Conjunctive Use of Surface and Groundwater for Coastal and Deltaic Systems

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Abstract: A regional conjunctive use model is developed for a near-real deltaic aquifer system, irrigated from a diversion system, with some reference to hydrogeoclimatic conditions prevalent in the east coastal deltas of India. Water resources are sufficiently available in these regions under average monsoon rainfall conditions, but their distribution in space and time has been ever challenging to water managers. Surface-water availability shows temporal fluctuations in terms of floods and droughts, and groundwater availability shows mainly spatial variability in terms of quality and quantity due to the hydrogeologic setting, boundary conditions, and aquifer properties. The combined simulation-optimization model proposed in this study is solved as a nonlinear, nonconvex combinatorial problem using a simulated annealing algorithm and an existing sharp interface model. The computational burden is managed within practical time frames by replacing the flow simulator with artificial neural networks and using efficient algorithmic guidance.

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Introduction

Conjunctive use has been defined in more ways than one, but in general it is defined as the allocation of surface water and groundwater in terms of quantity and/or quality so as to achieve one or more objectives while satisfying certain constraints. Coe (1990) defined conjunctive use with reference to stream diversion (or run-of-the-river) systems as the management of groundwater/surface water in a coordinated operation to the end that the total yield of such a system over a period of years exceeds the sum of yields of the separate components of the system resulting from an uncoordinated operation.

Management of water resources in coastal and deltaic regions irrigated by run-of-the-river schemes involves primarily two issues: First, availability of water resources in space and time, and second, seawater intrusion. Improper management arising out of excessive irrigation or increased groundwater exploitation often leads to waterlogging or seawater intrusion problems, respectively. Any conjunctive use model must address these two issues for application to coastal and deltaic regions.

Two general approaches have been used to simulate seawater intrusion in coastal aquifers. The freshwater and saltwater zones within an aquifer are separated by a transition zone in which there is a gradual change in density. The disperse interface approach explicitly represents the presence of this zone. Although disperse density dependent flow and transport models (Huyakorn et al. 1987; Das and Datta 1999) are presently available, their use in management models has been somewhat limited because of high computational burden. The second approach to the analysis of seawater intrusion problems is based on the simplifying assumption that the transition zone can be represented by a sharp interface (Bear and Dagan 1964; Polo and Ramis 1983; Essaid 1990; Bakker 2003).

Combined simulation-optimization models (Gorelick 1983) have been widely used to address the management issues. Willis and Finney (1988), Emch and Yeh (1998), and Das and Datta (1999), among others, have proposed a number of groundwater management models applicable for coastal aquifers. Although a number of studies have been reported on management issues related to coastal aquifers in general, not much attention has been paid to the issues unique to groundwater management in deltaic regions.

Also, as stated by Emch and Yeh (1998), the objectives and constraints in a coastal or deltaic aquifer management model are typically nonlinear, and therefore use of gradient-based methods for solving the optimization problem is beset with difficulties. Gradient-based methods for these problems are liable to get trapped in a local minimum. During the last 10 to 15 years, heuristic or nonexact methods have been developed for the purpose of correcting this problem. Among these, simulated annealing (SA) (Dougherty and Marryott 1991) and genetic algorithm (GA) are the two most popular methods. More recently, Wang and Zheng (1998) and Cunha (1999) have demonstrated the application of SA to hypothetical groundwater management problems in noncoastal regions.

Description of Study

Several coastal deltas of east India evolved during the Quaternary period with the deposition of sediment from large river basins

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over geologic time. The deltas are highly productive aquifers with intense agricultural activity. Traditionally, the land use has been mostly rice cultivation during the two main seasons, each of 6 months' duration, that is, the monsoon (Kharif crop) and nonmonsoon (Rabi crop) seasons. The Rabi crop is fully dependent on irrigation from diversion schemes or from groundwater sources. A third crop is also grown in some areas but is not very common. The silty-clayey alluvial aquifers are mostly unconfined, with a shallow water table that exhibits flat gradients. The spatial variability of fresh groundwater in deltas depends mainly upon hydrogeologic setting, proximity to the sea, and the presence of paleochannels. The hydrology of deltas is largely influenced by erratic monsoon rainfall that affects groundwater recharge during the monsoon season and surface-water availability during both seasons. This poses challenges to optimal utilization of water resources in the deltaic areas.

In the present study, an operational planning model is developed on a regional basis (macro level) for conjunctive use of surface water and groundwater in a coastal and deltaic environment under existing irrigation from a diversion system. In this study, conjunctive use is defined as the allocation of surface water and groundwater such that groundwater storage is induced at suitable locations during surplus (mainly monsoon season) surface-water flows and depleted during both periods. Single- and dual-objective management models are formulated as combinatorial optimization problems. The two objectives are minimization of the operational cost of supplying and providing surface water and groundwater at the demand centers, and maximization (or conservation) of groundwater storage in space and time.

One of the main focuses of this study is to address the fluctuation in the availability of surface-water and groundwater resources in space and time through conjunctive use. Flow simulation is accomplished using the SHARP interface model (Essaid 1990), which is suitable for a coastal environment on a regional basis. The computational time burden associated with a heuristic search method is reduced by replacing the SHARP simulator with a trained artificial neural network (ANN) at points of interest and through efficient algorithmic guidance. Although the study area has some reference to the hydrogeoclimatic conditions prevalent in the east coastal deltas of India, only hypothetical or idealized examples are used to illustrate the application of the concept and methodology.

Model Formulation

The model is formulated as an operational planning model, and the infrastructure is assumed already in place. The management model is formulated to meet the consumptive demands through optimal allocation of surface water and groundwater in space and time. The irrigation requirements are met from a diversion structure (surface water) and groundwater sources in the delta system. The water table is assumed to be below the root zone, such that groundwater is available only by pumping. The surface-water availability shows temporal fluctuations in terms of floods and drought conditions. The fresh groundwater availability shows spatial variability due to the hydrogeologic setting, aquifer properties, and boundary conditions. These spatial and temporal variations in the surface water and groundwater common to deltaic systems are embedded as constraints within the management model. The water-quality constraint is met indirectly by ensuring that well screens are above the freshwater-saltwater interface.

In the course of developing and testing the proposed methodology, hypothetical data sets representing a simplified unconfined

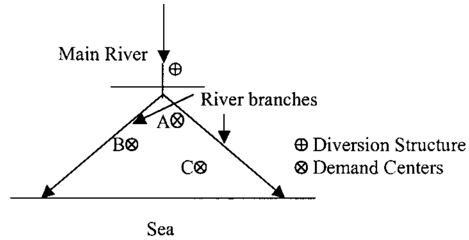

Fig. 1. Definition sketch of deltaic system

deltaic aquifer system (Fig. 1) are used. The aquifer system is assumed to be unconfined, with fresh water and saltwater separated by a sharp interface. The SHARP model, as discussed later, simulates the unconfined aquifer flow system as a single layer in an isotropic, homogeneous system. The river branches and sea are assumed to be constant head boundaries. Average rainfall recharge, surface-water availability, and demand during each time period (monsoon or nonmonsoon season) are inputs to the model, which uses time steps of 6 months for macro level operational planning and hence uses volumes as input rather than rates. However, the volumes are converted into rates while running the SHARP model over the seasonal time step.

The management model of this aquifer system involves two conflicting objectives and constraints pertaining to decision and state variables. The model is required to conjunctively allocate groundwater (decision variable) and surface water at each of the demand centers representing a response zone in the deltaic region so as to minimize operational costs, maximize groundwater reserves (or minimize drawdown volumes as a surrogate objective) at the end of the planning horizon, and satisfy a given set of constraints in space and time. The groundwater pumpages are allowed to take negative, zero, or positive values within a specified range. It is important to note that groundwater pumpages take negative values (injection) only if surplus surface water is available. Further, at a specific time and location (demand center), either injection or pumpage can occur; both cannot occur simultaneously.

Surface-water allocation takes only positive values so as to meet the balance of demand and is also subject to a certain minimum flow. The minimum-flow constraint reflects operational considerations that the canal cannot be operated unless a certain minimum discharge is maintained. The state variables include freshwater heads, saltwater heads, and interface elevations. The objectives and constraints of the problem are formulated either directly as functions of decision variables, indirectly as a function of state variables, or both. The problem has a nonlinear objective function and nonlinear constraints. Since two conflicting objectives have to be optimized, the final choice has to be made from a tradeoff curve. Mathematically, the two objectives are formulated as follows:

Cost Objective

$$\text{Min } Z_1=\sum_{n=1}^{N}\left[\sum_{k=1}^{K} C_{s\cdot k}\{D_{k,n}-Q_{g\cdot k,n}\}+\sum_{k=1}^{K} C_{g\cdot k}|Q_{g\cdot k,n}|\right] \tag{1}$$

Drawdown Volume (Surrogate) Objective

$$\text{Min } Z_2=\sum_{i=1}^{I}\sum_{j=1}^{J}[(L_{i,j}-h_{f\cdot i,j,N})A] \tag{2}$$

subject to:

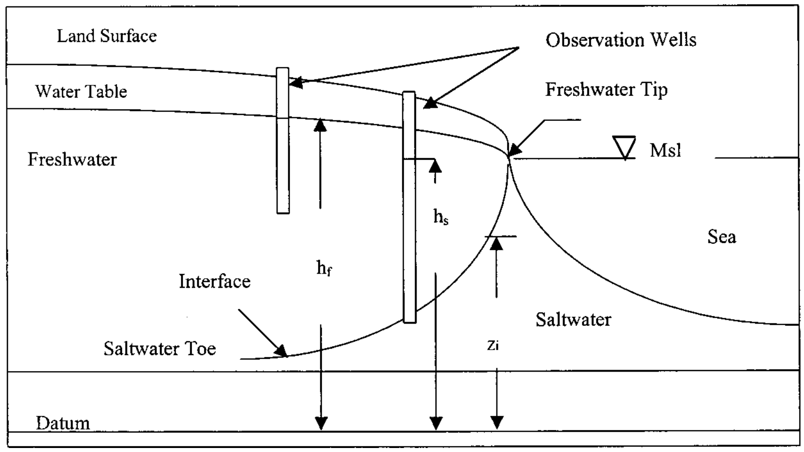

Fig. 2. Coastal aquifer system as represented in SHARP model

1. Surface-water minimum-flow constraint at each demand location and time period

$$Q_{s\cdot k,n}=D_{k,n}-Q_{g\cdot k,n}\geqslant Q_{\min} \quad \forall \ k \text{ and } n \tag{3}$$

2. Surface-water availability constraint for each time period

$$\sum_{k=1}^{K}(D_{k,n}-Q_{g\cdot k,n})\leqslant Q_n \quad \forall \ n \tag{4}$$

3. Drawdown elevation (head) constraint at each node

$$\mathbf{h}_{f(i,j,n)}\geqslant \mathbf{H}_{\min(i,j)} \quad \forall \ i,\ j, \text{ and } n \tag{5}$$

4. Waterlogging constraint at each node

$$\mathbf{h}_{f(i,j,n)}\leqslant \mathbf{H}_{\max(i,j)} \quad \forall \ i,\ j, \text{ and } n \tag{6}$$

5. Interface elevation constraint at coastal nodes

$$\mathbf{z}_{(i,j,n)}\leqslant \mathbf{Z}_{\min(i,j)} \quad \forall \ i,\ j, \text{ and } n \tag{7}$$

6. Hydraulic response equations (continuity) of the simulator in which, C_s=cost of providing unit surface water; C_g=cost of providing unit supply from ground water source; D_k=demand at center k $[L^3]$; $L_{i,j}$=ground elevation at node $i,j[L]$; $h_{f(i,j)}$=fresh water head at node i,j $[L^3]$; Q_g=volume of water supplied from ground water source $[L^3]$; Q_s=volume of water supplied from surface source $[L^3]$; N=number of the time periods (seasonal); I=number of rows; J=number of columns; K=number of demand centers; Q_n=surface-water availability for nth time period; A=area of the grid node $[L^2]$; $Q_{\min}$=minimum flow in the canal $[L^3]$; $H_{\min(i,j)}$=prescribed minimum head at node i,j [L]; $H_{\max(i,j)}$=prescribed maximum head at node i,j[L]; $z_{i,j,n}$=interface elevation at node i,j and the end of period n [L]; and $z_{\max(i,j)}$=prescribed interface elevation at node i,j[L].
7. As all facilities are assumed to be in existence, no capital costs are considered and all unit costs pertain only to energy, operation, and maintenance. The cost function is linear for surface-water supplies but nonlinear for groundwater pumpages, as it is a function of both discharge (unit volume) and head (or depth). The unit cost of groundwater is assumed to be cheaper than surface water at some midpoint in the delta. The unit costs at other locations are computed relatively. The unit cost of injected water is much more expensive as it involves both surface-water transport and injection under pressure. A high unit cost of injected water is assumed in this study, independent of location and depth.

Solution Methodology

The methodology adapted in this study uses a combined simulation-optimization approach (Gorelick 1983). A code was developed by interfacing the SA algorithm (optimizer) with the SHARP model as a subroutine. The SHARP simulator is subsequently replaced with the ANN model to reduce computational burden. The simulator (SHARP model), the optimizer (SA algorithm), and the ANN are briefly discussed below.

Flow Model—SHARP as Simulator

SHARP (Essaid 1990), the multilayered, two-fluid sharp interface model used in this study, is a quasi-3D finite-difference model. It simultaneously solves the freshwater and saltwater flow equations coupled by the boundary condition at the interface such that the pressures are equal on either side (Fig. 2). The coupled, nonlinear partial differential equations given below must be solved at each node:

$$[S_fB_f+n(\alpha+\delta)]\frac{\partial h_f}{\partial t}-n(1+\delta)\frac{\partial h_s}{\partial t} =\frac{\partial}{\partial x}\left(B_{fx}K_{fx}\frac{\partial h_f}{\partial x}\right)+\frac{\partial}{\partial y}\left(B_{fy}K_{fy}\frac{\partial h_f}{\partial y}\right)+Q_f \tag{8}$$

$$[S_sB_s+n(1+\delta)]\frac{\partial h_s}{\partial t}-n\delta\frac{\partial h_f}{\partial t} =\frac{\partial}{\partial x}\left(B_{sx}K_{sx}\frac{\partial h_s}{\partial x}\right)]+\frac{\partial}{\partial y}\left(B_{sy}K_{sy}\frac{\partial h_s}{\partial y}\right)+Q_s \tag{9}$$

$$z_i=(1+\delta)h_s-\delta h_f \tag{10}$$

in which, n=effective porosity; $\alpha=1$ for an unconfined aquifer, 0 for confined aquifer; h_f,h_s=fresh and saltwater hydraulic heads [L]; S_f,S_s=fresh and saltwater specific storage $[L^{-1}]$; $K_{fx},K_{sx},K_{fy},K_{sy}$=fresh and saltwater hydraulic conductivity in x and y directions $[L\,T^{-1}]$; B_f,B_s=fresh and saltwater saturated thickness [L]; Q_f,Q_s=fresh and saltwater source/sink terms (pumpage, recharge $[L\,T^{-1}]$; z_i=interface elevation [L]; $\delta=\gamma_f/(\gamma_s-\gamma_f)$; and γ_f,γ_s=fresh water and saltwater specific weights $[M\,L^{-2}\,T^{-2}]$.

Simulated Annealing Algorithm

In this study a simulated annealing (SA) algorithm is used so that a near-optimal solution can be obtained for the management problem. The SA algorithm with the perturbation procedure is suitable for discrete values of decision variables. Therefore, although groundwater pumpages and surface-water allocations are typically continuous variables, they are treated as discrete variables in this study. Also, these variables could be considered as discrete variables from practical and operational considerations. First, for an allocation model using seasonal time steps, the primary objective is only to arrive at a broad macro-level operational planning schedule; second, pumps have discrete capacities and canals must be operated for certain minimum flows.

It may also be noted here that the inaccuracies arising out of the above treatment can be further reduced by implementing the SA algorithm along with the bracketing procedure (Dougherty and Marryott 1991). Therefore, in the present study groundwater pumpages are considered discrete variables and the model is solved as a combinatorial problem. In the proposed conjunctive use model, since groundwater pumpages/injections assume discrete values, the surface water (although not a decision variable) also assumes discrete values because it must meet the balance of the demand (through constraints) at any point in space and time. It is assumed that the pump capacities and canal capacities are consistent within the range of decision variables used for pumping/injection and delivery of surface water from the diversion point to the demand center.

SA is a heuristic algorithm to find near-optimum solutions (Kirkpatrick et al. 1983). In this method, each decision variable is restricted to a set of possible discrete values. Each combination of decision variables, that is, the decision vector, is called a configuration. For example if there are five decision variables and each is allowed to take a value from a set of 10 possible discrete values, then there would be 10^5 configurations. The set of all possible combinations constitutes the configuration space. The basic idea of the method is to generate a random configuration (trial point) iteratively through perturbation and evaluate the objective function and the constraints after determining the state variables by using the simulator.

If the trial point results in infeasibility, that is, if the constraints are violated, it is rejected and a new point is generated. If the trial point is feasible and the objective functions value is smaller than the current best value (for a minimization problem), then the point is accepted and the record for the best value is updated. If the trial point results in feasibility but the objective function is higher than the current best value, then the trial point is either accepted or rejected using the Metropolis criterion (Metropolis et al. 1953). This is implemented by generating a random deviate, uniformly distributed on the interval (0,1). If the random deviate thus generated is smaller than the acceptance probability, then the uphill move is accepted.

In computing the probability for the acceptance of an uphill move, a parameter called temperature is used. For the optimization problem, this temperature can be a target value for the cost function corresponding to a global minimum. Initially, a larger temperature or target value is selected. As the trials progress, this value is progressively reduced using a cooling factor. The acceptance probability of uphill moves steadily decreases to zero as the temperature is reduced. Thus, in the initial stages the method is likely to accept worse configurations, while in the final stages, the worse designs are almost always rejected.

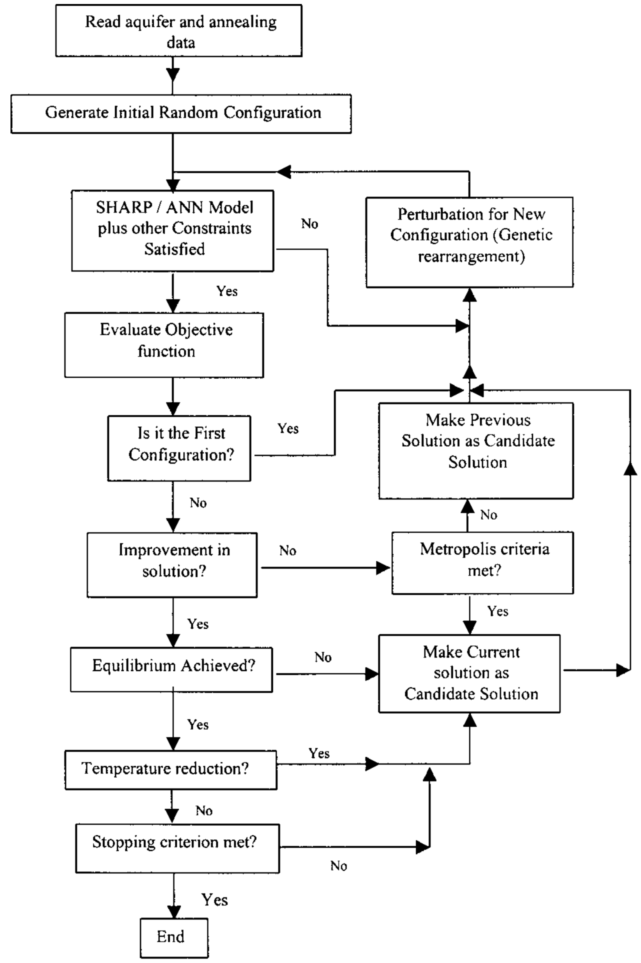

Fig. 3. Scheme of solution procedure using simulated annealing

The entire process is terminated after performing a fairly large number of trials or chains (iterations). The strategy avoids getting trapped in a local minimum. The initial temperature, cooling factor, chain length, and termination criteria are referred to as annealing parameters. These are difficult to determine (Wang and Zheng 1998), but certain guidelines have been defined by Dougherty and Marryott (1991), Cunha (1999), and others, for choosing the values of these parameters.

Algorithmic Guidance

The algorithmic representation of the SA along with the SHARP simulation model is shown in Fig. 3. The SA code generates a random configuration, modifies the SHARP input file, executes the simulation model, and verifies the constraints during each iterative step (referred to as a chain). Thus the optimizer (SA) calls the external simulator (SHARP) repeatedly. The SA explores the objective function's nonconvex surface randomly and tries to optimize the function while moving both uphill and downhill. Once the termination criterion is met, the optimal solution corresponds to the minimum cost configuration. As the number of decision variables and constraints increases or as the constraints get tighter, the number of infeasible solutions also increases. Therefore, two procedures are adopted to reduce the computational time burden. First, an efficient algorithmic guidance is used to generate only feasible configurations. This includes a perturbation procedure called excursion limiting for early convergence

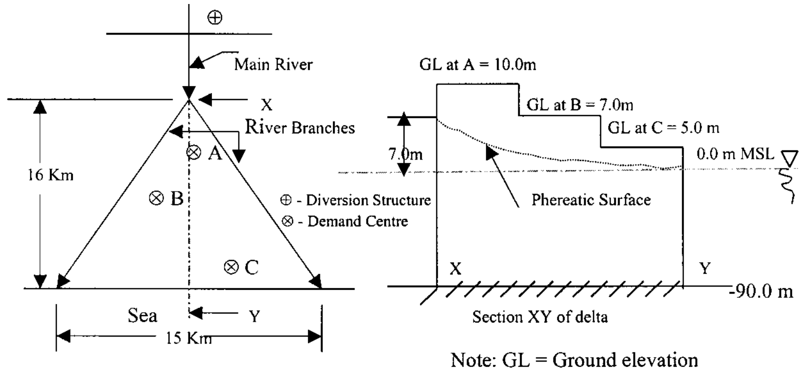

Fig. 4. Sketch of hypothetical deltaic system

(Dougherty and Marryott 1991). Second, the simulator is replaced by a trained ANN to obtain the values of the state variables at points of interest.

Algorithmic guidance ensures that only feasible configurations are generated before the simulator is called. This is made possible by terminating trial configurations that are infeasible at the earliest stage, rather than verifying supply and demand constraints after generating the whole configuration. The perturbation procedure used in this study (genetic rearrangement) has been discussed in detail by Dougherty and Marryott (1991).

Artificial Neural Network as Simulator

The optimization process involves calling the simulator several thousands of times to verify the constraints. This involves a significant amount of computational time. The computational burden is generally high in all combined simulation-optimization models and more so with heuristic methods, and hence there is a need to reduce this computational time. This is largely achieved in this study by replacing the SHARP model with trained neural networks.

ANN is discussed in detail in ASCE Task Committee (2000), Aly and Peralta (1999), and the ANN toolbox of *MATLAB* (2000). In this study a feedforward, error back-propagation network is used wherein the goal of ANN is to establish a relation of the form

$$(Y^m)=f(X^n) \tag{11}$$

where $X^n=n$-dimensional input vector consisting of $x_1,x_2,\ldots,x_n$; $Y^m=m$-dimensional output or target vector consisting of resulting variables of interest $y_1,y_2,\ldots,y_n$; and $f(.)$ $=$commonly used sigmoidal transfer function given by

$$f(t)=1/[1+\exp(-t)] \tag{12}$$

The network is trained to determine the weights and biases so as to minimize the error function given by

$$E=\sum_P \sum_p (y_i-t_i)^2 \tag{13}$$

where y_i=ANN output; t_i=desired output; p=number of output nodes; and P=number of training patterns or data sets.

Results and Discussions

The conjunctive use allocation model is discussed in three stages with the help of simple illustrative examples. The purpose here is to obtain a clear understanding of concept and methodology. The three stages are

1. Conjunctive use in space for the cost objective (one time period with several demand centers—Example 1);
2. Conjunctive use in time for the drawdown volume objective (one demand center with several time periods—Example 2); and
3. Conjunctive use in space and time for both cost and drawdown volume objectives (combination of 1 and 2 above—Example 3).

The conjunctive-use-in-space model with the cost objective primarily seeks to address the spatial variability in the availability of surface-water and groundwater resources and their resulting costs. The unit cost of surface water varies linearly in space and is based on transport and losses that increase with distance of the delivery point from the diversion point. The unit cost of groundwater, however, does not vary with space (except depth). The conjunctive-use-in-time model seeks to address the temporal fluctuations in monsoon rainfall input and consequent variations in surface-water availability and groundwater recharge. This model stores or conserves groundwater in the subsurface (or saturated) zone through injection during surplus periods and pumps during both periods to meet the demand in space and time. This is achieved by minimizing the drawdown volume objective. The conjunctive-use-in-space-and-time model seeks to combine both objectives that must be commonly addressed in any real system.

Conjunctive Use in Space

In Example 1, the simplified deltaic system shown in Fig. 1 is considered for one time period with three demand centers at A, B, and C. The availability and extent of fresh groundwater in deltaic systems may be limited from various considerations, depending on boundary conditions such as proximity to the sea, depth of interface, depth of drawdown (based on sustainable recharge), and aquifer properties. In this example the interface depth constraint applies only to wells closer to the coast.

The idealized unconfined aquifer system was discretized in the form of a delta (triangle), as shown in Fig. 4. The delta is assumed to be sloping toward the sea in a stepped manner, with ground elevations at A, B, and C being 10, 7, and 5 m, respectively, with respect to mean sea level. The boundary conditions encountered in near-real conditions are considered, as shown in Fig. 5. The river branches were assigned constant head boundary conditions, with head ranging from 7 to 0 m, decreasing linearly toward the sea. The sea boundary was similarly represented as a constant head boundary with zero elevation. Table 1 shows the aquifer properties used as input for the SHARP model. The initial groundwater levels were set at steady-state conditions.

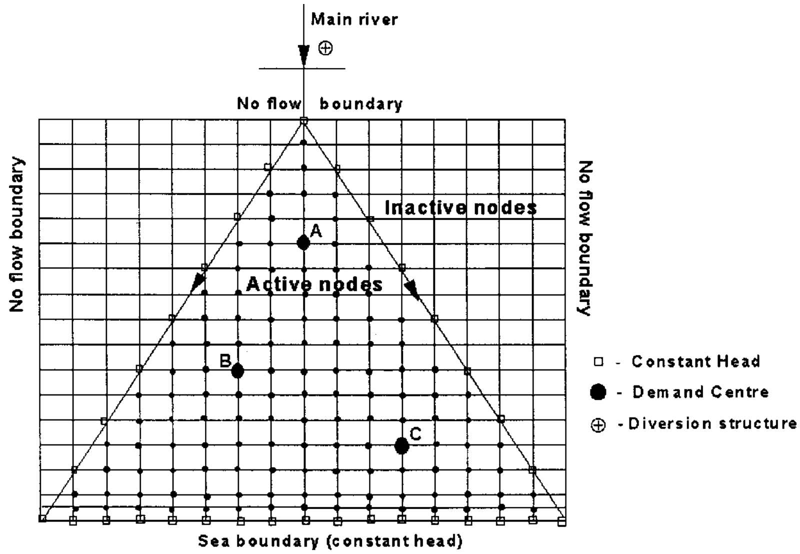

Fig. 5. Finite-difference grid for SHARP model

In this example, only three demand centers are considered, which implies three decision variables. Each decision variable was allowed to take 10 possible values (Table 2), resulting in 10^3 configurations. The supply and demand at the three points for one season (183 days) in terms of volume are shown in Figs. 6(a–c). The surface-water supply was restricted to 4.5 units ($1\text{ unit} = 10^6\text{ m}^3$); the demand at A, B, and C was assumed as 3.5, 2.5, and 2 units, respectively; and the depth of each well was assumed as 50 m (that is, location of screen) from ground level. The interface depth was constrained at an elevation of −60 m for the coastal node (that is, $Z_{\min}$ at location C), and the drawdown elevations ($H_{\min}$) at A, B, and C were restricted to 3, 2, and 1 m, respectively. Similarly, the upper bounds, that is, $H_{\max}$ from waterlogging considerations, were set at 9, 6, and 4 m, respectively. A minimum flow of 0.5 unit of surface water at each location was also imposed. The unit costs (in some monetary units) of surface water and groundwater are listed in Table 3.

The conjunctive-use-in-space model was implemented for the cost objective using the SA procedure presented in Fig. 3. As the problem is of small size (three decision variables), the optimal solution obtained by the proposed model could be verified directly by enumeration. The annealing parameters corresponding to initial temperature, reduction factor, chain length, and termination criterion were set at 5, 0.2, 300, and 4, respectively. The selection of annealing parameters is not of much significance for the small problem considered here, but for problems involving many decision variables, they need to be chosen judiciously, as discussed in later sections.

The global optimal minimum cost solution obtained using SA and SHARP and also from enumeration was 20.5 monetary units (MU) and is shown in Fig. 6(d). At A the model tries to allocate maximum surface water, since it is nearest to the surface-water source, where the relative cost of surface water is lower than groundwater. However, it restricts surface water to only 2 units and groundwater to 1.5 units, since groundwater availability is restricted by drawdown elevation constraint at B and C. At B the relative cost of surface water and groundwater enables the model to choose 1.5 units of surface water and 1 unit of groundwater at B. At location C the drawdown constraint forces the model to allocate one unit each for surface water and groundwater. The negative pumpages (injection), however, do not enter the optimal solution as they involve high cost. The negative pumpages and waterlogging constraint become relevant for the conjunctive-use-in-time model, where cost is not a consideration, as discussed in the next section.

The groundwater levels at A, B, and C at the beginning of simulation (that is, at the steady-state condition) were 5.02, 2.82, and 1.7 m, respectively, and the interface elevation at C for the steady-state condition was −68.1 m. The simulated water levels at the end of the time period, corresponding to the optimal groundwater pumpages, were 4.11, 2.36, and 1.20 m at A, B, and C, respectively, and also the interface depth at C was −67.6 m. It is important to note that at the coastal node C, while the groundwater level fell from 1.7 to 1.2 m (drawdown elevation constraint set at 1.0 m), the interface moved upward from −68.1 to −67.6 m (interface elevation constraint set at −60.0 m) for the optimal configuration. If, however, the drawdown elevation constraint is relaxed from 1.0 to 0.75 m, an improved optimal solution

Table 1. Aquifer Properties Used as Input to SHARP Flow Model (Example 1)

Parameter	Value
1. Area	120 km^2
2. Hydraulic conductivity	2.05E−04m/s
3. Specific storage of fresh/saltwater	1.0E−07/m
4. Porosity	0.3
5. Areal recharge (10% of mean rainfall)	5.7E−09 m/s[a]
6. Grid spacing (Δx)	1,000 m
7. Grid spacing (Δy except last two rows being 500 m)	1,000 m
8. Time step (Δt = one season = 6 months)	183 days
9. Specific gravity of seawater	1.025
10. Aquifer thickness	100 m

[a]Varies with mean rainfall for examples 2 and 3.

Table 2. Discrete Values of Groundwater (10^6 m^3) Decision Variables (Extraction/Injection in Volume)

Starting value	Step	Ending value	Number of values
−2.0	0.5	2.5	10

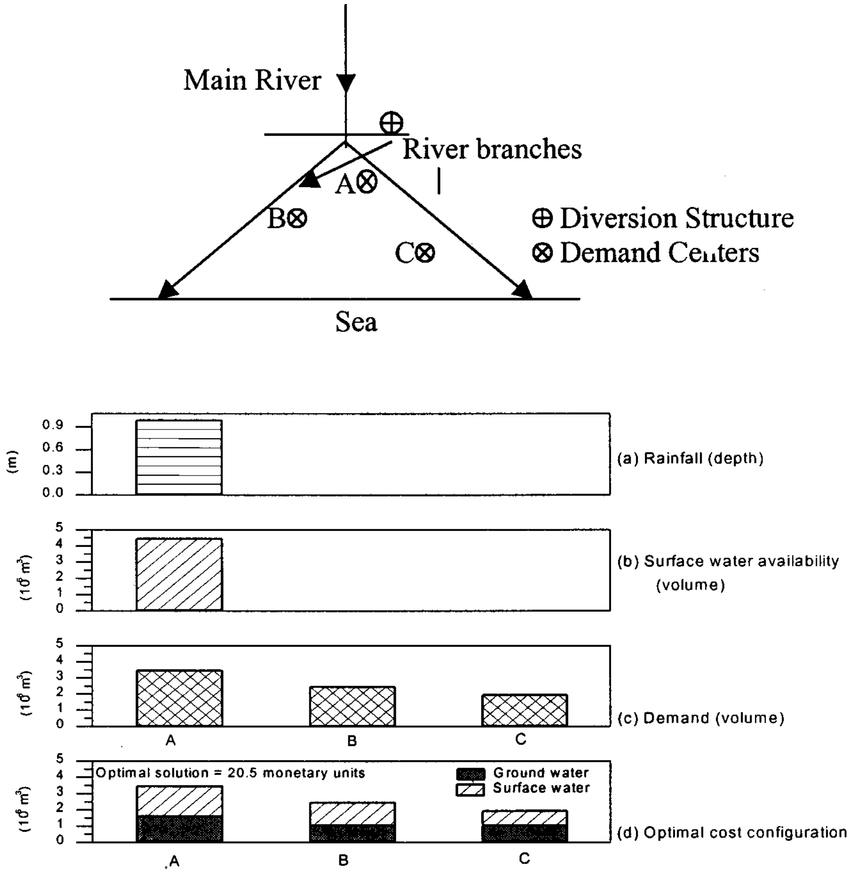

Fig. 6. Conjunctive use in space for hypothetical delta

(19.4 MU) was obtained with an increase in groundwater pumping at C (that is, 1.5 units instead of 1.0 unit). For this solution the drawdown elevation and interface elevation at C were computed as 0.8 and −67.3 m, respectively. Thus the optimal solutions in the present case are limited by the drawdown elevation constraint rather than the interface elevation constraint. In general, interface responses are slow, relative to head.

Conjunctive Use in Time

In Example 2, the simplified deltaic aquifer system discussed in the previous section is considered with one demand center. Under conditions typical of the Indian subcontinent, the monsoon rainfall is not dependable and often results in surplus (flood) or deficit (drought) conditions. The conjunctive-use-in-time model is primarily intended to address this temporal fluctuation in rainfall input and the resulting surface-water flows and groundwater recharge. A uniform rainfall recharge was assumed (10% of rainfall) over the study area during the monsoon season. It is assumed that no rainfall occurs during the nonmonsoon season and hence there is no recharge; however, surface water is available due to base flow.

In Fig. 7, the diversion structure feeds the delta region with surface-water sources. The region is assumed to have one demand center located at some point B within the delta, and the groundwater is assumed to be injected/discharged from the well located at B. While a number of scenarios could be possibly imagined, a typical case of two normal years followed by a flood year and a drought year is considered. The surface-water availability, rainfall, and constant demand at B in terms of volume for each season are shown in Figs. 7(a–c). Each season was assumed to be of 6 months duration, consistent with the two cropping seasons (Kharif and Rabi seasons) practiced during monsoon and nonmonsoon seasons, respectively, in the coastal deltas. The planning horizon was assumed to be of 4 years duration with a time step of 6 months (that is, eight time periods). From Fig. 7, it is clear that surface water alone cannot meet the demand, especially during the nonmonsoon season. Groundwater must supplement the demand. Further, during the fourth year (drought period with low rainfall and low recharge), when surface water cannot meet the demand, there is a possibility that even groundwater may not be able to meet the demand without being depleted beyond sustainable levels.

Table 3. Unit Costs of Surface Water and Groundwater (in Monetary Units)

Location	Unit cost	Depth range (m)
Surface water		
A	1.0	—
B	1.5	—
C	2.0	—
Groundwater[a]		
—	1.10	2–3
—	1.15	3–4
—	1.20	4–5
—	1.25	5–6
—	1.30	6–7

[a]Unit cost of injection: 5.0 (independent of location and depth).

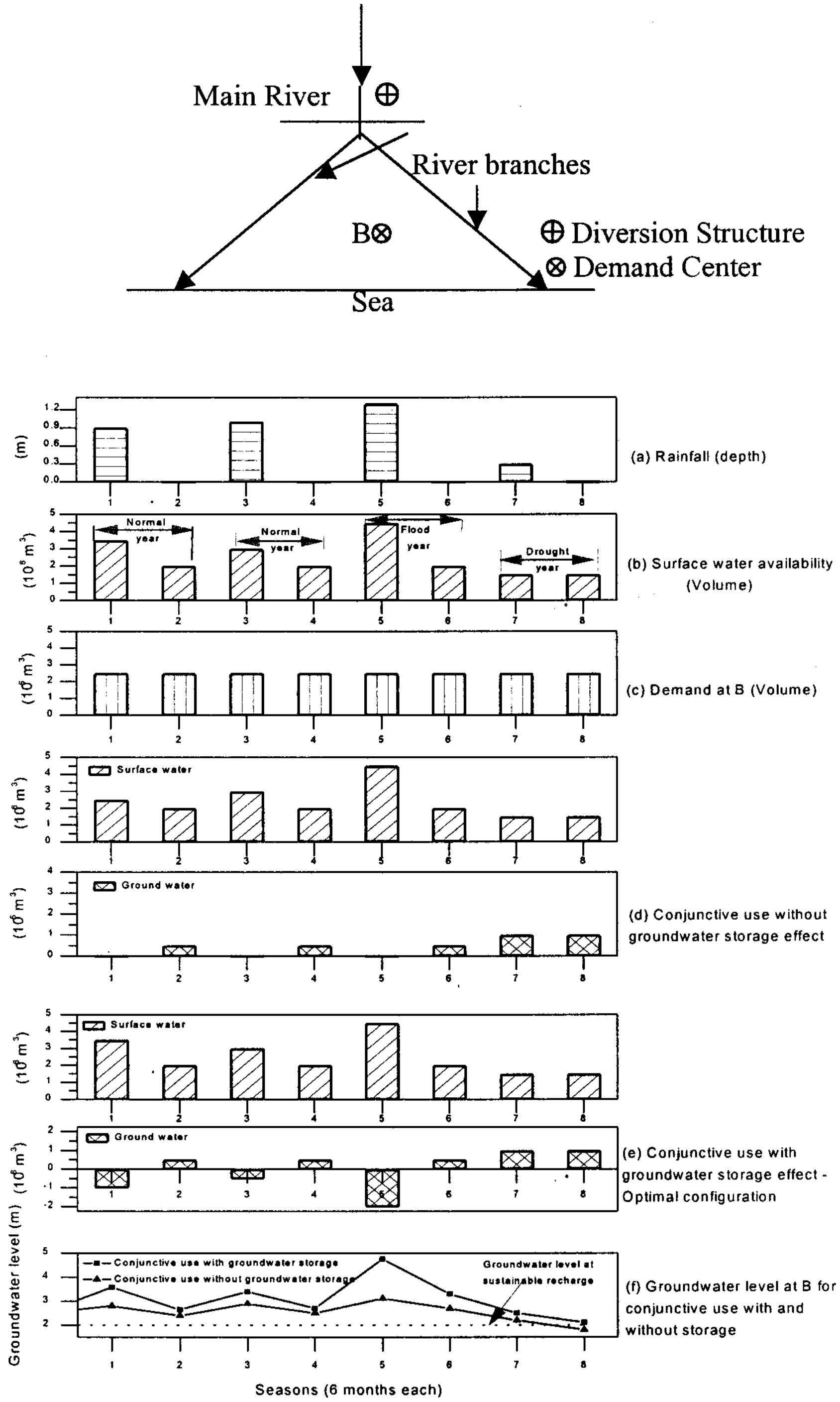

Fig. 7. Conjunctive use in time for hypothetical delta

The groundwater was allowed to take 10 discrete values, as in the previous example (Table 2). The drawdown, water logging, and interface depths were constrained at elevations of 1.75, 7, and -60 m with respect to mean sea level, respectively. A minimum flow of 1 unit of surface water was imposed during each time period. The conjunctive-use-in-time model involves eight decision variables (eight variables of groundwater at demand point B) for the eight time periods, and hence there are a total of 10^8 configurations. The objective function here is to minimize drawdown volume alone. This problem cannot be solved by enumeration. Therefore, the SA algorithm that embeds the SHARP model as shown in Fig. 3 was implemented. The annealing parameters were set based on guidelines suggested by Dougherty and Marryott (1991), Press et al. (1996), and Cunha (1999). The initial temperature (10^8) was set such that more than 80% of the configurations were accepted. It was assumed that equilibrium was achieved when no improvement in solution was found over 800 (100 times the number of decision variables) iterations (or chains) at any given temperature. The temperature was reduced with a cooling ratio equal to 0.10. It was assumed that the termination criterion was met if four successive temperature reductions did not yield any improvement in the solution. The solution took nearly 90 min of CPU time on a microcomputer.

Example 2 was designed such that the optimal solution was obvious and could be determined intuitively, and therefore the performance of the proposed SA-based optimization model could be evaluated. The optimal solution could be obtained in this case by inspecting the input data of Fig. 7 and Table 2. The trial solution is found by fully utilizing available surface water during each time period and maximizing injection while ensuring that supply-demand constraints are satisfied for each time period. The constraints pertaining to state variables for the trial configuration were verified separately by executing SHARP model independently and computing the drawdown volume. The resulting intuitive optimal solution is shown in Fig. 7(e). However, the solution obtained using the SA algorithm (Fig. 3), as discussed in the previous paragraph, was suboptimal with respect to the intuitive optimal solution. The annealing parameters were modified by trial and error, with limited success. The only option left was to increase the chain length and termination criteria. However, this substantially increases the computational burden. Therefore, the SHARP model was replaced with an ANN to facilitate longer chain lengths for obtaining improved solutions.

A feed-forward, three-layer network was trained using SHARP input/output data sets. The feed-forward network with the back-propagation algorithm consisted of three layers (input, hidden, and output), as implemented in *MATLAB* (2000). The network utilizes a sigmoidal transfer function and a pure linear function. The input (decision variables) included pumpage/injection for each time period at the demand center B, and the output included the corresponding SHARP responses in terms of freshwater head and interface depth at B and the sum of drawdown volumes for the delta at the end of the eighth (last) time period. The drawdown volume was computed for each grid cell as the volume of cell above freshwater head up to ground level. For this purpose, ground level for all cells was assumed to be 10.0 m for the entire delta.

For training, nearly 2,000 feasible input/output data sets (patterns) were randomly generated using SA/SHARP code (Fig. 3) for each time period under relaxed constraints pertaining to the state variables. The relaxation was to ensure that ANN was trained over a wide range of inputs/outputs. Random integers from 1 to 10 were generated using a library function. This integer was linked to the index level of discrete variables listed in Table 2 to assign pumping/injection at the demand center B for each time period. Of the 2,000 data sets, 1,500 were used for calibration and 500 for validation. The data sets were normalized and trained with a back-propagation algorithm as implemented in *MATLAB*.

A typical architecture (supervised training) of network training for the fourth time period is shown in Fig. 8. The network has four inputs (corresponding to injections/pumpages at the end of the fourth time period at B) in the first layer, six neurons in the second hidden layer, and two outputs (corresponding to head and interface depth and at demand center B) in the third output layer. The goal was to minimize the sum-of-square errors as discussed before. Thus, ANN weights and biases for eight training sets for the eight time periods were obtained. A small subroutine was coded to replace the SHARP model using the ANN weights and biases. The subroutine involves only matrix multiplication to compute SHARP responses at B and hence requires very little computational time.

The number of data sets or patterns required for training is generally important for ANN modeling and must be kept to a minimum. The goodness of fit (R^2) must be high and time consumed for training should be minimal. Unlike observed data that are prone to all types of errors (such as instrumental errors, measurement errors, etc.), the output responses to be modeled by the ANN here are generated by a physically based simulation model. As such, any number of data sets could be generated. In view of the above, the ANN mimics the SHARP model very well. This is evident from the goodness of fit for calibration (1,500 sets) and validation (500 sets), as shown in Fig. 9. The ANN takes less than 15 min of time (on a micro PC) for each of the 8 training sets and in most cases converges in less than 25 epochs.

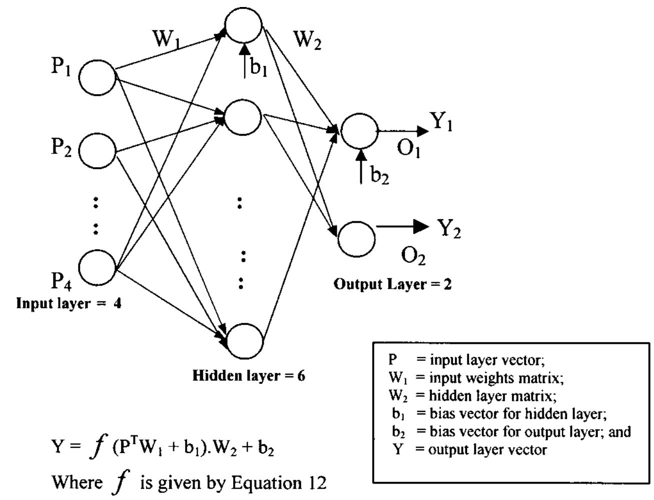

Fig. 8. Architecture of three-layer feed-forward network of an artificial neural network

With the ANN as the simulator, the computational time of simulation was very small and improved solutions were obtained when compared to those using the SHARP simulator. The optimal solution was also consistent with the intuitive solution discussed previously and shown in Fig. 7(e). The optimal solution with the ANN as simulator with very long chain lengths (200 times the number of decision variables, that is, 1,600) takes only a few minutes on a microcomputer compared to 90 min with the SHARP simulator. The evolution of the model solution for the SA procedure with the ANN as simulator (the same annealing parameter values were used as discussed earlier) is shown in Figs. 10 and 11.

The results of the conjunctive-use-in-time model help in comparing two scenarios. The first scenario is the one mostly in practice today, wherein surface water is utilized to its maximum extent, while groundwater is used only when the demand is not met from surface-water sources. Although this practice is also con-

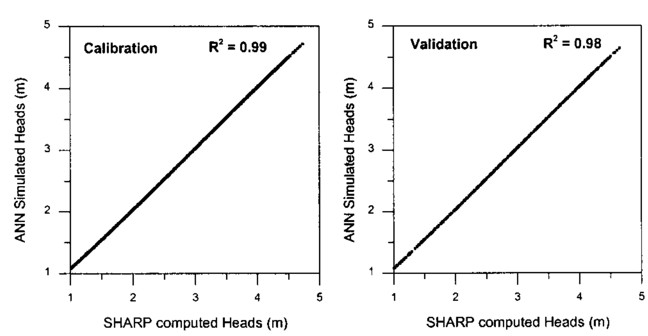

Fig. 9. Goodness of fit for typical data set using an artificial neural network (ANN) simulator

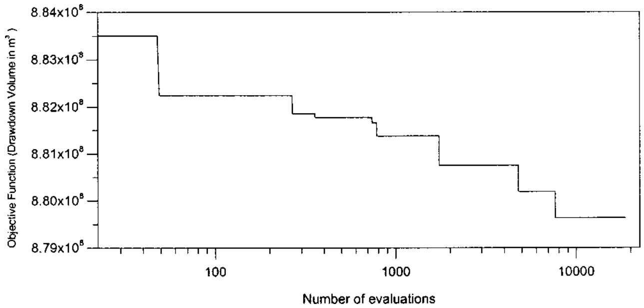

Fig. 10. Evolution of model solution

junctive use of surface water and groundwater, it involves wasting of surface water during surplus periods. The second scenario is the optimal solution obtained from the present model, which takes advantage of groundwater storage during surplus flows through injection and conjunctively allocates surface water and groundwater. The two solutions are shown in Figs. 7(d and e) for the purpose of comparison.

The two scenarios of conjunctive use with and without groundwater storage as shown in Fig. 7(f) must be viewed in the context of sustainability. The groundwater levels at demand center B for conjunctive use without groundwater storage fall below permissible levels of sustainable recharge (arbitrarily assumed here as 2.0 m) by the end of the planning horizon and normally should not be acceptable. In other words, no feasible solution is possible without depleting groundwater storage below acceptable levels. On the other hand, groundwater levels for conjunctive use with groundwater storage as proposed in the present model ensure that all constraints are met in space and time on a sustainable basis [Fig. 7(f)].

The model results are sensitive to the aquifer parameters. Modification of the aquifer parameters affects head and interface depth (state variables) and hence the feasible domain. Numerical experimentation has shown that the drawdown volume objective decreases with an increase in porosity and with a decrease in hydraulic conductivity. Therefore the proposed methodology is suitable to coastal deltas of east India that exhibit silty-clayey soils (with high porosity and low conductivity) such that groundwater is conserved during surplus periods and utilized later during drought periods. Further, the methodology could be formulated with shorter time steps (for example, 4 months) to raise a third crop during the year. This, however, could be more meaningful for sandy soils where storing water underground becomes more difficult because stored groundwater may drain into the sea anyway. In other words the planning horizon for conjunctive use in time must be consistent with aquifer properties.

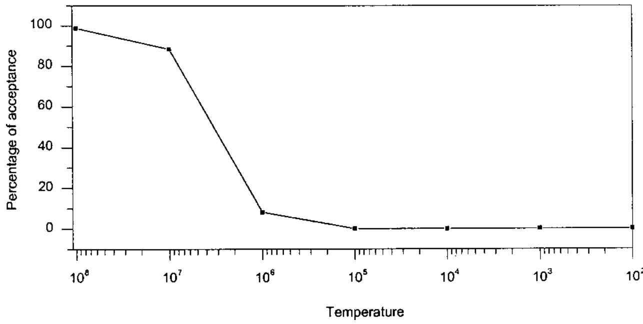

Fig. 11. Evolution of percentage of acceptance with temperature

Conjunctive Use in Space and Time

In Example 3, the real problems invariably come under the domain of conjunctive use in space and time. The space-time model has two objectives: To minimize the cost as well as to obtain the drawdown volume objective. In this case, as the number of variables and constraints is much more, resulting in a phenomenal increase in computational time. Thus the SA algorithm using the SHARP simulator is not practical unless the equilibrium and termination criterion are kept very small, which may lead to suboptimal solutions. The input data set is a combination of the previous two examples and is shown in Figs. 12(a–c). The range of discrete variables also remains the same as in Table 2. The demand center locations, constraints, and cost coefficients of Example 1, and the temporal variations in rainfall and the resulting surface flows during eight time periods and groundwater recharge, as in Example 2, were adopted. Thus, for illustration of the space-time model, three demand centers and eight time periods were considered (combination of examples 1 and 2). The demand was varied in space but was kept constant with respect to time, while a minimum surface-water flow of 0.5 units was ensured at each location and time step.

Considering the computational burden involved, the SHARP inputs/outputs for nearly 5,000 feasible configurations were used for ANN training using *MATLAB* for the eight time periods, on lines similar to those discussed in the previous section. The input included pumpage/injection for each time period. The output included SHARP responses in terms of heads (at locations A, B, and C), interface depth (at location C), and drawdown volume for the delta at the end of the eighth time period. Thus eight sets of ANN weights and biases were obtained corresponding to eight time periods. The ANN simulations stood justified with high correlations (0.98–0.99) and R^2 (0.96–0.98) values for calibration for all eight data sets.

The SA algorithm incorporated an ANN as the simulator to determine the optimal configurations, and the annealing parameters were set as in the previous section. Since the space-time model involves two objectives, one of them was imposed as an additional constraint such that the results could be interpreted from the tradeoff curve. Several optimal solutions were obtained by minimizing the drawdown volume objective while imposing the cost objective as a constraint. Arriving at a tradeoff curve involves stringent constraints. The tradeoff curve for this case is presented in Fig. 12(f). The midpoint of the tradeoff curve is most difficult to optimize and takes 2.5 h of CPU time on the microcomputer with a chain length set at 1,200 iterations (that is, 50 times the number of decision variables) and termination criteria at 4. The end points take only a few minutes, even with longer chain lengths of 2,400, since only one objective is optimized. The other points take intermediate computational time.

Point X on the tradeoff curve [Fig. 12(f)] corresponds to the minimum drawdown volume objective, unconstrained by the cost objective. The optimal configuration for surface and groundwater allocations corresponding to point X are shown in Figs. 12(d and e). Similarly, point Y was optimized for the cost objective, unconstrained by the drawdown objective. For each time period the optimal allocation of surface and groundwater for points X and Y are also listed in Table 4.

The explanation for point X in Fig. 12(f) and Table 4 is as follows. The surface-water allocation [Fig. 12(d)] during the first

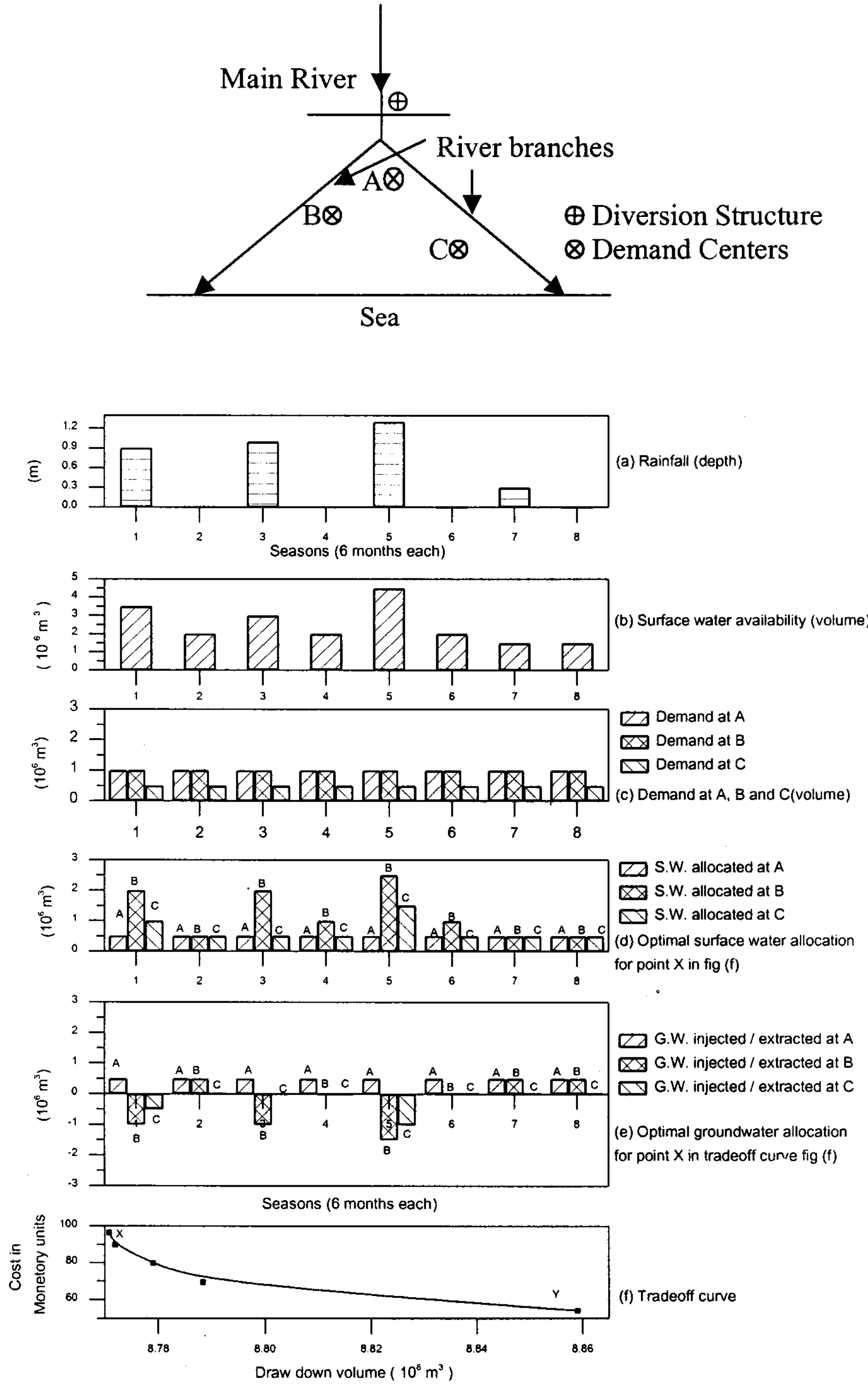

Fig. 12. Conjunctive use in space and time for hypothetical delta

time period (or season) at A, B, and C locations is 0.5, 2.0, and 1.0 unit, respectively. For the same time period the groundwater extraction (or injection shown with a negative sign) is 0.5, −1.0, and −0.5 units, respectively [Fig. 12(e)]. Thus the demand, supply, and minimum flow constraints at the three locations are satisfied during the period. Similarly the constraints are satisfied for the remaining time periods. The SHARP constraints pertaining to state variables were also verified independently in space and time for the optimal configuration.

At *X*, the management model tries to fully utilize surface water during all time periods and inject surplus surface water to the groundwater reservoir that could be used at later time periods. In other words, the model at *X* maximizes groundwater reserves as in Example 2 (conjunctive use in time) but results in high cost involving groundwater injection. On the other hand, point *Y* (as evident from Table 4) corresponds to the minimum cost solution, as emphasized in Example 1 (conjunctive use in space) but involves wastage of available surplus surface water during the 1st, 3rd, and 5th time periods. The relative cost of surface and groundwater fixes the optimal cost configuration with little or no room for injection involving high cost. Table 4 shows the minimum cost configuration (for point *Y*) with no injection to the groundwater reservoir. Thus the tradeoff curve enables the decision maker to prioritize be-

Table 4. Optimal Allocation of Surface and Groundwater in Volume (Million Cubic Meters)—Conjunctive-Use-in-Space-and-Time Model

Time periods (seasons)	1			2			3			4		
Surface water availability		3.5			2.0			3.0			2.0	
Location	A	B	C	A	B	C	A	B	C	A	B	C
Minimum drawdown volume objective (corresponding to point *X* on trade off curve)												
Demand	1.0	1.0	0.5	1.0	1.0	0.5	1.0	1.0	0.5	1.0	1.0	0.5
Surface water	0.50	2.00	1.00	0.50	0.50	0.50	0.50	2.00	0.50	0.50	1.00	0.50
Groundwater extraction	0.50	—	—	0.50	0.50	0.00	0.50	—	0.00	0.50	0.00	0.00
Groundwater injection	—	1.00	0.50	—	—	—	—	1.00	—	—	—	—
Minimum cost objective (corresponding to point *Y* on trade-off curve)												
Surface water	0.50	1.00	0.50	1.00	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
Groundwater extraction	0.50	0.00	0.00	0.00	0.50	0.00	0.50	0.50	0.00	0.50	0.50	0.00

Time periods (seasons)	5			6			7			8		
Surface water availability		4.5			2.0			1.5			1.5	
Location	A	B	C	A	B	C	A	B	C	A	B	C
Minimum drawdown volume objective (corresponding to point *X* on trade off curve)												
Demand	1.0	1.0	0.5	1.0	1.0	0.5	1.0	1.0	0.5	1.0	1.0	0.5
Surface water	0.50	2.50	1.50	0.50	1.00	0.50	0.50	0.50	0.50	0.50	0.50	0.50
Groundwater extraction	0.50	—	—	0.50	0.00	0.00	0.50	0.50	0.00	0.50	0.50	0.00
Groundwater injection	—	1.50	1.00	—	—	—	—	—	—	—	—	—
Minimum cost objective (corresponding to point *Y* on trade-off curve)												
Surface water	1.00	0.50	0.50	1.00	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
Groundwater extraction	0.00	0.50	0.00	0.00	0.50	0.00	0.50	0.50	0.00	0.50	0.50	0.00

tween cost and conservation of groundwater (drawdown volume) for the simple case of three demand centers and eight time periods.

Computational Time Reduction

Although enumeration or brute-force methods guarantee global optimal solutions for nonconvex combinatorial problems, this is not practically possible with presently available computing power. The SA procedure with SHARP as the simulator introduces a computational time burden that has two distinct components. The first component is due to the time consumed by the simulator (function calls). This can be reduced to near zero with the ANN as simulator. The second component is the average time consumed for generating a feasible configuration until equilibrium and termination criteria are met. This is significantly reduced through efficient algorithmic guidance in two stages: (1) A procedure for generating a configuration that is always feasible; and (2) a perturbation procedure called genetic rearrangement (or excursion limiting). At the initial temperature the number of iterations is large, mainly due to infeasible solutions, while at the final temperature the uphill moves are too many. The sum of the first two components put together and multiplied by the total number of iterations (or chains) determines the total CPU time. The total number of iterations depends on the problem tackled and on the annealing parameters. Therefore, the CPU time is known only after the actual model run.

It is clear that the ANN can only reduce the computational burden arising from the first component but can do little in respect of the second, besides thermal equilibrium and termination criteria inherent to the SA procedure. Therefore, the proposed methodology has no limitation in terms of the areal extent of deltas, since the ANN virtually reduces the time taken by the simulator to near zero. However, the restriction is imposed in terms of the number of decision variables and constraints implied in all large delta systems.

The writers' experience in the course of developing the model showed that a shorter chain length of about 20 to 30 times the number of decision variables was generally adequate. The improvement in the solution thereafter was mostly marginal; Dougherty and Marryott (1991) also mentioned this in their paper. However, for the third example (conjunctive use in space and time), chain lengths of 100 times the number of decision variables were adopted in arriving at near-optimal solutions in the present study, except for the midpoint of the tradeoff curve where the chain length was limited to 50 times the number of decision variables (due to stringent constraints). The role of the perturbation procedure in most cases was found to be relatively small in terms of computational time and marginal in terms of objective function improvement.

While the ANN reduces the computational burden in terms of time and facilitates longer chain lengths and tighter constraints, it reduces the overall efficiency (with respect to global optimum) of the model. This is a result of the fact that the ANN mimics the SHARP model imperfectly, which in turn also mimics the real physical system. Although the simulations by the ANN are very good, the reproduction cannot be exactly the same, resulting in a slightly altered feasible domain that may or may not contain the optimal solution obtained with SHARP as the simulator. Johnson and Rogers (2000) have, however, concluded that the ANN virtually replaces the full model. This is indeed true only within the range of input values for which the ANN is trained, but not otherwise as an extrapolator (ASCE 2000). In general, there will be a successive dilution in the optimal solution with respect to the true global optimum. This is due both to the SA procedure itself, which provides only for near-optimal solutions, and to the ANN. Nevertheless, the methodology is justified in the present study for a macro level planning and operational model using seasonal time steps.

Summary and Conclusions

A simple conceptual conjunctive use planning model for allocation of surface water and groundwater to meet consumptive demands in deltaic regions is presented. The model is intended for macro level planning of deltaic regions using seasonal time steps. The model seeks to address spatial and temporal variability in the availability of useful water resources. The allocation model is cast as a combinatorial problem for a simplified, unconfined, hypothetical deltaic aquifer system. The nonlinear, nonconvex problem is solved using the SA algorithm. A sharp interface model suitable to a coastal and deltaic environment is used to simulate the aquifer response. Although the framework of the proposed model is general, the performance of the model is illustrated using idealized examples with some reference to east coastal deltas of India. Also, the methodology is demonstrated for a single-layer aquifer system. However, it can be easily extended to multiaquifer systems already included in the SHARP model.

The computational burden is managed within practical timeframes by replacing the flow simulator with an ANN and using efficient algorithmic guidance. Since no method exists that guarantees global optimality, SA with an ANN should be useful for real problems of modest size (possibly 30 to 40 decision variables) in arriving at high-quality, practical, and near-optimal solutions on a microcomputer. However, for problems of larger size involving hundreds of variables, faster parallel processors may be required.

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