{95\%} = \{(\tilde{z}_{min} \leq \tilde{z}_{ss} + \alpha P z \leq \tilde{z}_{max}) \mid \|z\|_2 \leq 1\} \tag{14}$$

where α depends on the confidence limit, e.g., for a limit of 95%, $\alpha = 2$. This ellipsoid containment constraint (14) makes the MBOP selection problem as a infinite dimensional one. However, this could be transformed into a finite dimensional problem using S - procedure (see Boyd and Vandenberghe (2004)) as LMIs of the form

$$\begin{bmatrix} -\tau_i - {h_i}^T \tilde{z}_{ss} - t_i & \frac{\alpha}{2} {h_i}^T P \\ (\frac{\alpha}{2} {h_i}^T P)^T & \tau_i I \end{bmatrix} \succeq 0; \tau_i > 0; \ i = 1 \cdots 2n_z \tag{15}$$

where h_i's, t_i's are the respective vectors and scalars of the bound constraints written in the form of ${h_i}^T \tilde{z}_{ss} + t_i \leq 0$. Thus the matrix $H = [Z_x | Z_u; -Z_x | -Z_u]$ and vector $t = [\tilde{z}_{max}; -\tilde{z}_{min}]$. Now the MBOP selection problem with the same set of decision variables is reformulated in terms of LMI constraints as :

$$min \ {J_x}^T \tilde{x}_{ss} + {J_u}^T \tilde{u}_{ss} \tag{16a}$$

$$s.t. \ 0 = A\tilde{x}_{ss} + B\tilde{u}_{ss} \tag{16b}$$

$$\tilde{z}_{ss} = Z_x \tilde{x}_{ss} + Z_u \tilde{u}_{ss} \tag{16c}$$

$$(AX + BY) + (AX + BY)^T + G\Sigma_d G^T \prec 0 \tag{16d}$$

$$\begin{bmatrix} \Sigma_z - Z_d \Sigma_d {Z_d}^T & Z_x X + Z_u Y \\ (Z_x X + Z_u Y)^T & X \end{bmatrix} \succeq 0 \tag{16e}$$

$$P = \Sigma_z^{1/2} \tag{16f}$$

$$\begin{bmatrix} -\tau_i - {h_i}^T \tilde{z}_{ss} - t_i & \frac{\alpha}{2} {h_i}^T P \\ (\frac{\alpha}{2} {h_i}^T P)^T & \tau_i I \end{bmatrix} \succeq 0; \tau_i > 0 \tag{16g}$$

where $\tilde{x}_{ss}$, $\tilde{u}_{ss}$, $\tilde{z}_{ss}$, Y, $X \succeq 0$, $\Sigma_z \succeq 0$ and $P \succeq 0$ are the decision variables. The objective function and all the constraints in the above formulation (16) except (16f) are convex. Thus, the formulated minimum back off operating point selection problem is a non linear non convex program. However, this problem could be solved using the solution methodology developed in Section 3.

2.2 General case: Partially constrained

To determine the BOP that result in the minimum loss, the economic objective is linearized in terms of states and inputs. In addition, a quadratic penalty for inputs are included in the cost to account for the unconstrained degrees of freedom. Now the MBOP problem (16) is reformulated by replacing (16a) with (17) but with same set of constraints and decision variables as follows:

$$min \quad {J_x}^T \tilde{x}_{ss} + {J_u}^T \tilde{u}_{ss} + \tilde{u}_{ss}^T J_{uu} \tilde{u}_{ss} \tag{17}$$

where $J_{uu} \succeq 0$ can be obtained by numerically perturbing the unconstrained inputs and hence denotes the economic penalty for backing off the inputs from the nominal optimal value. Note that this cost function considers only the steady state effect on economics. Since the disturbances are assumed to be Gaussian and zero mean, this implies that the cost accounts only for the nominal steady state value of disturbances. This restriction of considering only the steady state effect in cost also applies to the cost in (13a)and (16a) but here the restriction is less severe as long as the optimal constraints remain the same. The last term must be included in the partially constrained case to get a meaningful solution. Also note that the optimal controller uses the unconstrained degrees of freedom for minimizing the linear cost (backoff from the active constraints) plus the quadratic cost in terms of changing the steady state operating point. As mentioned previously, the formulation is non-linear and non convex. Since not all constraints could be convexified, we propose a simple two stage iterative procedure that reduce the variability of the economically important (i.e., active constrained) variables by progressively increasing the variability of the economically unimportant variables at each iteration.

3. SOLUTION METHODOLOGY

The basic idea in the first stage is to find the smallest (in terms of trace) feasible ellipsoid Σ_z and a suitable multivariable controller L. In the second stage, this covariance ellipsoid is used to determine the closest possible MBOP ($\tilde{z}_{ss}$) to the OOP ($x_0^*, u_0^*, \bar{d}_0$) such that the EDOR is feasible. Information from the second stage (i.e., BOP) is used to create lower bounds on the variances by defining the parameter δ describing the closeness to OOP. This information is used as bounds to individual variances to recompute Σ_z and L in the first stage. This process is iterated until convergence. In the solution of the first stage, we impose the following constraints on the individual variances for obtaining the Σ_z and L that ensures feasibility in the second stage,

$$\sigma_{z,i}^2 < \frac{1}{4\alpha^2}\,(\tilde{z}_{max,i} - \tilde{z}_{min,i})^2; i = 1 \cdots n_z \qquad (18)$$

where $\sigma_{z,i}^2$ is the variance of the i^{th} component of z, viz., z_i. Additionally, we define the following constraints with respect to variance of the j^{th} variable $\sigma_{z,j}^2$,

$$\sigma_{z,i}^2 > \frac{\delta_{i,j}^2}{\alpha^2}\sigma_{z,j}^2; i = 1, j-1, j+1, n_z \qquad (19)$$

where the iterative parameters $\delta_{i,j}^2$ are chosen such that the BOP selected in stage 2 is used to select the new minimum variance ellipsoid that forces the BOP close to OOP. The parameter $\delta_{i,j}$ is defined as

$$\delta_{i,j} = \frac{distance\ of\ variable\ i\ from\ its\ closest\ bound}{distance\ of\ variable\ j\ from\ its\ closest\ bound} \qquad (20)$$

Physically, it tries to exploit the available manipulated input space to be utilized to find the MBOP and controller.

3.1 Stage 1

$$\begin{array}{ll} \min\limits_{X\succeq 0, \Sigma_z \succeq 0, Y} & Tr(\Sigma_z) \\ \text{s.t.} & (AX+BY)+(AX+BY)^T + G\Sigma_d G^T \prec 0 \\ & \begin{bmatrix} \Sigma_z - Z_d \Sigma_d {Z_d}^T & Z_x X + Z_u Y \\ (Z_x X + Z_u Y)^T & X \end{bmatrix} \succeq 0 \\ & \sigma_{z,i}^2 < \frac{1}{4\alpha^2}\,(\tilde{z}_{max,i} - \tilde{z}_{min,i})^2; i = 1 \cdots n_z \\ & \sigma_{z,i}^2 > \frac{\delta_{i,j}^2}{\alpha^2}\sigma_{z,j}^2; i = 1, j-1, j+1, n_z \end{array}$$

Solution of Stage 1 results in a feasible covariance ellipsoid Σ_z. Let $P = \Sigma_z^{1/2}$. This is used to find the approximation to the MBOP in stage 2 as follows.

3.2 Stage 2

$$\begin{array}{ll} \min\limits_{\tilde{x}_{ss}, \tilde{u}_{ss}, \tilde{z}_{ss}} & {J_x}^T \tilde{x}_{ss} + {J_u}^T \tilde{u}_{ss} + \tilde{u}_{ss}^T J_{uu} \tilde{u}_{ss} \\ \text{s.t.} & 0 = A\tilde{x}_{ss} + B\tilde{u}_{ss} \\ & \tilde{z}_{ss} = Z_x \tilde{x}_{ss} + Z_u \tilde{u}_{ss} \\ & \begin{bmatrix} -\tau_i - {h_i}^T \tilde{z}_{ss} - t_i & \frac{\alpha}{2}{h_i}^T P \\ (\frac{\alpha}{2}{h_i}^T P)^T & \tau_i I \end{bmatrix} \succeq 0; \\ & \tau_i \succeq 0;\ i = 1, \cdots 2n_z \end{array}$$

The δ's are updated based on the new MBOP and used to resolve Stage 1. It is to be noted that P is not a decision variable since Σ_z is known from first stage. Now it can be easily recognized that both the stages contains only convex constraints which could be easily solved using `CVX`, a package for specifying and solving convex programs (Grant and Boyd (2011)). Initializing $\delta_{i,j}$ to zero and given two successive iterates, $\tilde{z}_{ss}^{iter-1}$ and $\tilde{z}_{ss}^{iter}$ this process is iterated until the convergence criteria $\|\tilde{z}_{ss}^{iter} - \tilde{z}_{ss}^{iter\text{-}1}\|_2 \leq \epsilon$ is satisfied where ϵ being the prescribed tolerance limit.

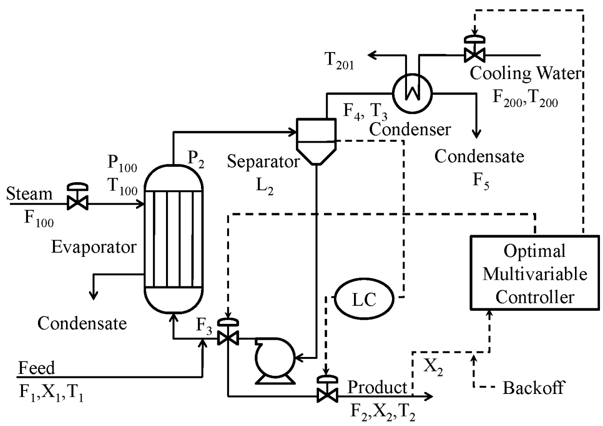

Fig. 2. Evaporator system

4. EVAPORATION PROCESS

The proposed back-off approach is applied to the evaporation process of Newell and Lee (1989). The forced-circulation evaporator system is depicted in Fig. 2, where the concentration of the feed stream is increased by evaporating the solvent through a vertical heat exchanger with circulated liquor. The overhead vapor is condensed by the use of process heat exchanger. The details of the mathematical model can be found in the original reference. The separator level is assumed to be perfectly controlled using the exit product flow rate F_2 which also eliminates the integrating nature of the state. The economic objective is to maximize the operational profit [\$/h], formulated as a minimization problem of the negative profit (Kariwala et al. (2008)). The first three terms of (21) are utility costs relating to steam, coolant and pumping respectively. The fourth term is the raw material cost, whereas the last term is the product value.

$$J = 600F_{100} + 0.6F_{200} + 1.009(F_2 + F_3) + 0.2F_1 - 4800F_2 \qquad (21)$$

The process has the following constraints related to product specification, safety, and design limits:

$$X_2 \geq 35\% \qquad (22)$$
$$40\ kPa \leq P_2 \leq 80\ kPa \qquad (23)$$
$$P_{100}\ \leq 400\ kPa \qquad (24)$$
$$0\ kg/min \leq F_{200} \leq 400\ kg/min \qquad (25)$$
$$0\ kg/min \leq F_1 \leq 20\ kg/min \qquad (26)$$
$$0\ kg/min \leq F_3 \leq 100\ kg/min \qquad (27)$$

Nominal operating point. The nominal steady state values are obtained by solving a nonlinear optimization problem for the nominal values of disturbances and the profit is found to be $J = \$693.41/h$ and the nominal values are shown in Table 1. At the nominal optimal point, there are two active constraints: product composition, $X_2 = 35\%$ and steam pressure, $P_{100} = 400\ kPa$. And, the corresponding Lagrange multipliers are 229.36 \$/% h and -0.096685 \$/$kPa\ h$ respectively.

Degree of freedom analysis. The process model has seven degrees of freedom. Inlet conditions of the feed (flow rate, composition, temperature) and inlet temperature of

Table 1. Variables and Nominal optimal values

Variables	Description	Nominal value
	States (x)	
X_2	product composition	35.00 %
P_2	operating pressure	56.15 kPa
	Inputs (u)	
F_3	recirculating flow rate	27.70 kg/min
P_{100}	steam pressure	400 kPa
F_{200}	cooling water flow rate	230.57 kg/min
	Disturbances (d)	
F_1	feed flow rate	10.00 kg/min
X_1	feed composition	5.00 %
T_1	feed temperature	40.00 °C
T_{200}	inlet temperature of cooling water	25.00 °C
	Dependent variables	
F_2	product flow rate	1.43 kg/min
F_4	vapor flow rate	8.57 kg/min
F_5	condensate flow rate	8.57 kg/min
F_{100}	steam flow rate	9.99 kg/min
T_2	product temperature	90.91 °C
T_3	vapor temperature	83.47 °C
T_{100}	steam temperature	151.52 °C
T_{201}	outlet temperature of cooling water	45.45 °C
Q_{100}	heat duty	365.63 kW
Q_{200}	condenser duty	330.00 kW

the condenser are considered as disturbances (i.e., $d = [F_1\ X_1\ T_1\ T_{200}]^T$). There are three manipulated inputs, $u = [F_3\ P_{100}\ F_{200}]^T$. The disturbance range is assumed to be 10% variation of the nominal value (i.e., $\Sigma_d = diag([1\ 0.25\ 16\ 6.25])^2$) and the set of active constraints do not change in the whole range of disturbances . It is important to note that there is one unconstrained degrees of freedom.

Linearized steady state model. Linear approximation of the process model at the nominal optimum yields,

$$A = \begin{bmatrix} -0.16709 & -0.17185 \\ -0.013665 & -0.043132 \end{bmatrix};$$

$$B = \begin{bmatrix} 0.44083 & 0.04217 & 0 \\ 0.062976 & 0.0060243 & -0.0016249 \end{bmatrix};$$

$$G = \begin{bmatrix} -1.2211 & 0.5 & 0.031818 & 0 \\ 0.039837 & 0 & 0.0045455 & 0.03665 \end{bmatrix}$$

The performances z are defined by the matrices,

$$Z_x = [I_{2\times2}|0_{2\times3}]^T; Z_u = [0_{3\times2}|I_{3\times3}]^T; Z_d = [0_{4\times5}]^T$$

and the bound constraints written in the form of $h_i{}^T \tilde{z}_{ss} + t_i \leq 0$ are obtained from the rows of the matrix H and elements of vector t, $H = [I_{5\times5}| - I_{5\times5}]^T$; $t = [-5\ \ -23.849\ \ -72.299\ 0\ \ -169.43\ 0\ \ -16.151\ \ -27.701\ \ -200\ \ -230.57]^T$. The linearized negative profit function is

$$J_x = [-293.23\ \ -526.8]^T; J_u = [1368.9\ 130.85\ 0.6]^T$$

As the input P_{100} is constrained, the quadratic penalty is included only for the other inputs and the numerical perturbation of inputs F_3 and F_{200} yield,

$$J_{uu} = \begin{bmatrix} 4.4953 & 0.00010226 \\ 0.00010226 & 0.0052699 \end{bmatrix}$$

Results. For the case of full state information, the amount of back off required to remain feasible for 10% variation in the nominal disturbances is tabulated in Table 2. It is to be noted that the amount of back-off for steam pressure (P_{100}) is zero as expected as it is a input variable. However, the assumed disturbances have significant effect

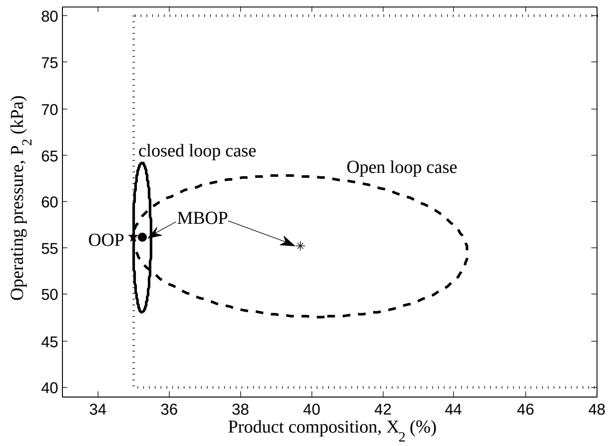

Fig. 3. Product composition vs operating pressure. a) Open loop case: F_3 and F_{200} are constant. b) Closed loop case: F_3 and F_{200} are used for control of X_2

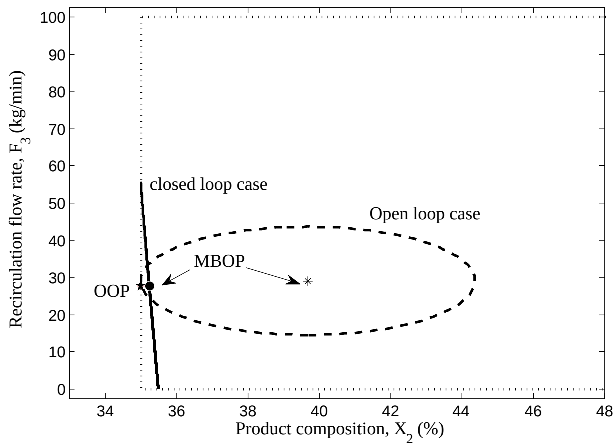

Fig. 4. Product composition vs recirculation flow rate

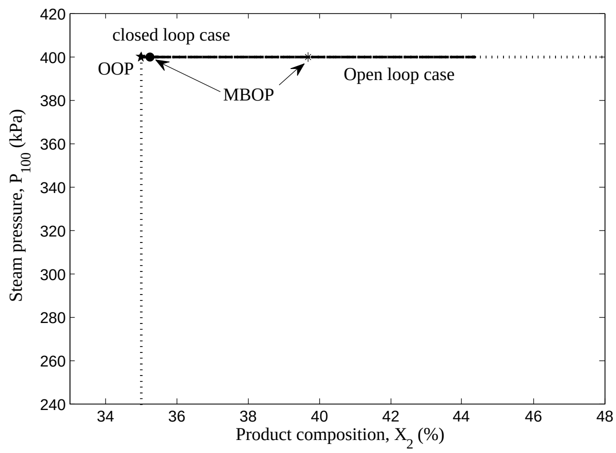

Fig. 5. Product composition vs steam pressure

on product exit composition, X_2. The MBOP solution and EDOR for the open loop and closed loop case are shown as ellipses in Figures 3-6. The loss obtained for operating the evaporator at this backed off operating point is \$58.65/h which corresponds to the achievable profit of \$634.76/h. In other words, the loss we incur

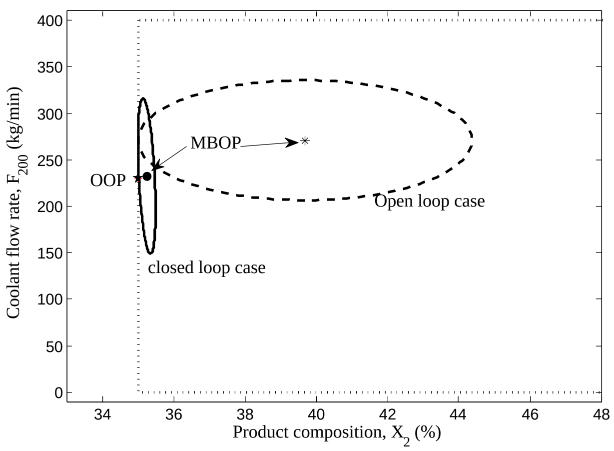

Fig. 6. Product composition vs coolant flow rate

Table 2. Nominal and Back-off operation

Variables	Units	Nominal value	MBOP solution(17) closed loop (proposed)	open loop ($u = 0$)
		States		
X_2	%	35.00	35.26	39.75
P_2	kPa	56.15	56.10	55.16
		Inputs		
F_3	kg/min	27.70	27.78	29.12
P_{100}	kPa	400.00	400	400
F_{200}	kg/min	230.57	232.71	271.65
Profit	$\$/h$	693.41	634.76	-414.92

to ensure feasible operation with 95% confidence interval is $58.65/h$. Indeed, the back-off estimated is the best possible lower bound for the product composition to ensure feasibility because of the simultaneous consideration of controller in the formulation. This could be inferred from Table 2 by comparing the closed loop solution with the open loop solution. The multivariable feedback controller ($u = Lx$) to be implemented to operate the system profitably is

$$L = \begin{bmatrix} -108.5643 & 0.3868 \\ -0.0606 & 0.0002 \\ -123.2216 & 97.3625 \end{bmatrix}$$

Without the controller (open loop case), the amount of back off required is higher and also the process would incur a loss of $414.92/h$. Note that the optimal controller is using both F_3 and F_{200} to control the product composition with the aim of minimizing the overall cost. This feedback gain could be used to determine the appropriate objective function weights using the inverse optimality results of Chmielewski and Manthanwar (2004) and could be successfully implemented using Model Predictive Control. The back off operating point determined above are given as set points to the control system. It is important to note that without the quadratic term, the MBOP solution obtained by solving formulation (16) is $[x^T u^T] = [35.41\ 76.53\ 35.80\ 399.99\ 0.01]$. Note that for instance, F_{200} is changed from 230.57 to 0.01 kg/min, which is unrealistic. This corresponds to the lower left corner in Fig. 6. Hence, the quadratic term in the cost function is important in the partially constrained case to get a meaningful solution.

5. CONCLUSION

A multivariable controller obtained from the proposed formulation which when implemented to operate the evaporation process at the back-off operating point will ensure feasible operation and also yields the maximum achievable profit. The proposed solution strategy has been successfully demonstrated using the evaporation process. This formulation could be extended to include measurement noise as an additional source of uncertainty.

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