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RAPID PUBLICATION

Development of a Mathematical Model to Predict the Fatigue Life of FCAW Cruciform Joints

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A mathematical model has been successfully developed to predict the fatigue life of Flux Cored Arc Welded (FCAW) cruciform joints containing lack of penetration (LOP) defects. Design of Experiments concept has been utilized to optimize the experimental conditions. Analysis of Variance (ANOVA) technique has been applied to check the validity of the developed model. By using the developed model the fatigue life can be predicted at a 95% confidence level.

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1. Introduction

Fillet welds are the most common ones in various structures including offshore and nuclear. Although the quality of the welding has been improved over the past decades, welding discontinuities are still unavoidable. The sizes of internal discontinuities present in the weld have a large effect on the measured fatigue life. The most important variable in determining the fatigue life of a flawed weldment would seem to be the nature of internal flaws contained within the weld and the manner in which these flaws interact with the stress field in and around the weld during its fatigue life.[1] Linking the effects of weld discontinuities and the failure analysis of weldments, it is evident that the most of the disruptive failures are due to fatigue alone.[2]

The fatigue properties of weld metal are better or equal to the base metal but the problems can arise when there is an abrupt change in section caused by excess weld reinforcement, undercut, inclusion of slag or lack of penetration (LOP) or fusion.[3] Root cracking and toe cracking are the two types normally involved in fillet welded joints. But the root failures cannot be prevented unless the weld dimensions are appropriate to the plate thickness.[4] A LOP defect will affect the fatigue behaviour of the fillet welds when it exceeds a critical value of half of the plate thickness to be welded.[5]

Fatigue life prediction of welded joints is complex, costly and time consuming. This is due to the multiplicity of stress concentration locations and heterogeneity of the weld metal properties. The traditional approach is to apply the S-N curve covered by BS 5400 or IIW documents.[6] However, the fatigue life estimation of a welded joint with defects can be made by performing a crack growth experiment and subsequently the fatigue life is evaluated in terms of crack growth parameters such as $\mathrm{d}a/\mathrm{d}N$ versus ΔK. Such data merely indicate the fatigue crack growth behaviour of the component and do not predict the actual fatigue life.[7] Hence, in this paper an attempt has been made to develop a mathematical model for predicting the fatigue life of Flux Cored Arc Welded (*FCAW*) cruciform joints based on its dimensions.

2. Experiment

A high strength quenched and tempered steel (ASTM 5I7'F' Grade) of weldable quality in the form of rolled plates of 8 mm thickness was used as the base material throughout the investigation. This material is widely used for welded constructions of all kinds such as pressure vessels, pen stocks, bridges and structures as well as transport vehicles, hoisting and earth moving equipments which are utilized in different type of conditions.[8] FCAW process with matching weld metal consumable (AWS E100 K_5T_4) was used for fabricating the cruciform joints. The details of cruciform joint fabrication and specimen sectioning were given elsewhere.[9] The dimensions of the test specimen are shown in Fig. 1 (Note 2B = specimen width).

Based on the previous work done in our laboratory[9,10] and from the literature[11-13] the predominant factors which are

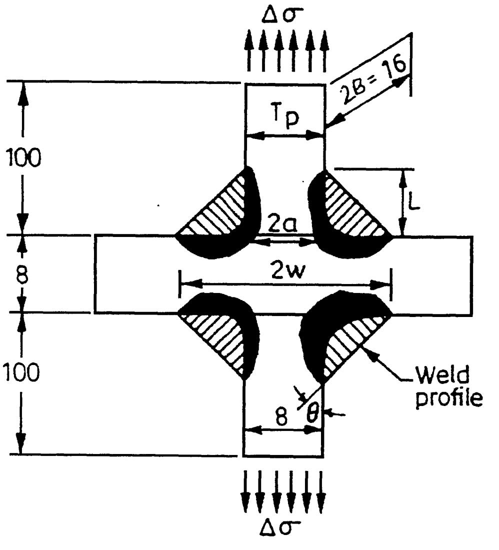

Fig. 1 Dimensions of cruciform specimen.

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Table 1 Important factors and their levels.

S. No.	Factor	Notation	Unit	Levels				
				(−2)	(−1)	(0)	(+1)	(+2)
1	a/W	A	—	0.25	0.3	0.35	0.4	0.45
2	L/T_p	L	—	0.4	0.6	0.8	1.0	1.2
3	θ	P	deg	22.5	30	37.5	45	52.5
4	$\Delta\sigma$	S	MPa	120	160	200	240	280

Table 2 Design matrix and experimental results.

Experiment number	A (X_1)	L (X_2)	P (X_3)	S (X_4)	N_f (life) (Y) $\times 10^5$ cycles
1	−1	−1	−1	−1	4.4
2	+1	−1	−1	−1	2.0
3	−1	+1	−1	−1	11.2
4	+1	+1	−1	−1	7.5
5	−1	−1	+1	−1	6.8
6	+1	−1	+1	−1	4.2
7	−1	+1	+1	−1	13.6
8	+1	+1	+1	−1	9.5
9	−1	−1	−1	+1	0.7
10	+1	−1	−1	+1	0.5
11	−1	+1	−1	+1	2.0
12	+1	+1	−1	+1	1.5
13	−1	−1	+1	+1	1.1
14	+1	−1	+1	+1	0.6
15	−1	+1	+1	+1	2.3
16	+1	+1	+1	+1	1.9
17	−2	0	0	0	5.2
18	+2	0	0	0	1.3
19	0	−2	0	0	2.1
20	0	+2	0	0	3.5
21	0	0	−2	0	1.6
22	0	0	+2	0	4.4
23	0	0	0	−2	17.2
24	0	0	0	+2	0.6
25	0	0	0	0	4.5
26	0	0	0	0	4.6
27	0	0	0	0	3.6
28	0	0	0	0	5.2
29	0	0	0	0	4.8
30	0	0	0	0	3.4
31	0	0	0	0	4.1

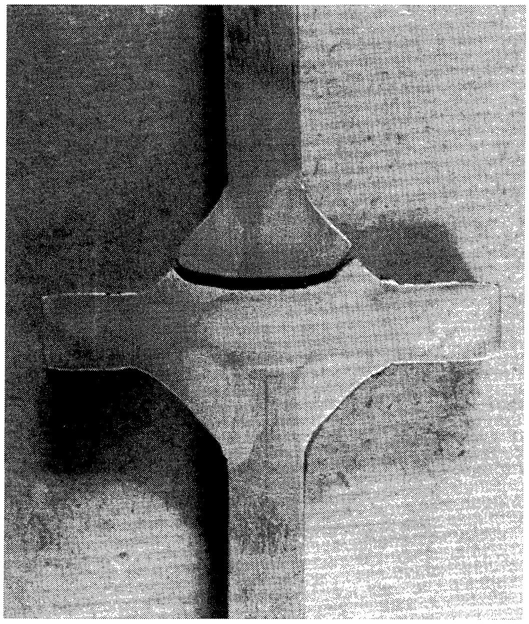

Fig. 2 A typical failed specimen.

having influence on fatigue life of cruciform joints have been identified. They are: (i) the ratio between initial LOP size ($2a$) and fillet width ($2W$), (ii) the ratio between leg length (L) and plate thickness (T_p), (iii) fillet angle (θ) and (iv) stress range ($\Delta\sigma$). The feasible range of the above factors were identified and shown in Table 1. For the convenience of recording and processing the experimental data, the upper and lower levels of the factors are coded as +2 and −2, respectively and the coded values of any intermediate levels can be calculated by using the expression[14]

$$X_i = [2X - (X_{\max} + X_{\min})]/[(X_{\max} - X_{\min})/2] \quad (1)$$

where X_i is the required coded value of a factor of any value X from $X_{\min}$ to $X_{\max}$; $X_{\min}$ is the lower level of the factor and $X_{\max}$ is the upper level of the factor.

To optimize the experimental conditions, Design of Experiments concept was utilized. Table 2 shows the chosen design matrix, a five level (factors are at five different levels), central composite, rotatable factorial type, consisting of 31 ($2^4 + 8$ star points + 7 centre points) set of experimental conditions. The method of designing such a matrix is dealt elsewhere.[14] The fatigue experiments were conducted as per the conditions dictated by the design matrix (Table 2), by using a mechanical resonance controlled universal fatigue testing machine (SCHENCK 200 kN capacity) with a frequency of 30 Hz under constant amplitude loading (R = stress ratio = $\sigma_{\min}/\sigma_{\max} = 0$). Care was taken to see that the load was axial and no bending component was present in the joint. The number of cycles to complete failure of each specimen was recorded and presented in the Table 2. Figure 2 shows a fatigue tested (failed) cruciform specimen.

3. Developing the Mathematical Model

Representing the fatigue life of welded cruciform joints containing LOP defects by N_f, then the response function can be expressed as[14]

$$N_f = f(a/W,\ L/T_p,\ \theta,\ \Delta\sigma) = f(A,\ L,\ P,\ S). \quad (2)$$

The model was a second degree response surface expressed

Table 3 ANOVA (Analysis of Variance) test results.

	First order terms	Second order terms	Error terms	Lack of fit	F_{ratio} (ms of lack of fit/ms of error terms)	$F_{(6,10,0.95)}$ from standard F-table	Remarks
sum of squares(ss)	352.57	96.19	2.78	10.84			Model is adequate
degrees of freedom(dof)	4	10	6	10	2.36	4.05	($F_{\text{calculated}} < F_{\text{standard}}$)
mean square (ms = ss/dof)	88.14	9.62	0.46	1.084			

as follows:

$$\begin{aligned} N_f &= N_0 + N_1(A) + N_2(L) + N_3(P) + N_4(S) \\ &\quad + N_{11}(A^2) + N_{22}(L^2) + N_{33}(P^2) + N_{44}(S^2) \\ &\quad + N_{12}(AL) + N_{13}(AP) + N_{14}(AS) + N_{23}(LP) \\ &\quad + N_{24}(LS) + N_{34}(PS). \end{aligned} \tag{3}$$

The values of the coefficients were calculated by regression with the help of the following equations[15]

$$N_0 = 0.142857\Sigma(Y) - 0.035714\Sigma(X_i^2 \cdot Y); \tag{4}$$

$$N_i = 0.041667\Sigma(X_i \cdot Y); \tag{5}$$

$$\begin{aligned} N_{ii} &= 0.03125\Sigma(X_i^2 \cdot Y) + 0.003720\Sigma(X_i^2 \cdot Y) \\ &\quad - 0.035714\Sigma(Y); \end{aligned} \tag{6}$$

$$N_{ij} = 0.0625\Sigma(X_{ij} \cdot Y); \tag{7}$$

where i, j will take the values of 1, 2, 3, 4 and Y will take the fatigue life value for the corresponding X values (refer Table 2).

Student's t-test[14,15] was applied to eliminate the insignificant coefficients without sacrificing much of the accuracy to avoid cumbersome mathematical labour. According to this test, when the calculated value of 't' corresponding to a coefficient exceeds the standard (table) value of the desired level of probability (say 95%) the coefficient becomes significant. After determining the significant coefficients, the final model was developed including only those coefficients and it is given below:

$$\begin{aligned} \text{Fatigue life, } N_f &= \{4.4 - 0.93(A) + 1.33(L) + 0.66(P) \\ &\quad - 3.41(S) - 0.29(A^2) - 0.41(L^2) \\ &\quad - 0.36(P^2) + 1.12(S^2) + 0.7(AS) \\ &\quad - 1.23(LS) - 0.49(PS)\} \times 10^5 \text{ cycles.} \end{aligned} \tag{8}$$

The adequacy of the above model was then checked by using the Analysis of Variance (*ANOVA*) technique.[14,15] As per this technique, if the calculated value of the F_{ratio} of the developed model does not exceed the standard (F tables) value of F_{ratio} for a desired level of confidence (say 95%), then the model is considered to be adequate within the confidence limit. ANOVA test results are presented in the Table 3. Further, the experimental data and the predicted data by using the above model are plotted as shown in Fig. 3, which indicates a good correlation.

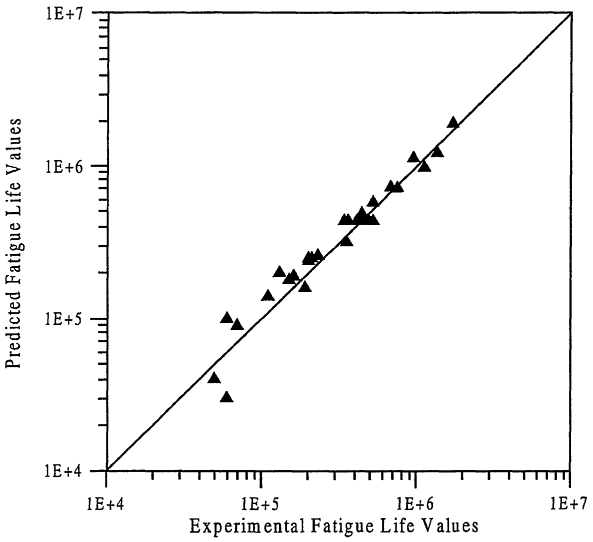

Fig. 3 Correlation graph.

4. Conclusions

(1) By using the developed model, the fatigue life of FCAW cruciform joints containing LOP defects can be predicted at 95% confidence level. But the validity of the model is limited to the range of the factors considered for the investigation.

(2) Factorial experimentation technique is economical to predict the effects of various factors on fatigue life by conducting optimum number of experiments. Further, the ANOVA technique is more convenient to check the validity of developed model.

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