Design of an active suspension system for a quarter-car road vehicle model using model reference control

S S Parthasarathy and **Y G Srinivasa***
Department of Mechanical Engineering, Indian Institute of Technology Madras, Chennai, India

The manuscript was received on 4 August 2004 and was accepted after revision for publication on 15 September 2005.

DOI: 10.1243/095965105X33671

Abstract: The use of skyhook damping for reduction of the vibration in the suspension systems is well documented. The drawback is that it is not practically feasible to obtain the equivalent of a reference point in the sky. The alternative method of using a damper between the sprung mass (the vehicle hull) and the unsprung mass (the wheel assembly) leads to a deterioration in the performance of the unsprung mass. In this paper a new method is proposed that tries to overcome the limitation of the practical skyhook damping method by the use of the model reference control (MRC) method, where the control input is utilized to achieve near-ideal skyhook sprung mass performance in a practical skyhook damping set-up. The MRC method is also applied to a quarter-car suspension model, where the control input is used to achieve the response of a system with damping and stiffness values different from those of the actual system. This allows for variation of the system constants as dictated by dynamic response needs. The proposed methods are applied on a typical quarter-car suspension system and the necessary time and frequency domain simulations are carried out to validate the theoretical predictions.

Keywords: model reference control, skyhook damping, quarter-car suspension

1 INTRODUCTION

The use of suspension systems in vehicles is to provide rider comfort, ensure contact with the road terrain and of course to carry the weight of the chassis and the riders. Conventional suspension systems normally used in vehicles consist of a spring and damper system with fixed dynamic characteristics, i.e. passive in nature.

Over the years there has been a great increase in the operational velocity of passenger vehicles and the demand for better ride comfort. This has led to the development of active suspension systems with an additional actuator or a variable damper element along with the traditional spring and damper system. The actuator and the variable damper are controlled by an appropriate control algorithm, which is designed to meet the specified design objectives by supplying a suitable controller force. The key problem in the implementation of the active suspension systems has been the design of an appropriate implementable controller algorithm that can satisfy the conflicting requirements of achieving driver comfort as well as better vehicle handling performance. Some of the widely used controller algorithms are the PID controller, the full state feedback optimal controller, the semi-active controller, etc. [**1–3**].

One of the most popular and implemented controllers in commercial applications is the skyhook damping concept [**4–6**]. In the skyhook damping process a damper is placed between the sprung mass, i.e. the hull and the rider mass supported by the chassis, and an imaginary point in the sky. This is equivalent to the negative feedback of the sprung mass velocity with appropriate amplification such that there is no force applied to the unsprung mass (the wheel and tyre assembly). Such a scheme is shown to be very effective in controlling the sprung mass acceleration and is attractive because of its inherent simplicity from a practical point of view.

* *Corresponding author: Precision Engineering and Instrumentation Lab, Department of Mechanical Engineering, Indian Institute of Technology, Madras, Chennai 600036, India. email: ygs@iitm.ac.in*

The key issue with the skyhook approach is that it is not practically implementable, because finding an imaginary point in the sky for fixing the damper is not possible. The practical implementation calls for the use of an actuator between the sprung and the unsprung masses. However, this leads to deterioration of the unsprung mass dynamic performance as the controller force input has to be applied on both the sprung as well as the unsprung masses. Thus the dynamic response of the practical skyhook damping system is considerably worse than that of the ideal skyhook-based suspension system [**4**, **5**].

In this paper a new method is proposed that improves the sprung mass performance compared to the practical skyhook case by using a control force input in the practical skyhook set-up. The requisite

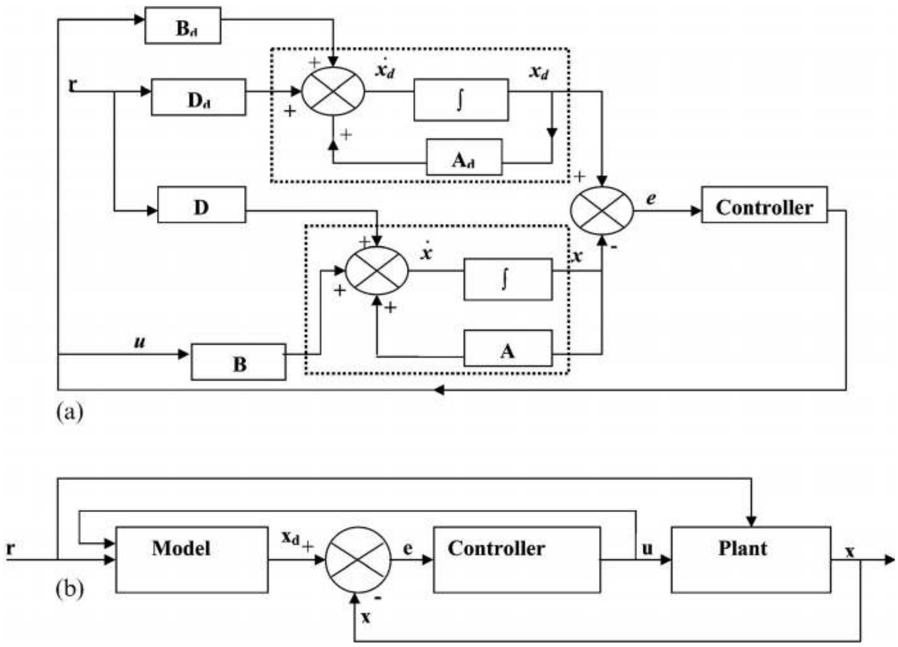


Fig. 1 Model reference control system

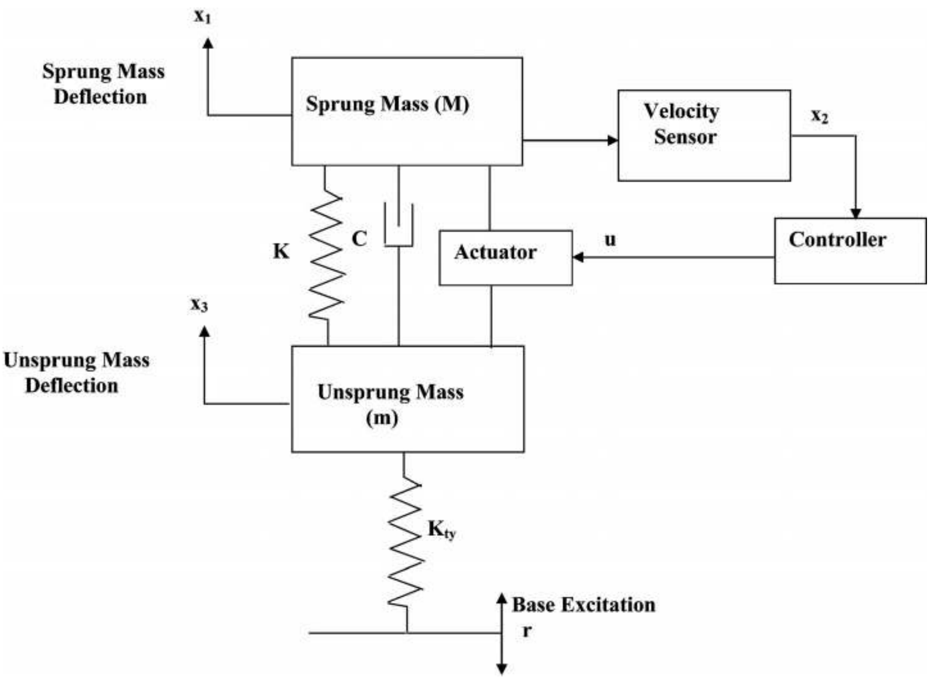


Fig. 2 Quarter-car with practical skyhook damping

controller force input to achieve this objective has been obtained by application of model reference control (MRC) theory. The work has been carried out for a quarter-car model of vehicles that can model the vertical motion of the sprung mass alone. A comparison of the time and frequency domains of the simulations for the different models proves the superiority of the MRC-based system over the passive as well as the practical skyhook-based system responses.

The application of MRC theory is not limited to that of the practical skyhook case in active suspension systems. There are many cases in active suspension systems where the design and practical constraints force the designer to use springs and dampers that are suboptimal as far as the dynamic response is concerned. In such cases it is possible to adopt an active suspension based on the MRC concept where the controller can force the system to behave in an optimal manner. This possibility has also been explored in this paper, where a passive suspension with a given damping coefficient is forced to behave as a system with a lower damping coefficient, leading to a better dynamic response.

2 MODEL REFERENCE CONTROL

Model reference control (MRC) operates on the basic principle of making a given system behave as a desired system by the application of a suitable

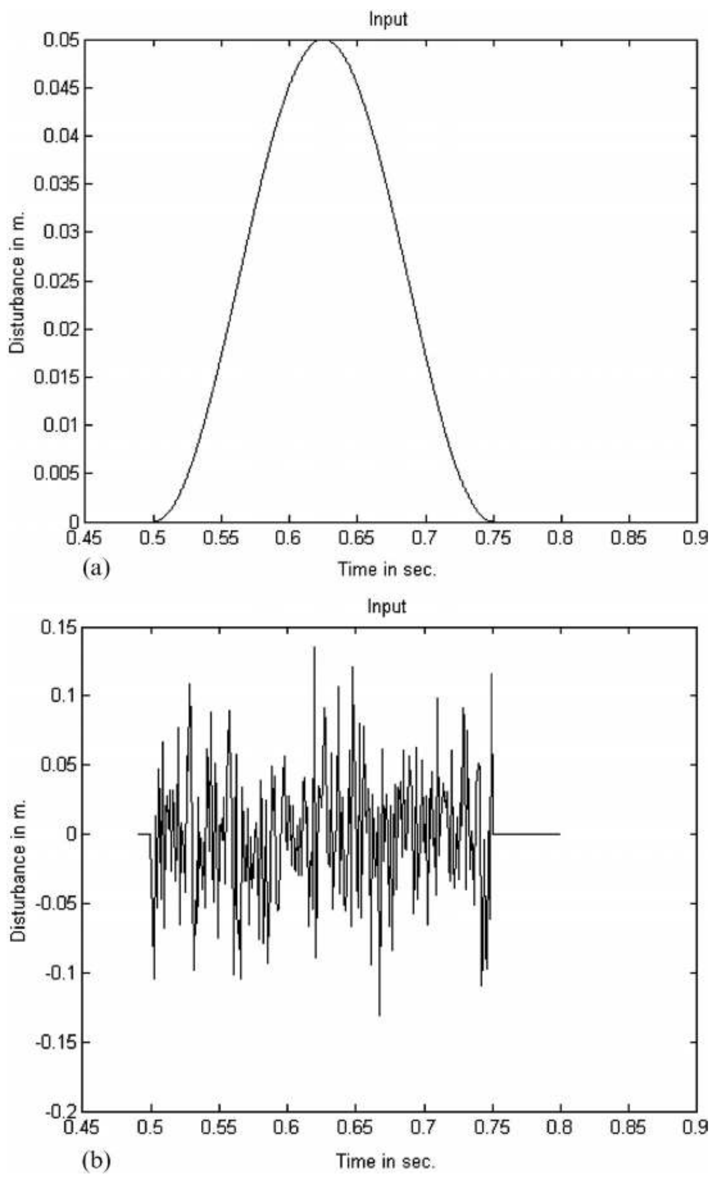


Fig. 3 Inputs to the system

control force. The desired system is referred to as the model reference system. The versatility of the MRC method lies in the fact that the model reference system does not need to be a practical system, but can be any ideal mathematical model, and does not need to be practically feasible [**5**]. The output of such a model is compared to that of the actual system response and the difference error signal is used to generate the required controller input. A brief theoretical outline of the MRC method is given below. Figure 1 shows the block diagram of the model reference control system.

The plant is considered to be characterized by the following equation

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, t) = \mathbf{A}x + \mathbf{B}u + \mathbf{D}r \tag{1}$$

where

$\boldsymbol{x}$ = state vector of the plant
$\boldsymbol{u}$ = control vector
$\boldsymbol{f}$ = vector-valued function
$\mathbf{A} = n \times n$ constant matrix
$\mathbf{B} = n \times r$ constant matrix
r = road disturbance input

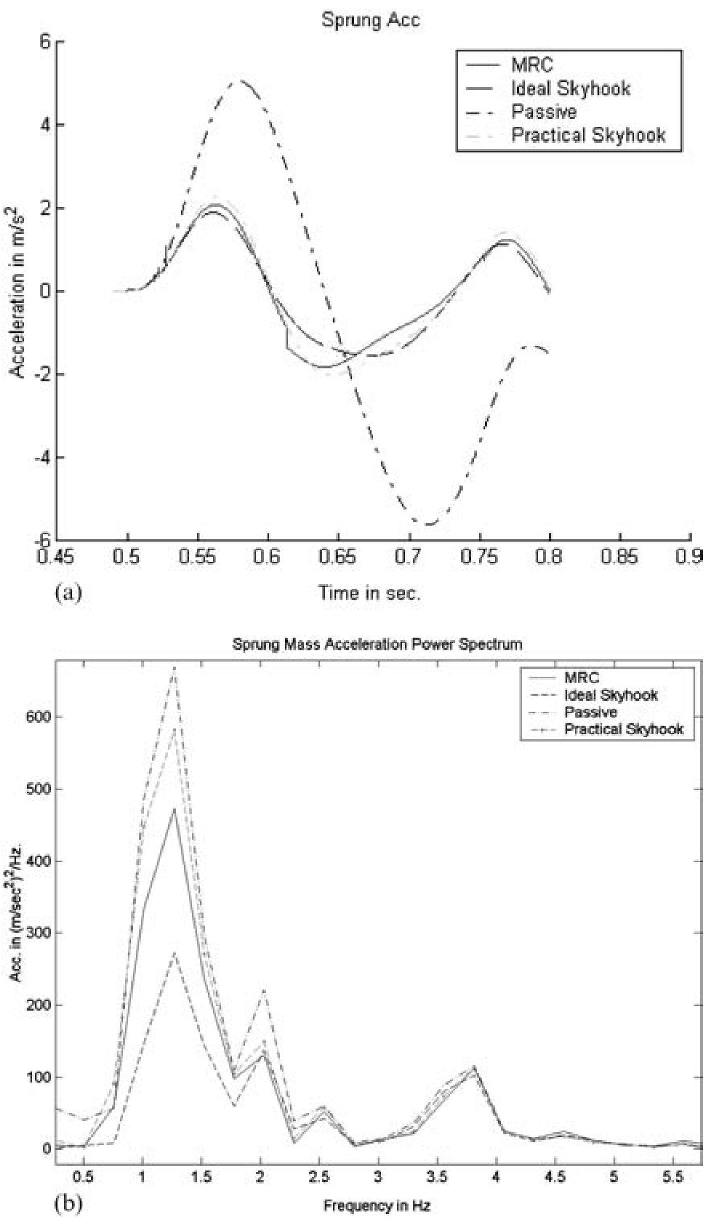


Fig. 4 Sprung mass acceleration values

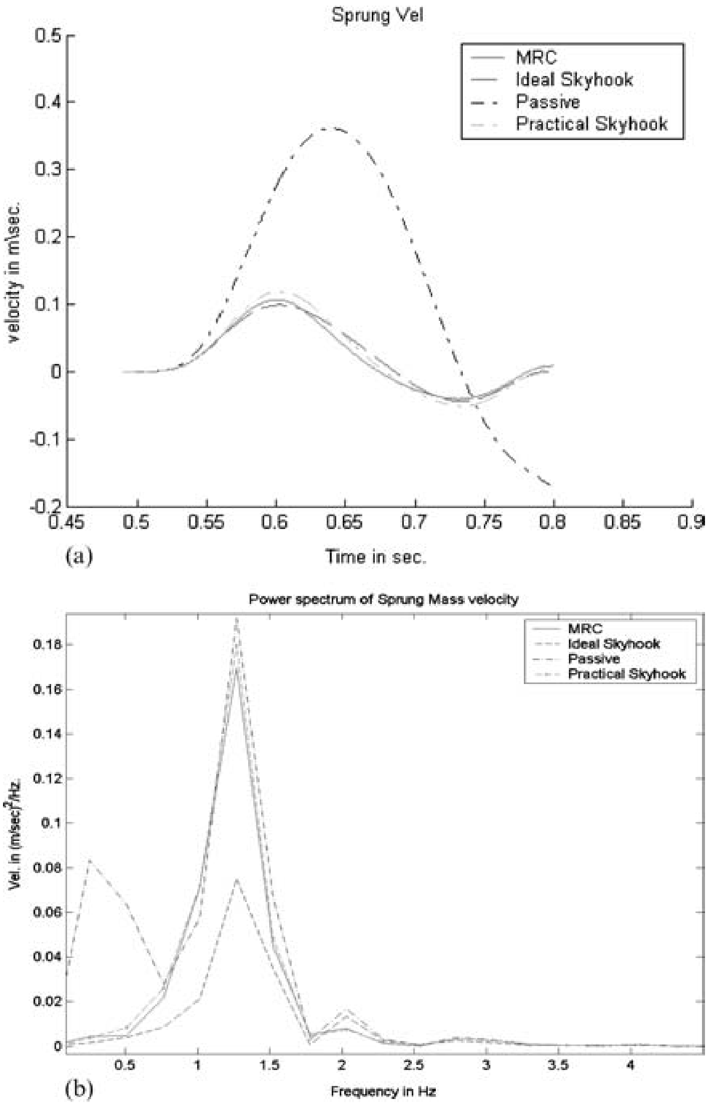


Fig. 5 Sprung mass velocity

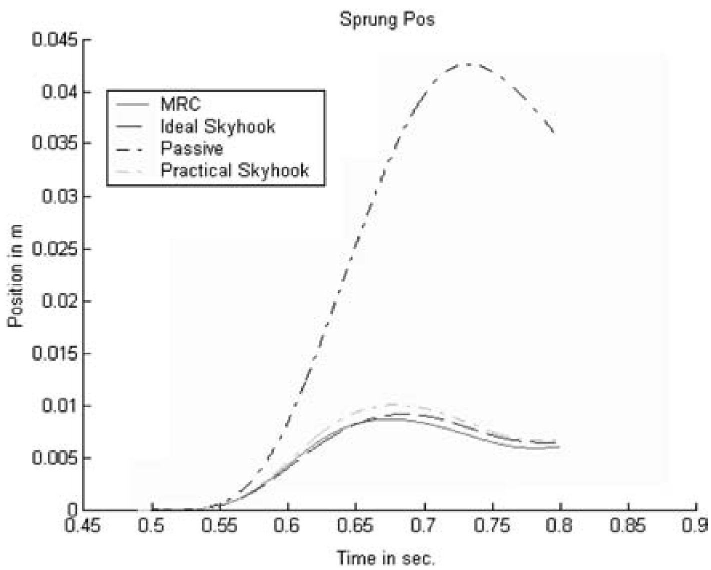


Fig. 6 Sprung mass position

It is assumed that the model reference system is given by

$$\dot{x}_d = \mathbf{A}_d x_d + \mathbf{B}'_d v = \mathbf{A}_d x_d + \mathbf{B}_d u + \mathbf{D}_d r \qquad (2)$$

where

x_d = state vector of the model
$\mathbf{A}_d = n \times n$ constant matrix
$\mathbf{B}'_d = n \times r$ constant matrix
r = road disturbance input

and

$$v \text{ is a vector} = \begin{bmatrix} 0 \\ u \\ 0 \\ r \end{bmatrix}$$

It is assumed that the eigenvalues of $\mathbf{A}_d$ in equation (2) have negative real parts, ensuring an asymptotically stable equilibrium state. The error vector e is defined by

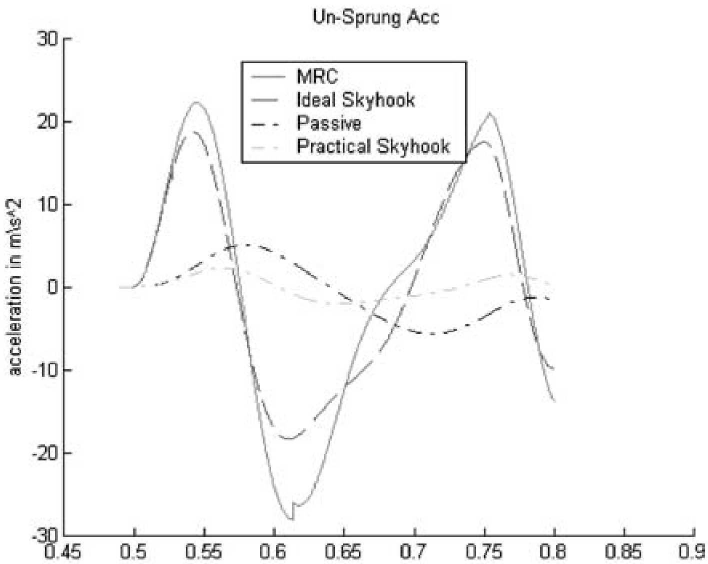
$$e = x_d - x \qquad (3)$$

From equations (1) and (2)

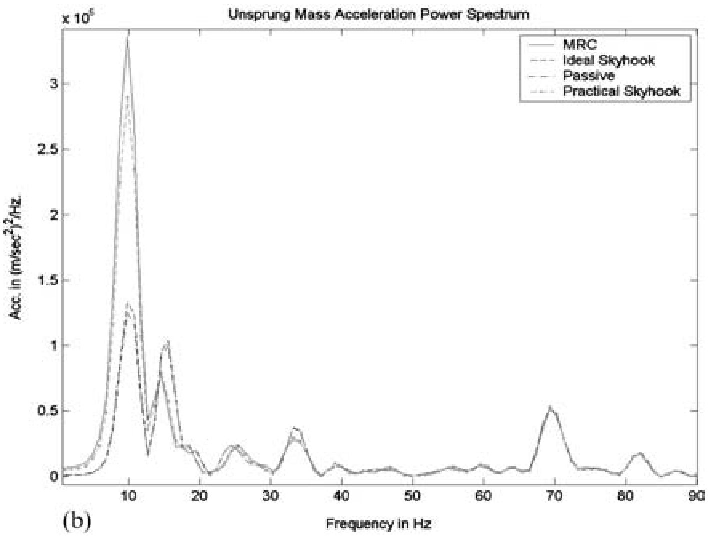
$$\dot{e} = \dot{x}_d - \dot{x} = \mathbf{A}_d x_d + \mathbf{B}'_d v - f(x, u, t)$$

$$= \mathbf{A}_d e + \mathbf{A}_d x - f(x, u, t) + \mathbf{B}'_d v \qquad (4)$$

The aim of the controller is to ensure that at the steady state $x = x_d$ and $\dot{x} = \dot{x}_d$ and $e = \dot{e} = \mathbf{0}$. This can



(a)



(b)

Fig. 7 Unsprung mass acceleration

be achieved by ensuring the Lyapunov stability of the error equation system. Consider the following Lyapunov function V and its derivative as given in the following equations

$$\begin{aligned} V(\boldsymbol{e}) &= \boldsymbol{e}^{\mathrm{T}}\mathbf{P}\boldsymbol{e} \\ \dot{V}(\boldsymbol{e}) &= \dot{\boldsymbol{e}}^{\mathrm{T}}\mathbf{P}\boldsymbol{e} + \boldsymbol{e}^{\mathrm{T}}\mathbf{P}\dot{\boldsymbol{e}} \\ &= [\boldsymbol{e}^{\mathrm{T}}\mathbf{A}_{\mathrm{d}}^{\mathrm{T}} + \boldsymbol{x}^{\mathrm{T}}\mathbf{A}_{\mathrm{d}}^{\mathrm{T}} - \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}, \boldsymbol{u}, t) + \boldsymbol{v}^{\mathrm{T}}\mathbf{B}_{\mathrm{d}}'^{\mathrm{T}}]\mathbf{P}\boldsymbol{e} \\ &\quad + \boldsymbol{e}^{\mathrm{T}}\mathbf{P}[\mathbf{A}_{\mathrm{d}}\boldsymbol{e} + \mathbf{A}_{\mathrm{d}}\boldsymbol{x} - \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, t) + \mathbf{B}_{\mathrm{d}}'\boldsymbol{v}] \\ &= \boldsymbol{e}^{\mathrm{T}}(\mathbf{A}_{\mathrm{d}}^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{A}_{\mathrm{d}})\boldsymbol{e} + 2M_1 \end{aligned} \tag{5}$$

where $\mathbf{P}$ is a positive-definite Hermitian or real symmetric matrix and M_1 is given by

$$M_1 = \boldsymbol{e}^{\mathrm{T}}\mathbf{P}[\mathbf{A}_{\mathrm{d}}\boldsymbol{x} - \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, t) + \mathbf{B}_{\mathrm{d}}'\boldsymbol{v}] = \text{scalar} \tag{6}$$

It can be seen that the expression for M results in a scalar quantity.

The Lyapunov stability criterion is satisfied if the following conditions are satisfied: if $\mathbf{A}_{\mathrm{d}}^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{A}_{\mathrm{d}} = -\mathbf{Q}$ is a negative definite matrix and if the control input $\boldsymbol{u}$ in the expression for M_1 can be so chosen as to make the value of M_1 negative. This will ensure that the equilibrium state around the point $\boldsymbol{e} = \mathbf{0}$ is asymptotically stable. The controller input, which would be a solution of an inequality, would force the plant to behave as the model system in the dynamic state.

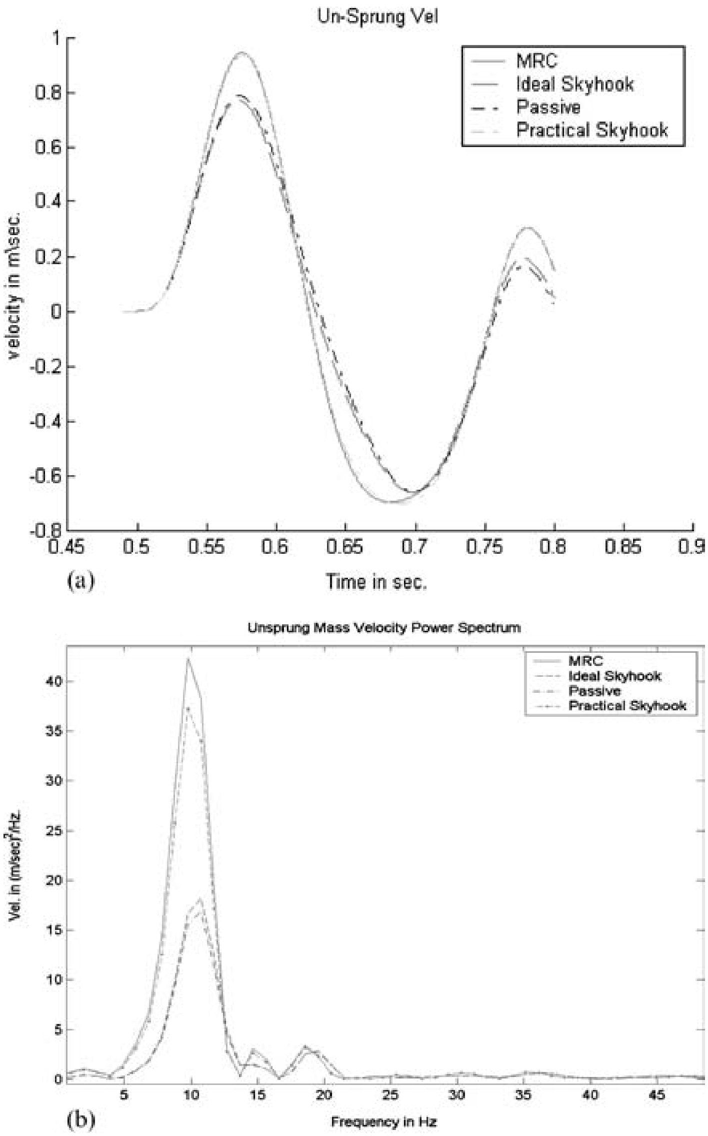


Fig. 8 Unsprung mass velocity

3 QUARTER-CAR VEHICLE MODEL

The active suspension design calls for choosing an appropriate model of the vehicle. For the purpose of making a dynamic evaluation several vehicle models have been developed. Of these the quarter-car model is the simplest and most amenable for intuitive analysis [1, 7]. The quarter-car model of the suspension is a two degree of freedom (DOF) model, which models the vertical or the heave motion of the vehicle alone. As the design goal of most of the active suspension is to reduce the vertical acceleration, the quarter-car model is sufficient from the controller design point of view. Hence the quarter-car system has been chosen as the vehicle model in the present study for the MRC design.

The dynamic equations for the quarter-car system (plant) can be written as follows

$$\dot{x}_1 = x_2 \tag{7}$$

$$\dot{x}_2 = \frac{1}{M}[K(x_3 - x_1) + C(x_4 - x_2) + u] \tag{8}$$

$$\dot{x}_3 = x_4 \tag{9}$$

$$\dot{x}_4 = \frac{1}{m}[K_{\text{ty}}(r - x_3) - K(x_3 - x_1) - C(x_4 - x_2) - u] \tag{10}$$

In equations (7) to (10) x_1 refers to the sprung mass position, x_2 represents the sprung mass velocity, x_3 represents the unsprung mass position, and x_4

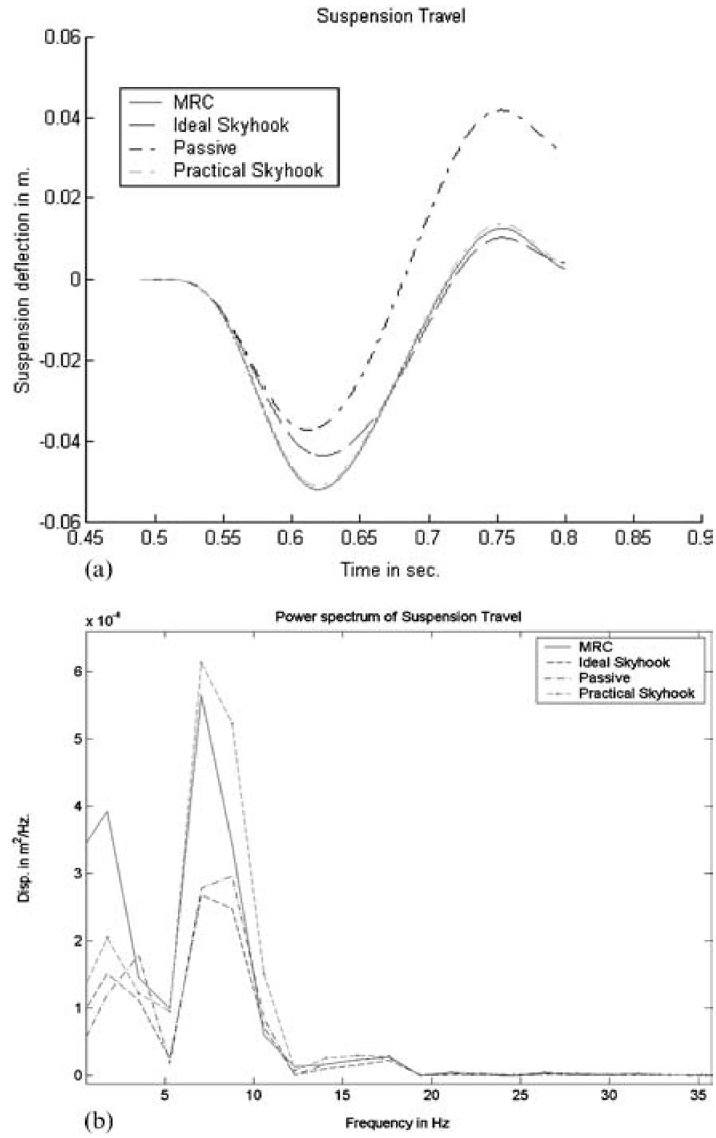


Fig. 9 Suspension travel

represents the unsprung mass velocity. The road disturbance input is given by r. The suspension stiffness, damping constant, and tyre stiffness are represented by the variables K, C, and K_{ty} respectively; u represents the force developed by the actuator based on the controller algorithm. For the schematic shown in Fig. 2 the practical skyhook damping case is adopted. Hence the controller force is given by

$$u = -K_{sky} x_2 \tag{11}$$

where K_{sky} refers to the amplification constant used. In the case of an ideal skyhook system the u term in the expression for the unsprung mass acceleration (equation (10)) would not be present. For the MRC-based system the expression for the value of u is obtained in the next section.

4 MRC PROBLEM FORMULATION – SKYHOOK CASE

The formulation of the MRC problem for the quarter-car case would be carried out within the framework described in section 2. The equations (7) to (11) represent the plant dynamic equation as described in equation (1). The model system equations are those of the ideal skyhook suspension system. Hence

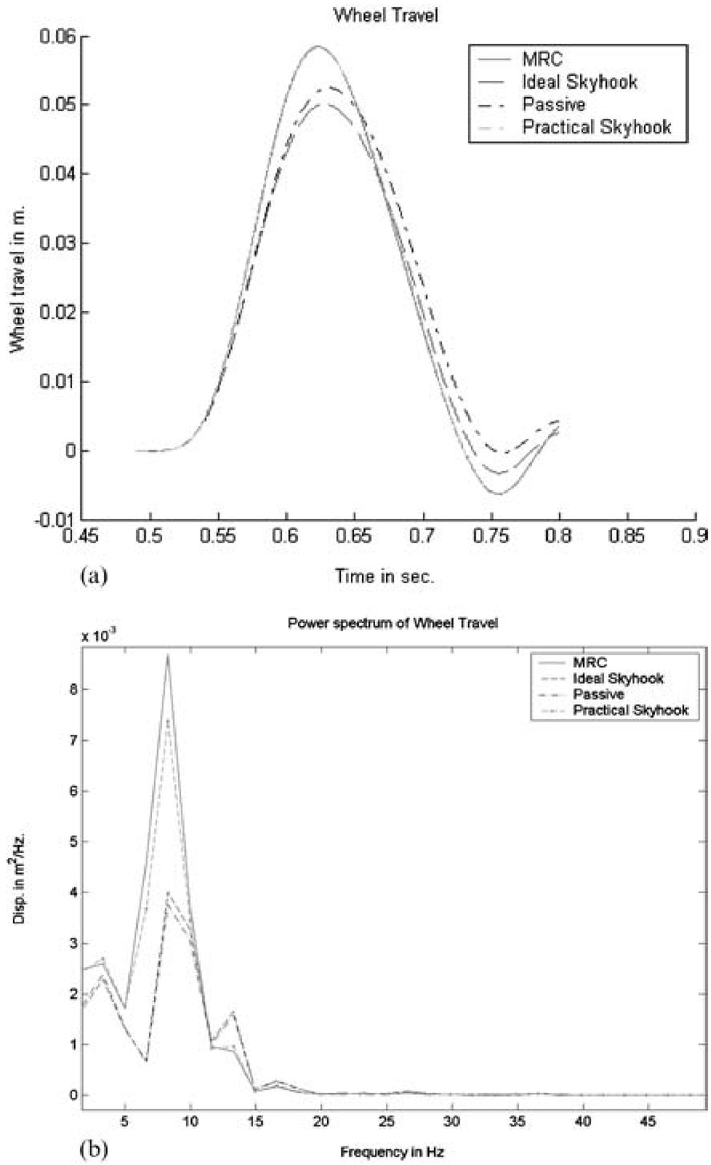


Fig. 10 Wheel travel

they are given as

$$\dot{x}_{1d} = x_{2d} \tag{12}$$

$$\dot{x}_{2d} = \frac{1}{M}[K(x_{3d} - x_{1d}) + C(x_{4d} - x_{2d}) + u] \tag{13}$$

$$\dot{x}_{3d} = x_{4d} \tag{14}$$

$$\dot{x}_{4d} = \frac{1}{m}[K_{ty}(r - x_{3d}) - K(x_{3d} - x_{1d}) - C(x_{4d} - x_{2d})] \tag{15}$$

The expressions in equations (1) and (2) can be rewritten as by separating the input vector $\boldsymbol{v}$ to its control input and road disturbance input components respectively

$$\dot{\boldsymbol{x}} = \mathbf{A}\boldsymbol{x} + \mathbf{B}\boldsymbol{u} + \mathbf{D}r \tag{16a}$$

$$\dot{\boldsymbol{x}}_d = \mathbf{A}_d\boldsymbol{x}_d + \mathbf{B}_d\boldsymbol{u} + \mathbf{D}_d r \tag{16b}$$

In equations (16) $\boldsymbol{u}$ represents the MRC control input and r the road disturbance input; $\mathbf{A}_d$ and $\mathbf{B}_d$, as

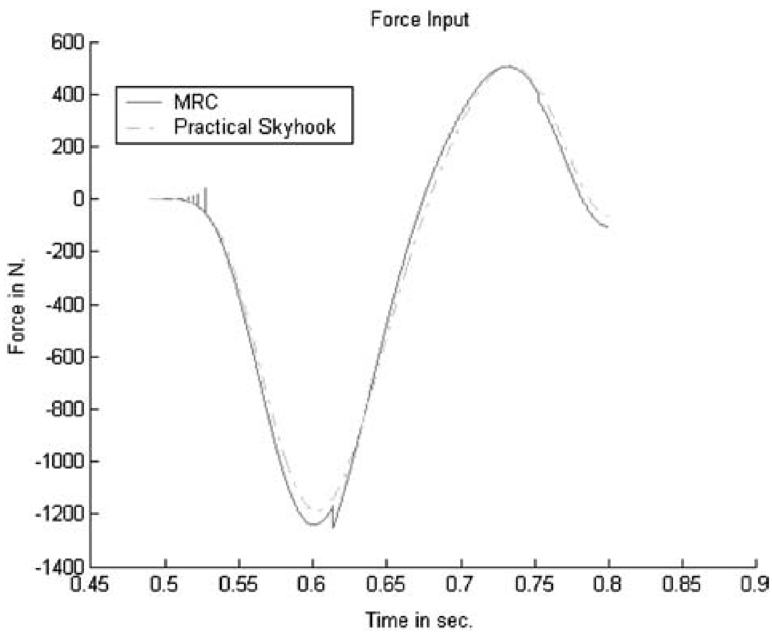


Fig. 11 Force input

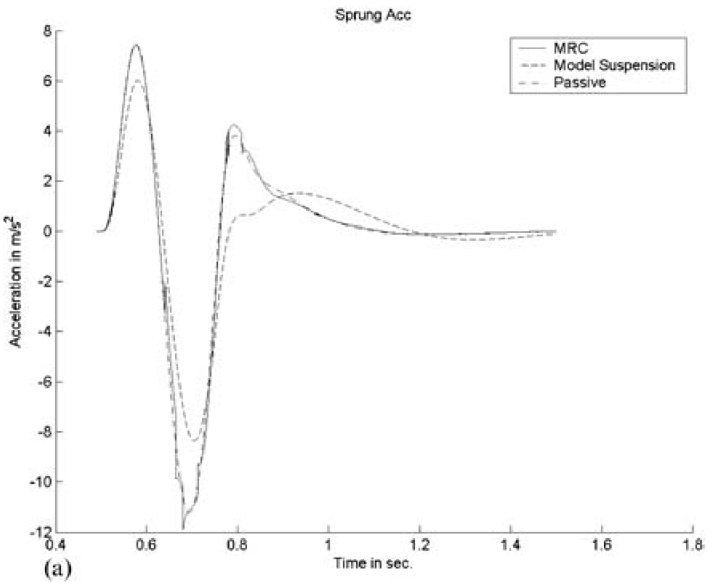
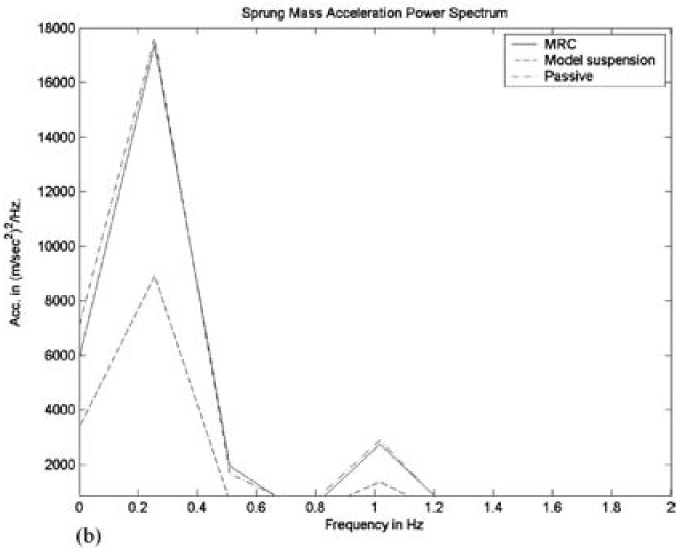


Fig. 12 Sprung mass acceleration



(b)

Fig. 12 (Continued)

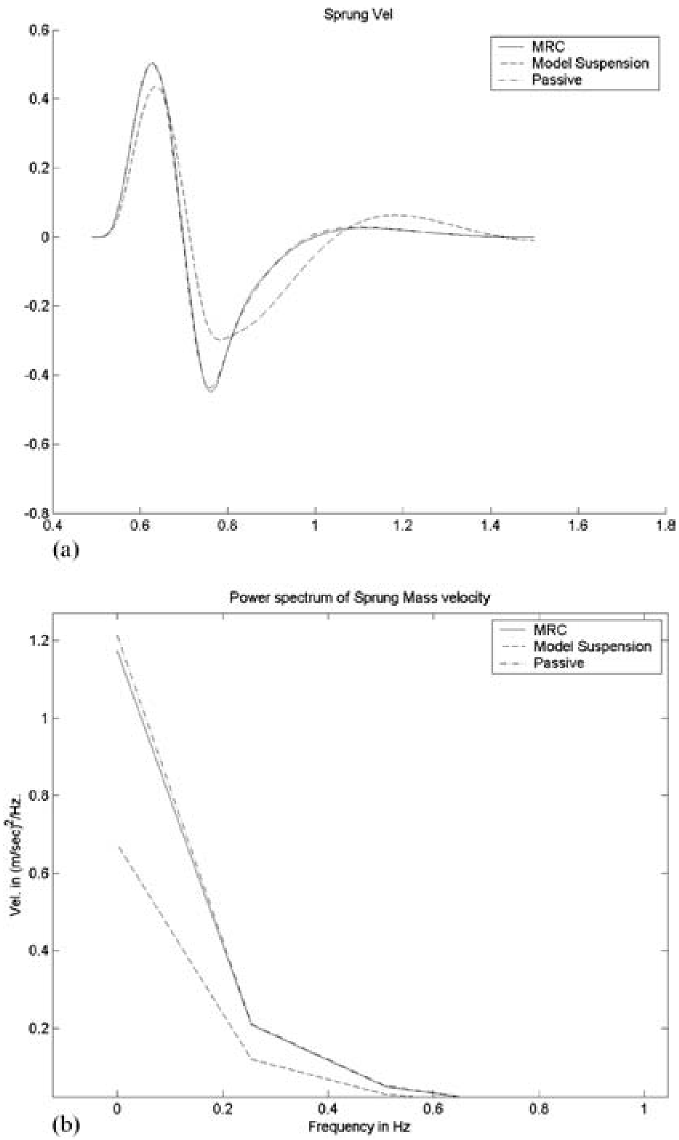


Fig. 13 Sprung mass velocity

described in equations (16), are given by

$$\mathbf{A}_\mathrm{d} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ \frac{-K}{M} & \frac{-C}{M} & \frac{K}{M} & \frac{C}{M} \\ 0 & 0 & 0 & 1 \\ \frac{K}{m} & \frac{C}{m} & \frac{-(K+K_\mathrm{ty})}{m} & \frac{-C}{m} \end{pmatrix} \qquad (17)$$

$$\mathbf{B}_\mathrm{d} = \begin{pmatrix} 0 \\ \frac{1}{M} \\ 0 \\ 0 \end{pmatrix} \quad \mathbf{D}_\mathrm{d} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{K_\mathrm{ty}}{m} \end{pmatrix} \qquad (18)$$

Following the discussion in equations (5) and (6), the **Q** and **P** can be chosen as any (4×4) positive definite real symmetric matrix. Hence the expression for $\dot{V}(\boldsymbol{e})$ can be written as

$$\dot{V}(\boldsymbol{e}) = -(q_{11}e_1^2 + q_{22}e_2^2 + q_{33}e_3^2 + q_{44}e_4^2) + 2M_1 \qquad (19)$$

where

$$M_1 = [\boldsymbol{e}]_{(1\times 4)}[\mathbf{P}]_{(4\times 4)} \times [\mathbf{A}_\mathrm{d}\boldsymbol{x} + \mathbf{B}_\mathrm{d}\boldsymbol{u} + \mathbf{D}_\mathrm{d}r - \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, t)]_{(4\times 4)} < 0 \qquad (20)$$

In equation (20), $\boldsymbol{e}$ represents the error vector for all four-state variables as defined in equation (3). By the Lyapunov stability criteria the expression for M_1 must be negative by the choice of an appropriate $\boldsymbol{u}$. Upon simplifying the expression for M_1 the following equation is obtained

$$\left(\sum_{i=1}^{4} e_i p_{2i}\right)\left[\frac{-1}{M}(K_\mathrm{sky}x_2 + u)\right] + \left(\sum_{i=1}^{4} e_i p_{4i}\right)\frac{1}{m}u < 01 \qquad (21)$$

In equation (21), e_i and p_{ji} refer to the ith element and the (j, i)th elements of the error matrix $\boldsymbol{e}$ and the positive definite matrix **P** respectively. The above inequality degenerates into a system of four inequalities based on the sign of $(\sum_{i=1}^{4} e_i p_{2i})$ and $(\sum_{i=1}^{4} e_i p_{4i})$, each giving an upper or lower range of the value of u. Denoting $(\sum_{i=1}^{4} e_i p_{2i})$ as k_1 and $(\sum_{i=1}^{4} e_i p_{4i})$ as k_2, the following expressions for the value of u are obtained

For $k_1 > 0, k_2 > 0$

$$u > \frac{-K_\mathrm{sky}x_2}{M}\left(\frac{1}{1/m - 1/M}\right)$$

For $k_1 > 0, k_2 < 0$

$$u > \frac{-K_\mathrm{sky}x_2}{M}\left(\frac{1}{1/m + 1/M}\right)$$

For $k_1 < 0, k_2 > 0$

$$u < \frac{-K_\mathrm{sky}x_2}{M}\left(\frac{1}{1/m + 1/M}\right)$$

For $k_1 < 0, k_2 < 0$

$$u < \frac{-K_\mathrm{sky}x_2}{M}\left(\frac{1}{1/M - 1/m}\right) \qquad (22)$$

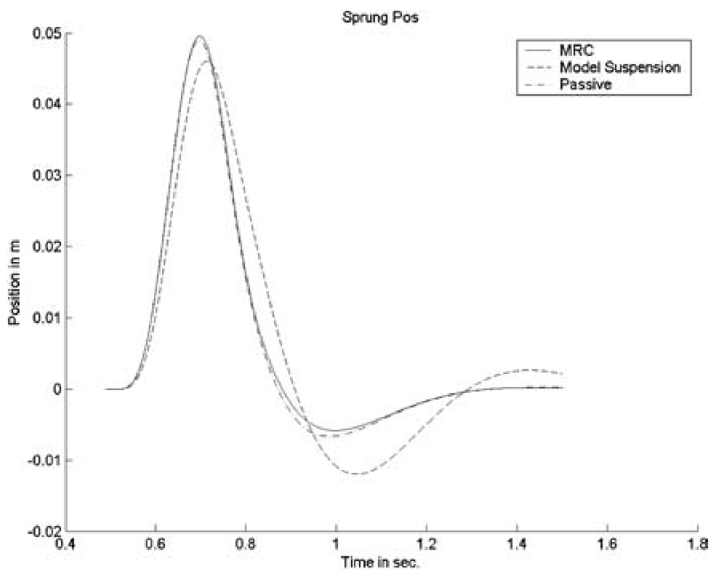


Fig. 14 Sprung position

This expression for the value of u in the different ranges of k_1 and k_2 denotes the solution for the MRC case that would force the response of a practical skyhook-based active suspension system to behave like an ideal skyhook-based active suspension system. As the expression for the control input is in terms of inequalities, suitable constant amplification factors may be chosen after evaluating the dynamic response. It can be noted from the expression for the controller input for the MRC case that it depends only on the sprung mass velocity like in the skyhook case. Thus the simplicity of the skyhook controller is preserved while ensuring better dynamic performance.

5 MRC–SKYHOOK CASE: SIMULATION RESULTS

The theoretical formulation as presented in the previous section has been tested by the time domain and frequency domain simulations of the passive, practical skyhook, ideal skyhook, and the MRC-based controller. The time domain simulation is for the passage of the quarter-car suspension over a sinusoidal bump of 5 cm radius and the frequency domain simulation is for a band-limited white noise input; these are shown in Figs 3(a) and 3(b) respectively. The sprung mass (M) value is chosen to be 200 kg, the unsprung mass (m) value to be 40 kg, the suspension

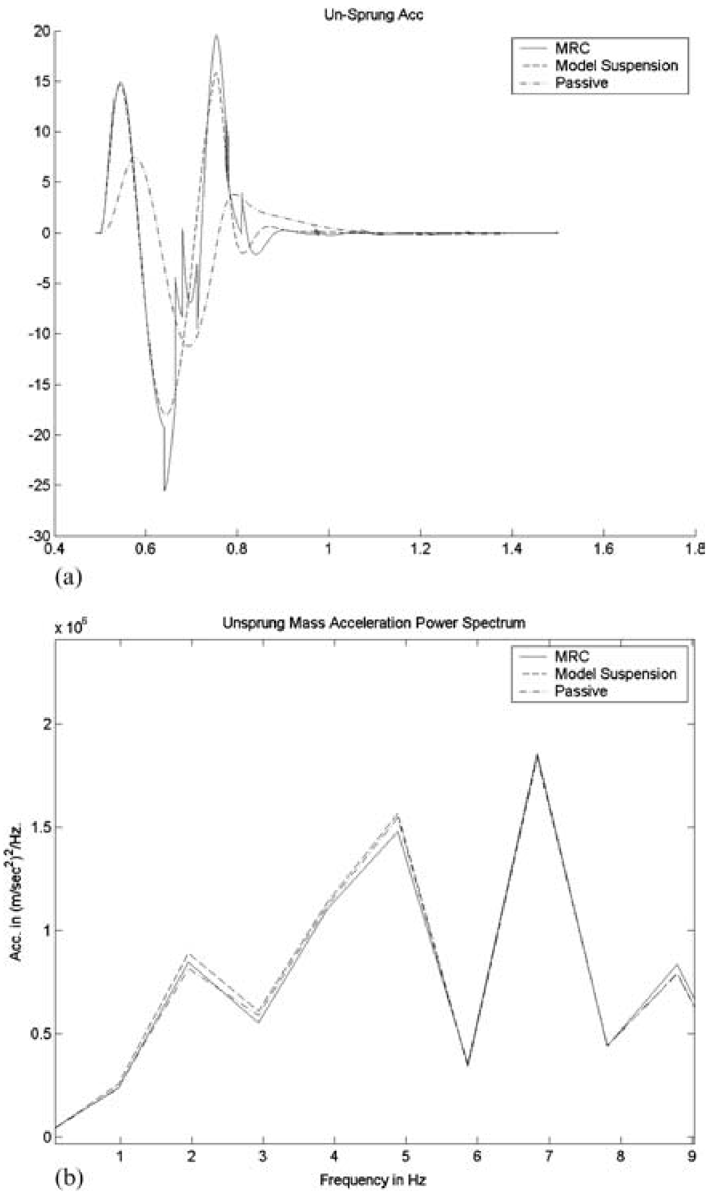


Fig. 15 Unsprung mass acceleration

stiffness (K) value to be 16 000 N/m, the suspension damping coefficient to be 980 N s/m, and the tyre stiffness to be 160 000 N/m for the purpose of the simulations. The results are filtered by a lowpass filter to remove any high-frequency components. Some of the simulation results are shown below.

It can be seen from Figs 4(a), 5(a), and 6 obtained from the time domain simulation and Figs 4(b) and 5(b) from the frequency domain simulation that the sprung mass response is considerably better for the MRC case as compared to the passive suspension response. The MRC response is better than the practical skyhook case, as expected, and tends to that of the ideal skyhook response.

From Figs 7(a), 8(a), 9(a), and 10(a) of the time domain simulation and Figs 7(b), 8(b), 9(b), and 10(b) of the frequency domain simulation it can be seen that the unsprung mass response is worse than that of the practical skyhook case for the acceleration and velocity case, while in the suspension travel and the wheel position cases the response is nearly the same as that of the ideal and practical skyhook cases in the time domain. This is expected due to the unavoidable application of the reaction of the controller force input on the unsprung mass.

The controller force for the MRC case as compared to that of the skyhook case is given in Fig. 11 for the domain case.

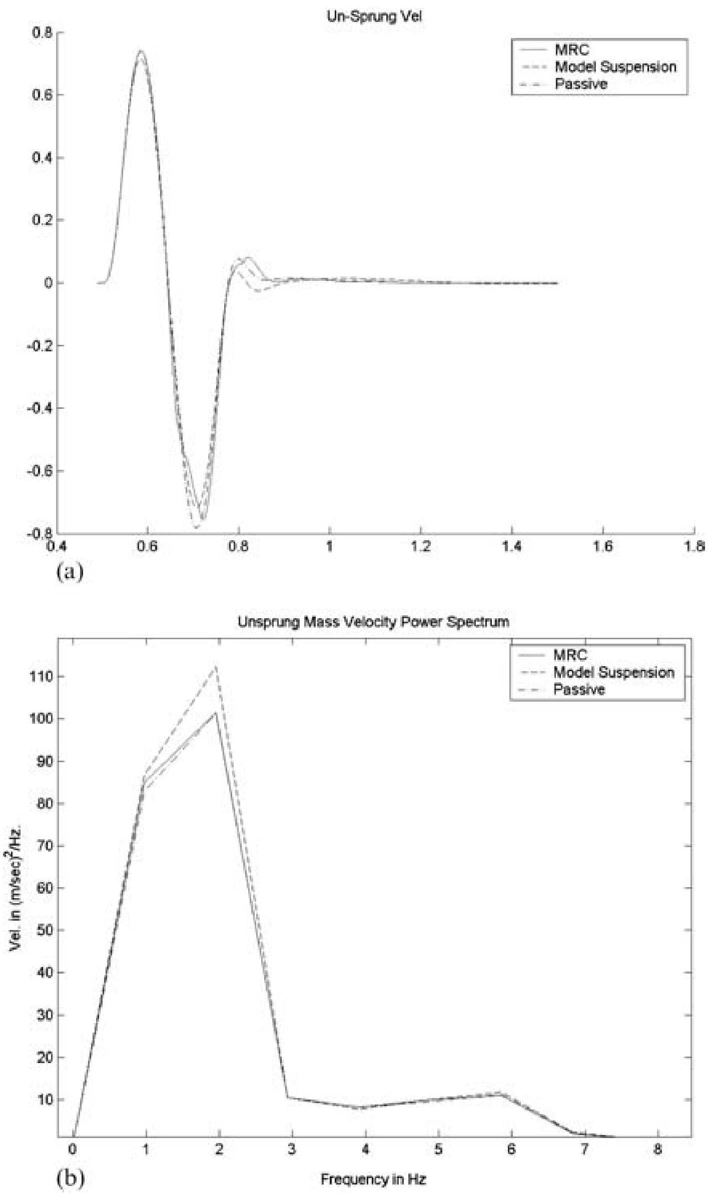


Fig. 16 Unsprung velocity

6 MRC PROBLEM FORMULATION: VARIABLE DESIGN CONSTANTS CASE

The MRC-based approach can also be used to make a given suspension behave as a suspension with different design constants to achieve a better dynamic system response. This is important in the case where practical constraints impose restrictions on the designer to choose an appropriate damping constant or spring stiffness. The dynamic equations for the quarter-car plant are the same as those described in equations (8) to (11). For the model reference system the equations would be similar to the following equations, with only the damping, stiffness values being different from that of the plant

$$\dot{x}_{1\mathrm{d}} = x_{2\mathrm{d}} \tag{23}$$

$$\dot{x}_{2\mathrm{d}} = \frac{1}{M}[K(x_{3\mathrm{d}} - x_{1\mathrm{d}}) + C_\mathrm{d}(x_{4\mathrm{d}} - x_{2\mathrm{d}})] \tag{24}$$

$$\dot{x}_{3\mathrm{d}} = x_{4\mathrm{d}} \tag{25}$$

$$\dot{x}_{4\mathrm{d}} = \frac{1}{m}[K_{\mathrm{ty}}(r - x_{3\mathrm{d}}) - K(x_{3\mathrm{d}} - x_{1\mathrm{d}}) - C_\mathrm{d}(x_{4\mathrm{d}} - x_{2\mathrm{d}})] \tag{26}$$

As a representative case only the damping coefficient of the model system is chosen to be different from that of the plant. Following the treatment given in section 4 for the ideal–practical skyhook system the expression for $\dot{V}(\boldsymbol{e})$ is obtained as

$$\dot{V}(\boldsymbol{e}) = -(q_{11}e_1^2 + q_{22}e_2^2 + q_{33}e_3^2 + q_{44}e_4^2) + 2M_1 \tag{27}$$

where

$$M_1 = [e]_{(1\times 4)}[\mathbf{P}]_{(4\times 4)} \times [\mathbf{A}_\mathrm{d}\boldsymbol{x} + \mathbf{B}_\mathrm{d}\boldsymbol{u} + \mathbf{D}_\mathrm{d}r - f(\boldsymbol{x}, \boldsymbol{u}, t)]_{(4\times 4)} < 0 \tag{28}$$

In equation (28), $\boldsymbol{e}$ represents the error vector for all four state variables as defined in equation (3). By the Lyapunov stability criteria the expression for M_1 must be negative by the choice of an appropriate $\boldsymbol{u}$. Upon simplifying the expression for M_1 the following inequality is obtained

$$\frac{-(\sum_{i=1}^4 e_i p_{2i})}{M}[(C_\mathrm{d} - C)(x_2 - x_4) + u] + \frac{(\sum_{i=1}^4 e_i p_{4i})}{m}[(C_\mathrm{d} - C)(x_2 - x_4) - u] < 0 \tag{29}$$

In equation (29), e_i and p_{ji} refer to the ith element and the (j, i)th elements of the error matrix $\mathbf{e}$ and the positive definite matrix $\mathbf{P}$ respectively. The above inequality degenerates into a system of four inequalities based on the signs of the $(\sum_{i=1}^4 e_i p_{2i})$ and the $(\sum_{i=1}^4 e_i p_{4i})$ terms, which upon solution give the expressions for the control force input. Denoting $(\sum_{i=1}^4 e_i p_{21})$ as k_1 and $(\sum_{i=1}^4 e_i p_{41})$ as k_2, the following expressions for the value of u are obtained

For $k_1 > 0$, $k_2 > 0$

$$u < \left(\frac{N}{M} - \frac{N}{m}\right)\left(\frac{1}{1/m - 1/M}\right)$$

For $k_1 > 0$, $k_2 < 0$

$$u > -\left(\frac{N}{M} + \frac{N}{m}\right)\left(\frac{1}{1/m + 1/M}\right)$$

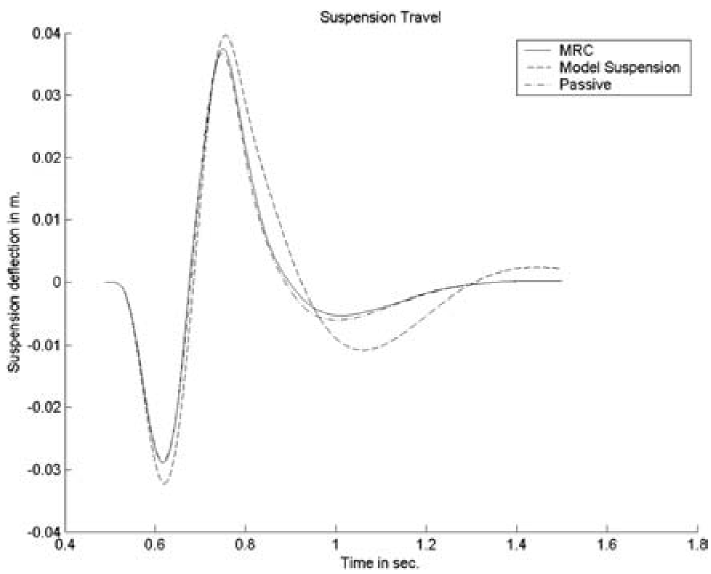


Fig. 17 Suspension travel

For $k_1 < 0$, $k_2 < 0$

$$u < \left(\frac{N}{m} - \frac{N}{M}\right)\left(\frac{1}{1/M - 1/m}\right)$$

For $k_1 < 0$, $k_2 > 0$

$$u < -\left(\frac{N}{M} + \frac{N}{m}\right)\left(\frac{1}{1/m + 1/M}\right) \quad (30)$$

In equation (30), N stands for $(C_d - C)(x_2 - x_4)$.

This expression for the value of u in the different ranges of k_1 and k_2 denotes the solution for the MRC case that would force the response of a given suspension system with a damping constant C to behave like a suspension system with a damping constant of C_d. As the expression for the control input is in terms of inequalities suitable constant amplification factors may be chosen after evaluating the dynamic response. It can be seen from the expression for the controller input that it depends only on the velocity variables of the sprung mass as well as the unsprung mass as only the damping value of the model system is different from that of the plant. This calls for a simpler controller than that of the traditional full state feedback controller, which calls for the measurement of all the state variables.

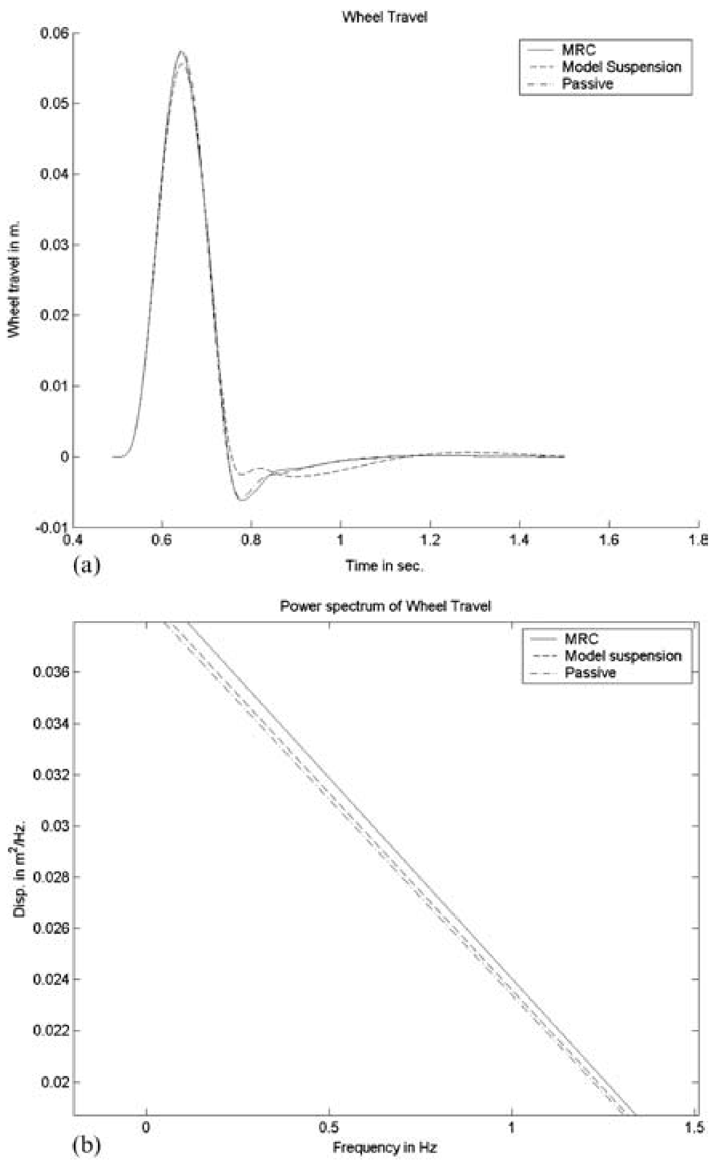


Fig. 18 Wheel travel

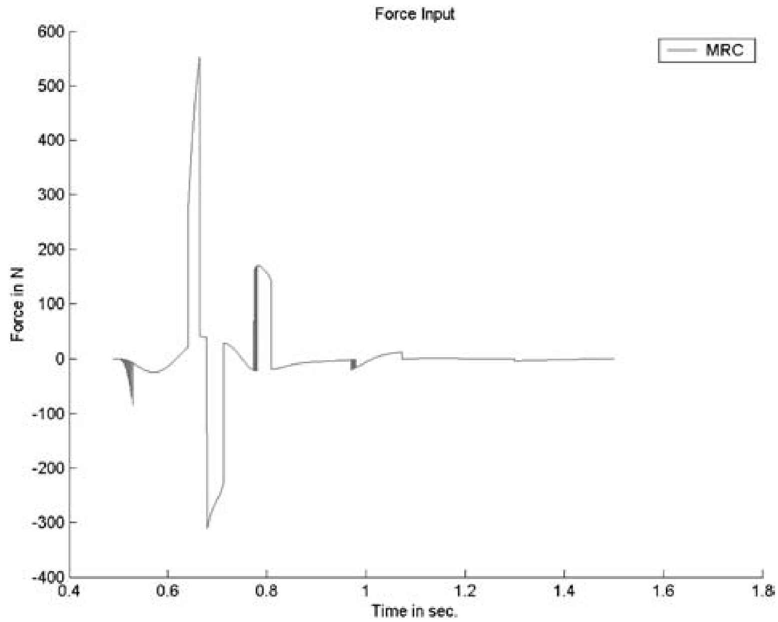


Fig. 19 Force input

7 MRC VARIABLE DAMPING CASE: SIMULATION RESULTS

The theoretical formulation as presented in the previous section has been tested by the time domain simulation of the passive, model suspension and the MRC-based controller. The plant damping, mass, and stiffness values are kept the same as those given in section 5. The model suspension is assumed to have a damping value C_d of 4670 N s/m. Some of the simulation results are shown below.

It can be seen from Figs 12(a), 13(a), and 14 of the time domain and Figs 12(b) and 13(b) of the frequency domain that the sprung mass response is considerably better for the MRC case as compared to the passive suspension response. The MRC response is naturally worse than that of the model suspension results.

From Figs 15(a) to 18(a) of the time domain and Figs 15(b), 16(b), and 18(b) of the frequency domain it can be seen that the unsprung mass response is worse than both the plant as well as the model suspension. The expression for the controller force for the MRC case is given in Fig. 19.

8 CONCLUSION

From the simulation results it is observed that the MRC-based systems provide a better dynamic performance without any increase in the complexity of the controller or the number of sensors required for the measurement of the different dynamic variables. This approach towards the design of active suspensions can be further pursued for a practical set-up that could validate the theoretical predictions and the simulation results.

REFERENCES

1 **Hrovat, D.** Survey of advanced suspension developments and related optimal control applications. *Automatica*, 1997, **33**(10), 1781–1817.

2 **Appleyard, M.** and **Wellstead, P. E.** Active suspension: some background. *IEE Proc. on Control Theory Applics*, 1995, **142**(2), 123–160.

3 **Srinivasa, Y. G.** and **Teja, R.** Investigations on the stochastically optimal PID controller for a linear quarter car road vehicle model. *Veh. System Dynamics*, 1996, **26**, 103–116.

4 **Karnopp, D.** Active damping in road vehicle suspension systems. *Veh. System Dynamics*, 1983, **12**, 291–316.

5 **Sunwoo, M.** and **Cheok, K.C.** Model reference adaptive control for vehicle active suspension systems. *IEEE Trans. Ind. Electronics*, 1991, **38**(3), 217–222.

6 **Nagai, M.** Recent researches on active suspension for ground vehicles. *Jap. Soc. Mech. Engrs Int. J. Ser. C*, 1993, **36**(2), 161–170.

7 **Kosut, R. L.** Sub-optimal control of linear time-invariant systems subject to control structure constraints. *IEEE Trans. Autom. Control*, 1970, **AC-15**(5), 557–563.

BIBLIOGRAPHY

Katsuda, K., Nobou, H., Doi, S., and **Yasuda, E.** Improvement of ride comfort by continuously controlled damper. *Trans. SAE 920276*, 1992, 356–363.

Sharp, R. S. and **Crolla, D. A.** Road vehicle suspension design – a review. *Veh. System Dynamics*, 1987, **16**, 167–192.

APPENDIX

Notation

$\mathbf{A}$, $\mathbf{A}_d$	$n \times n$ constant state matrix
$\mathbf{B}$, $\mathbf{B}_d$	$n \times r$ constant state matrix
C	damping constant (N s/m)
$\boldsymbol{e}$	error vector
e_i and p_{ji}	ith element and the (j, i)th elements of the error matrix $\mathbf{e}$ and the positive definite matrix $\mathbf{P}$ respectively
$\boldsymbol{f}$	vector-valued function
K	suspension stiffness (N/m)
K_{sky}	amplification constant [equation (11)]
K_{ty}	tire stiffness (N/m)
m	unsprung mass (kg)
M	sprung mass (kg)
$\mathbf{P}$	positive-definite Hermitian or real symmetric matrix
$\mathbf{Q}$	positive-definite matrix
r	road disturbance input
$\boldsymbol{u}$	control vector (r-vector)
$V(\boldsymbol{e})$	Lyapunov function
$\boldsymbol{x}$	state vector of the plant (n-vector)
$\dot{\boldsymbol{x}}$	first-order differential of vector $\boldsymbol{x} = \mathrm{d}x/\mathrm{d}t$
$\boldsymbol{x}_d$	state vector of the model (n-vector)
$\dot{\boldsymbol{x}}_d$	first-order differential of vector $\boldsymbol{x}_d = \mathrm{d}x_d/\mathrm{d}t$
x_1	sprung mass position (m)
x_2	sprung mass velocity (m/s)
x_3	unsprung mass position (m)
x_4	unsprung mass velocity (m/s)