

Complex system approach to investigate and mitigate thermoacoustic instability in turbulent combustors

Cite as: Phys. Fluids **32**, 061401 (2020); <https://doi.org/10.1063/5.0003702>

Submitted: 04 February 2020 . Accepted: 26 May 2020 . Published Online: 17 June 2020

 R. I. Sujith, and  Vishnu R. Unni



View Online



Export Citation



CrossMark

ARTICLES YOU MAY BE INTERESTED IN

[Coupled interaction between unsteady flame dynamics and acoustic field in a turbulent combustor](#)

Chaos: An Interdisciplinary Journal of Nonlinear Science **28**, 113111 (2018); <https://doi.org/10.1063/1.5052210>

[Thermoacoustic instability in a Rijke tube: Non-normality and nonlinearity](#)

Physics of Fluids **20**, 044103 (2008); <https://doi.org/10.1063/1.2895634>

[Synchronization transition from chaos to limit cycle oscillations when a locally coupled chaotic oscillator grid is coupled globally to another chaotic oscillator](#)

Chaos: An Interdisciplinary Journal of Nonlinear Science **30**, 033121 (2020); <https://doi.org/10.1063/1.5134821>

Physics of Fluids

SPECIAL TOPIC: Flow and Acoustics of Unmanned Vehicles

Submit Today!



Complex system approach to investigate and mitigate thermoacoustic instability in turbulent combustors

Cite as: *Phys. Fluids* **32**, 061401 (2020); doi: [10.1063/5.0003702](https://doi.org/10.1063/5.0003702)

Submitted: 4 February 2020 • Accepted: 26 May 2020 •

Published Online: 17 June 2020



R. I. Sujith^{1,a)}  and Vishnu R. Unni² 

AFFILIATIONS

¹Department of Aerospace Engineering, Indian Institute of Technology Madras, Chennai 600036, India

²Department of Mechanical and Aerospace Engineering, University of California San Diego, San Diego, California 92093-0411, USA

^{a)} Author to whom correspondence should be addressed: sujith@iitm.ac.in

ABSTRACT

Thermoacoustic instability in turbulent combustors is a nonlinear phenomenon resulting from the interaction between acoustics, hydrodynamics, and the unsteady flame. Over the years, there have been many attempts toward understanding, prognosis, and mitigation of thermoacoustic instabilities. Traditionally, a linear framework has been used to study thermoacoustic instability. In recent times, researchers have been focusing on the nonlinear dynamics related to the onset of thermoacoustic instability. In this context, the thermoacoustic system in a turbulent combustor is viewed as a complex system, and the dynamics exhibited by the system is perceived as emergent behaviors of this complex system. In this paper, we discuss these recent developments and their contributions toward the understanding of this complex phenomenon. Furthermore, we discuss various prognosis and mitigation strategies for thermoacoustic instability based on complex system theory.

Published under license by AIP Publishing. <https://doi.org/10.1063/5.0003702>

I. INTRODUCTION

Thermoacoustic instability, also known as combustion instability, refers to the occurrence of ruinously large amplitude acoustic oscillations resulting from a positive feedback between the heat release rate fluctuations from the combustion process and the acoustic field of the confinement. Rocket engines and gas turbine engines used for propulsion and power generation are particularly susceptible to thermoacoustic instability due to high energy densities in these systems; even a small fraction of this energy, when converted to acoustic oscillations, can sustain large amplitude pressure oscillations.^{1,2}

The consequences of thermoacoustic instabilities are often catastrophic. Thermoacoustic instability can lead to thrust oscillations, which can seriously compromise space missions. Thermoacoustic instability can result in large amplitude vibration of the engine and can lead to structural failure—either immediate or due to fatigue, and wear and tear. The increased heat transfer due to the large amplitude acoustic oscillations can overwhelm the thermal

protection system. Furthermore, electronic components in the control system or the payload can fail due to the high level of acoustic ambience. Many instances of thermoacoustic instability have been documented, including those in the Apollo space mission,³ cold war missiles,⁴ the soviet efforts in rocketry,⁵ and more recently in the gas turbine industry.⁶ Despite nearly half a century of research, thermoacoustic instability remains a nightmare for engineers.

A century and a half ago, Lord Rayleigh postulated that when heat is added in phase with unsteady pressure, sound is generated. When the amount of acoustic energy generated is more than the damping in the system, the oscillations grow in amplitude until the nonlinearities in the system cause the amplitude of oscillations to saturate.⁷ This positive feedback between the heat release rate and the acoustic field can result from a number of mechanisms, such as equivalence ratio fluctuations,⁸ flame surface area modulations,⁹ modulations in the mixing field,⁶ coherent structures in the flow,⁹ droplet and spray dynamics, and the resulting unsteady evaporation and entropy fluctuations.¹⁰ On the other hand, viscous and thermal losses, transfer of energy to vorticity fluctuations, radiation from

the combustor exit or nozzle, and wall vibrations contribute to the damping in the system.^{6,11}

Avoiding thermoacoustic instability requires us to establish the stability boundaries of the system. This is done by conducting acoustic pressure and acceleration measurements by performing elaborate testing before the hardware is transferred to the customer. For modeling, thermoacoustic systems have traditionally been treated as an acoustic system driven by combustion. The stability of the system is assessed by performing a linear stability analysis and examining the eigenvalues of the system.⁶ Although useful for determining the linear stability boundaries, linear stability analysis cannot be used to study how a system transitions through the stability boundaries or its dynamics beyond them. This is because such dynamics are controlled by the nonlinearities in the acoustic sub-system.

In thermoacoustic systems, both the acoustic sub-system and the unsteady flame can exhibit nonlinearities. However, in most thermoacoustic systems, where the acoustic pressure fluctuations are low compared to the mean pressure of the working fluid, such as in gas turbine combustors, the acoustics can be considered to be linear (this is not true in the case of rockets), whereas the flame response to acoustic perturbations is highly nonlinear. Consequently, while modeling sound waves is well established and is easy to implement, modeling the response of flames to acoustic waves, particularly in turbulent flows, is not as straightforward. The current methodology is to perturb the flame using velocity fluctuations and determine the flame transfer function (ratio between the heat release rate and the acoustic velocity fluctuations, normalized appropriately) experimentally, or numerically from computational fluid dynamics (CFD) simulations [e.g., large eddy simulations (LES)].¹² In summary, for the past several decades, it has been conventional to treat thermoacoustic systems as acoustic systems driven by combustion. However, this is not the only way to analyze thermoacoustic systems.

An alternate approach to studying thermoacoustic instability based on nonlinear dynamics (alternatively known as dynamical system theory) and complex system theory has emerged in recent years.

A. Dynamic behavior of thermoacoustic systems

Thermoacoustic systems are inherently nonlinear. Hence, studying a thermoacoustic system in the framework of nonlinear dynamics can offer new insights into its nonlinear behavior.

Let us examine the stability of small perturbations close to the onset of thermoacoustic instability. We see that at a parameter value prior to the onset of thermoacoustic instability, small perturbations in the system decay. Post the onset of thermoacoustic instability, small perturbations grow. In other words, the growth rate changes sign from negative to positive. Thus, we see a qualitative change in the behavior of the thermoacoustic system; such a qualitative change occurring when a parameter is changed smoothly is termed a bifurcation.¹³ The specific bifurcation that we see here is referred to as an Andronov–Hopf bifurcation, or simply Hopf bifurcation, in the language of dynamical system theory.¹⁴ The parameter value at which there is a change in the dynamic behavior of the system is called the Hopf point.

There are two types of Hopf bifurcations: supercritical and subcritical.¹³ In supercritical bifurcation, crossing the Hopf point results

in the occurrence of low amplitude stable limit cycle oscillations. The amplitudes grow slowly as the control parameter is varied. Furthermore, reversing the change in the control parameter causes the oscillations to disappear at the Hopf point. In contrast, for a subcritical Hopf bifurcation, crossing the Hopf point leads to a jump in the amplitude; large-amplitude oscillatory solutions are present right at the onset of instability. Furthermore, in the reverse path, the control parameter needs to be varied significantly past the Hopf point to bring the system back to a stable non-oscillatory state; this point where the system attains stability is called the fold point (also known as the saddle point).¹³ Thus, a subcritical bifurcation exhibits hysteresis.

In this hysteresis zone, the system is bistable. That is, two stable asymptotic states are possible: a stable fixed point and a stable limit cycle. The attained state depends on the magnitude and type of initial perturbation. If the initial perturbation is within the basin of attraction of the fixed point, the perturbation dies down; in contrast, if the initial perturbation is within the basin of attraction of the limit cycle, the perturbation evolves to a limit cycle. This phenomenon is referred to by the thermoacoustic community as triggering.^{1,2,15–17} Classical linear stability analysis cannot deal with bistability or indeed even predict the supercritical or subcritical nature of a Hopf bifurcation because it depends on the order and type of nonlinearity in the system.

B. Beyond limit cycles: Quasiperiodicity, intermittency, and chaos

The general view of the community until recently was that the asymptotic states of a thermoacoustic system were either fixed points or limit cycles. Indeed, pressure oscillations more complex than limit cycles have been reported in the context of thermoacoustic instability;^{18–22} however, this idea did not get traction until recently.

Using nonlinear time series analysis, Kabiraj *et al.*^{23–25} systematically characterized bifurcations in laminar thermoacoustic systems with simple ducted laminar flames. They showed that after attaining limit cycle oscillations, thermoacoustic systems can undergo further bifurcations and attain states, such as quasiperiodicity, frequency locked, period- n , intermittency, and chaos, even in such simple laminar thermoacoustic systems. They established the quasiperiodic²⁴ and frequency-locking quasiperiodic route to chaos.²⁵ Recently, in the same system, Premraj *et al.*²⁶ observed strange nonchaos. Kashinath *et al.*²⁷ observed similar routes to thermoacoustic instability in numerical experiments of a G -equation flame coupled with duct acoustics. Recently, Guan *et al.*²⁸ discovered an alternate route to chaos through intermittency in a laminar thermoacoustic system.

Thus, we see that even a simple, deterministic, thermoacoustic system, such as a ducted laminar premixed flame, can give rise to complexity arising from chaotic behavior. Next, we examine complexity in turbulent reacting flow systems.

C. Complex dynamics of thermoacoustic systems in turbulent combustors

Turbulence introduces lot more complexity to a thermoacoustic system, such as a wide range of scales and a large number of degrees of freedom. In addition, a thermoacoustic system has other

complexities, such as chemical kinetics, molecular mixing, and interaction with acoustic waves. Thermoacoustic instability in turbulent combustors displays many attributes that cannot be described by the classical view of it being a Hopf bifurcation. These attributes, which belong to a system that is indeed complex, are described in the following paragraphs.

During stable operation, thermoacoustic systems are characterized by low amplitude aperiodic fluctuations, traditionally, referred to as “combustion noise.” Recent studies have shown that combustion noise displays scale invariance and have a multifractal signature, confirming the absence of a single characteristic spatial or temporal scale in thermoacoustic systems during stable operation.²⁹ Multifractality reflects the complex nature of the dynamics that results from the highly nonlinear interaction between combustion, flow, and duct acoustics, whereas periodic oscillations during thermoacoustic instability are characterized by dominant time and length scales. This results in the loss of multifractality at the onset of thermoacoustic instability.^{29,30}

Traditionally, the stable operation of a thermoacoustic system was treated as a fixed point.^{10,14} However, as mentioned before, a system involving turbulent reacting flow is never really “silent.” Often, these fluctuations are treated as stochastic fluctuations.^{31–33} Tony *et al.*³⁴ recently characterized these low amplitude aperiodic pressure fluctuations occurring during the stable operation of a thermoacoustic system as deterministic and noted that the aperiodic oscillations correspond to high dimensional chaos, corrupted by white and colored noise. In contrast, thermoacoustic instability is characterized by periodic fluctuations, i.e., order. Thermoacoustic instability is then the transition from chaos to order. This transition happens via a state of intermittency, a state that comprises intermittent bursts of periodic oscillations that appear randomly amid epochs of low amplitude aperiodic fluctuations.³⁵ In other words, we see pockets of order amid disorder.

During stable operation, the flow field is characterized by small vortices that lead to chaotic dynamics.^{36,37} Similar dynamics is seen during the aperiodic epochs of intermittency. In contrast, during the periodic epochs of intermittency, the reactive flow field gets self-organized by a collective interaction of the small scale vortices resulting in the periodic emergence of large scale structures and coherent patterns.³⁷ This self-organization and the resulting coherent structures are sustained during thermoacoustic instability, and significant acoustic power production occurs in these structures, leading to self-sustained thermoacoustic oscillations.

Thermoacoustic instability is thus a collective phenomenon. Interestingly, many of these features can be seen in other turbulent flow systems, such as aeroacoustic,³⁸ aeroelastic,³⁹ and compressor surge, that lead to the occurrence of oscillatory instabilities.⁴⁰ That is, these different systems with different physical mechanisms display similar dynamical behavior. Traditional reductionist approaches aim at studying a system by analyzing the constituent elements; such an approach will not be successful in explaining these similarities in fundamentally different physical systems.⁴¹

Traditional approaches deal with thermoacoustic instability as a transition from fixed point to limit cycle and fail to account for the transition from chaos to limit cycle via the state of intermittency. Often, the system is studied by forcing using externally imposed oscillations. This approach emphasizes the flame dynamics

and ignores the process of self-organization of the thermoacoustic system. This typical reductionist approach studies thermoacoustic instability in different types of configurations by investigating the details of the flame dynamics in these configurations, and it does not emphasize the interaction between the elements and the resulting self-organization process. In a reductionist approach, we zoom into the constituent elements and analyze them in detail. On the contrary, in a complex system approach, the focus is on the interaction between the constituent elements.

A complex system is defined as a system with “multiple interacting components whose behavior cannot be simply inferred from the behavior of its components.”⁴² While adopting a complex system approach, we do not split things apart and zoom in on the component behavior; instead, we focus on the new phenomena that can emerge from a relatively simple set of components, interacting through a relatively simple set of rules. The key ingredients of complex systems are: (1) interacting elements, (2) nonlinear feedback that affects the behavior of these systems, and (3) exchange of energy and mass with surroundings.⁴³ A thermoacoustic system that involves turbulent reacting flow has these ingredients. We now list below some of the behaviors of typical complex systems,⁴³ which can indeed be observed in turbulent thermoacoustic systems.

D. Characteristics of complex systems

A high level of coherence is observed during thermoacoustic instability. Often, engineers and technicians refer to such a coherent thermoacoustic system as something that is “alive,” even using a terminology, such as “the combustor is breathing.” Large coherent structures emerge in the turbulent flow during the occurrence of thermoacoustic instability.^{44,45} Such emergent behavior is a tell tale sign of complex systems. The self-organization of the turbulent flow and the emergence of the ordered acoustic field and large coherent structures happen without any need for forcing, i.e., we do not need an “invisible hand.”

Emergence is a characteristic feature of complex systems. For example, we cannot explain the wetness of water by examining the individual molecules of H₂O and their properties. It is an emergent property that originates from the interaction of the H₂O molecules. From a reductionist and linear perspective, emergent phenomena are often surprising. Most interestingly, en route to the process of self-organization leading to thermoacoustic instability, we observe intermittency, wherein we see intermittent bursts of high amplitude periodic oscillations (order) that appear in an apparently random fashion amid epochs of aperiodic low-amplitude fluctuations (disorder).⁴⁶ Furthermore, the emergence of order from disorder happens through a chimera state where order and disorder coexist simultaneously.⁴⁷ This is a classical feature of complex systems that shows a mix of ordered and disordered behaviors.

E. Entropy defying emergence of order from chaos

Thus, we see that the occurrence of thermoacoustic instability in turbulent combustors can be viewed as order emerging from chaos in complex systems. Nature abounds such phenomena. Examples include synchronous flashing of fireflies and collective animal behaviors, such as flock of birds, school of fish, or bridges

made by ants. Life indeed is an example of self-organization. Examples involving wave phenomena include laser⁴⁸ and thermoacoustic instability.³⁷

The second law of thermodynamics tells us that the entropy of a system should increase, and the system proceeds toward disorder. Do the phenomena of self-organization and the subsequent emergence of order defy the second law of thermodynamics?

The key to resolving this puzzle lies in the nature of systems that undergo self-organization, including thermoacoustic systems. The second law of thermodynamics is applicable to closed systems. Self-organization occurs in open systems.⁴⁹ For example, in a thermoacoustic system, there are energy and matter flowing in and out of the system. To decrease entropy, work must be done.

Thermoacoustic systems are non-equilibrium systems. In such systems, energy dissipation itself determines the birth of new structures. Through energy dissipation, a part of the energy of the ordered motion is transformed into disordered motion, and finally to heat. Thus, ironically, dissipation leads to the origin of structures. Prigogine⁵⁰ was awarded the Nobel Prize in chemistry in 1977 for “for his contributions to non-equilibrium thermodynamics, particularly the theory of dissipative structures.” Even Rayleigh’s results are just a formal consequence of Prigogine’s results concerning open systems.^{51,52}

Thus, we can see that a thermoacoustic system has all the features of a complex system. Thermoacoustic instability can then be viewed as self-organization occurring in this complex system. Next, we will describe the onset of thermoacoustic instability in turbulent reacting flows in the language of self-organization. First, we will discuss the stable operation of a turbulent thermoacoustic system and then proceed to discuss the transition to thermoacoustic instability.

II. WHAT IS STABLE OPERATION?

The stable operation of a turbulent combustor has an acoustic signature that is characterized by low amplitude aperiodic pressure fluctuations that have a broadband spectrum with shallow peaks near to the acoustic modes of the combustor.^{53,54} These pressure fluctuations are referred to as combustion noise. Combustion noise can originate from various unsteady combustion and fluid dynamical processes in a combustor, such as fluctuations in the local heat release rate, accelerating entropy waves, inhomogeneities in vorticity, and local fluid dilatation.⁵³

Strahle, through a series of seminal papers, developed an analytical framework to describe the far field noise generated by a turbulent flame.^{55–57} This was largely based on Lighthill’s theory of aerodynamic noise⁵⁸ appropriately modified using order of magnitude arguments to account for the presence of combustion. These theories^{55–57} captured the dominance of low frequency oscillations also known as “roar” in combustion noise. Strahle suggested that combustion noise was largely unaffected by chemical kinetics. He also unraveled the directional nature of combustion noise for wavelengths comparable to the integral length scale of turbulence. Extending Strahle’s work, Hassan⁵⁹ derived scaling laws for combustion generated noise. The drawback of these studies as pointed out by Candel *et al.*⁶⁰ was that they considered only situations where flow dynamics was largely independent of the radiated sound. In practice, combustion happens in a confined environment where the flow, sound, and the local heat release from the flame are closely

coupled with each other. This coupling, when sufficiently strong, can result in thermoacoustic instability. Rajaram and Lieuwen⁶¹ argued that the spectral characteristics of the flame’s acoustic emissions are determined both by the underlying spectrum of heat release rate fluctuations and the transfer function relating these heat release rate and acoustic fluctuations.

Dynamically, the transition from the stable state of combustion noise to the unstable state of thermoacoustic instability is traditionally considered to be a transition from a fixed point to limit cycle oscillations.¹⁴ In this framework, combustion noise is characterized as the response of stochastic perturbations on an acoustic oscillator and is represented in the phase space as perturbations around a fixed point. Accordingly, combustion noise was often modeled as a random noise or noise with a measured power spectrum.^{31–33}

Recently, Nair *et al.*⁶² analyzed the deterministic nature of pressure oscillations during the occurrence of combustion noise using the Kaplan–Glass test⁶³ and surrogate analysis and suggested that combustion noise has the signature of deterministic chaos. With an even more rigorous surrogate analysis using various measures, such as the Hurst exponent, translational error, permutation entropy, permutation spectrum, correlation dimension, and correlation entropy, Tony *et al.*³⁴ proved that combustion noise is high dimensional chaos corrupted with white and colored noise. Nair and Sujith²⁹ further discovered that combustion noise exhibits complex scaling behavior and is multifractal in nature. This deterministic and multifractal nature of combustion noise is possibly the result of the deterministic nature of processes that are responsible for the generation of combustion noise and the fractal characteristics of the underlying turbulent flow.

The discovery of the deterministic and multifractal nature of combustion noise suggests that the phase space attractor corresponding to combustion noise cannot be considered as a mere fixed point and rather has a complex fractal topology. Hence, the traditional interpretation that the onset of thermoacoustic instability is a transition from a fixed point to limit cycle oscillation needs to be reconsidered.

III. INTERMITTENCY PRESAGES THERMOACOUSTIC INSTABILITY

Nair *et al.*^{35,46} investigated the transition from combustion noise to thermoacoustic instability and discovered that it is characterized by a dynamic regime known as intermittency. The pressure oscillations during intermittency consist of bursts of high amplitude periodic oscillations inter-spaced with epochs of low amplitude aperiodic oscillations. The switch between high amplitude periodic oscillations and low amplitude aperiodic oscillations happens at non-equal intervals. In Fig. 1, we can see the time series of pressure oscillations during intermittency in combustors with two different flame holding mechanisms. Figure 1(a) corresponds to the time series from a bluff body stabilized backward facing step combustor, and Fig. 1(b) shows the time series from a swirl stabilized backward facing step combustor. In both cases, during intermittency, we can see that the time series of pressure contains bursts of high amplitude periodic oscillations amid epochs of low amplitude aperiodic oscillations. As we approach thermoacoustic instability by changing the Reynolds number of the flow, the duration for which the large amplitude bursts persist increases, and eventually during

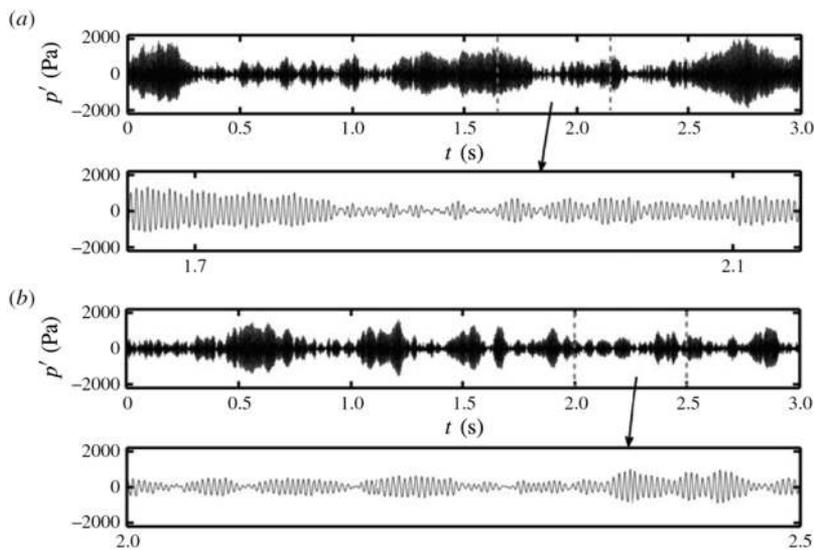


FIG. 1. Acoustic pressure fluctuations during the state of intermittency obtained from the two combustors: (a) acoustic pressure fluctuations during intermittency in a bluff body stabilized backward facing step combustor ($Re = 1.88 \times 10^4$, $\phi = 0.85$) and (b) acoustic pressure fluctuations during intermittency in swirl stabilized backward facing step combustor ($Re = 2.58 \times 10^4$, $\phi = 0.77$). Here, ϕ represents the global equivalence ratio, and Re is the flow Reynolds number. The zoomed regions of the signal (lower) indicate bursts of high-amplitude oscillations that appear amid low-amplitude aperiodic fluctuations. Reproduced with permission from V. Nair, G. Thampi, and R. I. Sujith, "Intermittency route to thermoacoustic instability in turbulent combustors," *J. Fluid Mech.* **756**, 470–487 (2014). Copyright 2014 Cambridge University Press.

thermoacoustic instability, the system exhibits periodic oscillations, otherwise known as limit cycle oscillations. The presence of intermittency was also reported subsequently by many others (e.g., by Karlis *et al.*⁶⁴ in a swirl combustor).

The presence of intermittency in the transition regime to thermoacoustic instability can also be observed in various other combustor configurations, such as spray combustors,⁶⁵ matrix burners,⁶⁶ after burners,⁶⁷ liquid rocket combustors,⁶⁸ and ducted inverse non-premixed flames.⁶⁹ Systems that exhibit flame intrinsic thermoacoustic instabilities also exhibit intermittency prior to the onset of thermoacoustic instability.⁷⁰ Interestingly, other fluid dynamic systems that exhibit oscillatory instabilities and embody turbulent flow also exhibit intermittency en route to oscillatory instability. Examples include systems that exhibit aeroelastic instability,³⁹ aeroacoustic instability,³⁸ and hydrodynamic instabilities.^{71,72}

A. Characterizing intermittency

In the parlance of dynamical system theory, intermittency is the irregular alternation either between laminar phase (periodic oscillations) and turbulent phase (chaotic oscillations)⁷³ or between two different chaotic phases.^{74,75} Intermittency is often observed as a system is approaching a bifurcation point. According to the underlying bifurcation, the characteristics of intermittency vary. Based on this, Pomeau and Manneville⁷³ characterized intermittency into three different types. Type 1 corresponds to an approach to saddle node bifurcation, type 2 to subcritical Hopf bifurcation, and type 3 to an inverse period doubling bifurcation. Many more types of intermittencies have been identified recently, and a discussion on the types of intermittency and their characteristics can be found in the work of Elaskar and Del Río.⁷⁶ Intermittency is common in turbulent systems and is often observed in confined convection, boundary layers, pipe flows, etc.⁷³ These intermittencies are generally associated with low amplitude laminar phases (periodic oscillations) and high amplitude chaotic phases (aperiodic or chaotic oscillations). A similar type of intermittency is observed in a laminar

combustor prior to flame blowout.²³ However, the intermittency observed in turbulent thermoacoustic systems prior to the onset of thermoacoustic instability is qualitatively different. Here, the chaotic phase has lower amplitude compared to the laminar phase.

The characteristics of intermittency can be analyzed by analyzing the topology of the corresponding phase space. Recurrence analysis is a powerful tool used to study the topology of a phase space. A recurrence plot is a 2D plot (t_i vs t_j) representing the times t_i for which the state vector (X_i) representing the state of the system visits the same neighborhood of the phase space as it was at time t_j .⁷⁷ Mathematically, the recurrence plot is represented as a matrix R , the elements of which are defined as

$$R_{ij} = \begin{cases} 1 & \text{if } \|\vec{X}_i - \vec{X}_j\| \leq \epsilon \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

The patterns in a recurrence plot can be used to characterize the type of intermittency.⁷⁸ Multiple studies have attempted to characterize the type of intermittency in turbulent combustors using recurrence analysis.^{35,36,46,65,69,79,80} A typical recurrence plot during thermoacoustic instability in a turbulent combustor is shown in Fig. 2. The methodology used in construction of the recurrence plot is detailed in the work of Pawar *et al.*⁶⁵

We see that the recurrence plot corresponding to intermittency consists of black patches [Fig. 2(b)]. These black patches correspond to the low amplitude aperiodic regimes of the time series. The periodic part of the time series corresponds to lines parallel to the diagonal of the recurrence plot [Fig. 2(d)]. When we zoom in to the black patch, we see perforated top-right corners of black patches from which lines parallel to the diagonal line emerge [Fig. 2(c)]. Such a pattern in the recurrence plot is indicative of a type 2 intermittency. From such observations, and recurrence quantitative analysis, previous studies have suggested that the intermittency prior to thermoacoustic instability may be of type 2.^{36,65,79} However, this inference is based on the characteristics of re-injection of phase space trajectory from laminar phase to turbulent phase and vice versa and does not

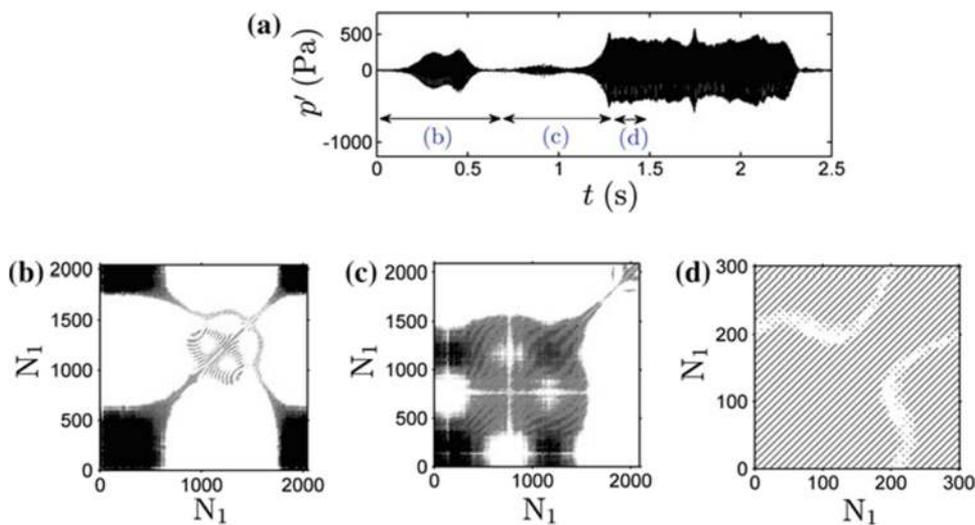


FIG. 2. (a) Time series of pressure corresponding to intermittency. [(b)–(d)] Recurrence plots (RPs) for different parts of the pressure time series. (c) RP shows a kite-like structure of type-II intermittency. Data points $N = 7500$, $\tau = 3$, $d = 10$, and $\epsilon = 20\%$ of the mean diameter of the attractor. Reproduced with permission from Pawar *et al.*, “Intermittency route to combustion instability in a laboratory spray combustor,” *J. Eng. Gas Turbines Power* **138**, 041505 (2016). Copyright 2016 American Society of Mechanical Engineers.

account for the relative amplitude of laminar and turbulent phases. Therefore, a rigorous analysis is required to correctly characterize the type of intermittency and the associated bifurcation.

B. Physics of intermittency

In turbulent combustors, intermittency is observed not only as a precursor to the onset of thermoacoustic instability but also prior to flame blowout.^{30,46} The presence of intermittency associated with flame blowout in turbulent combustors was first identified by Nair and Sujith.⁴⁶ A detailed analysis of the flame dynamics during intermittency prior to the onset of thermoacoustic instability and intermittency prior to flame blowout in a bluff body stabilized combustor was performed by Unni and Sujith.³⁶ Using high speed Mie scattering images, they showed that a periodic shedding of vortices from the flame stabilization mechanism is observed during thermoacoustic instability. Intermittency is characterized by periodic shedding of vortices from the flame stabilization mechanism during periodic epochs of intermittency and aperiodic vortex shedding during the aperiodic epochs³⁶ (Fig. 3). The primary difference in flame dynamics during intermittency prior to and post-thermoacoustic instability was attributed to the difference in the location where the flame is stabilized within the combustor.

In a backward facing step combustor, flow dynamics during intermittency were studied by Sampath and Chakravarthy.⁸¹ They obtained simultaneous particle image velocimetry (PIV) and CH^* chemiluminescence images to study the coupled interaction of the hydrodynamic field and the flame. They distinguished the flame and flow dynamics during intermittency from that during the occurrence of thermoacoustic instability. Intermittency was attributed to the prevalence of multiple hydrodynamic modes that interact with each other and with the acoustic mode of the duct, resulting in alternating relative dominance between the different modes of hydrodynamic fluctuations.

Flame dynamics during intermittency was studied by Pawar *et al.*⁸² in a laboratory spray combustor. They showed that thermoacoustic instability is the result of collective synchronization between

the individual flamelets that are anchored at the flame holder. During intermittency, the flamelets exhibited collective behavior during the periodic epochs and desynchronized behavior during the aperiodic epochs.

C. Modeling intermittency

Nair *et al.*³⁵ showed that the onset of thermoacoustic instability is always presaged by intermittent bursts of high amplitude oscillations that appear in an apparently random manner amid epochs of low amplitude oscillations. While intermittency in thermoacoustic systems was characterized as a specific dynamic state only recently, verbal description of dynamics resembling intermittency can be found in earlier literature as well.

Calvin *et al.*³¹ remarked “The pressure–time traces observed in combustion chambers of liquid-propellant rocket engines exhibit acoustic oscillations, which sometimes have an erratic evolution of the amplitude (Roudakov, 1993; Yang, 1994, especially Dranovsky *et al.*, 1994). This random character is more pronounced in the vicinity of critical conditions for the occurrence of high-frequency combustion instabilities and may exhibit itself there in the form of random pressure bursts of strong amplitude.” Hong *et al.*⁸³ reported a low-frequency instability mode, where an alternating flame behavior between periodic oscillations (unsteady flame behavior) and low amplitude fluctuations was observed as a result of large equivalence ratio modulation. In a premixed gas turbine combustor model, Arndt *et al.*⁸⁴ observed unpredictable transition between a thermoacoustically unstable (“noisy”) state and a state without pulsations (“quiet” state). They also observed that this transition was also correlated with a change in the configuration of flame stabilization within the combustor.

Such bursting oscillations were also accounted for and observed in the low order models developed to describe transition to thermoacoustic instability. The initial interest was to study the bursting oscillations observed in rocket engines. Clavin *et al.*³¹ argued that noise introduced by turbulence causes a thermoacoustic system to exhibit bursts, if the system undergoes a subcritical bifurcation.

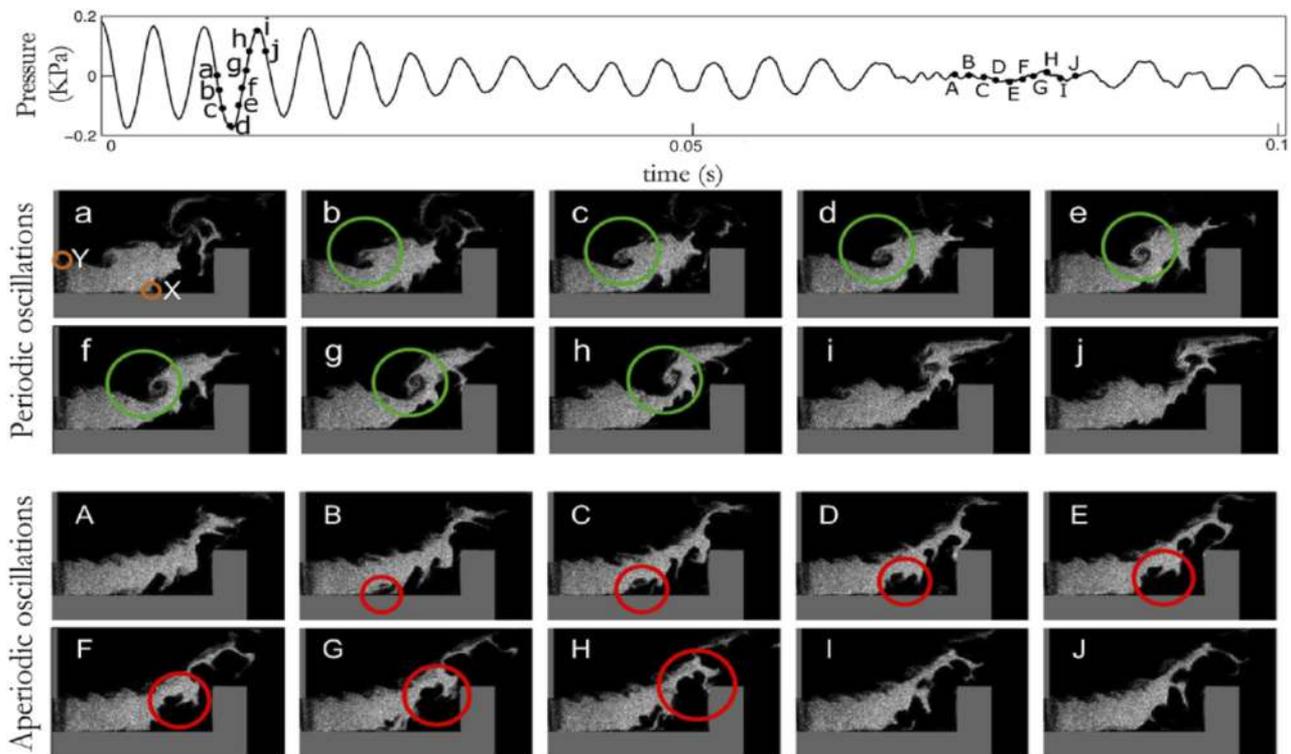


FIG. 3. Mie scattering images representing the flame dynamics during intermittent oscillations prior to the onset of thermoacoustic instability. During the periodic part of intermittency, the shear layer along which the flame is stabilized exhibits periodic fluctuations of the flame front due to periodic shedding of vortices from the dump plane that produces periodic roll-up of the flame front. During aperiodic epochs of intermittency, the shear layers exhibit aperiodic fluctuations. Large scale oscillations in the inner shear layer as shown in the figure are observed only during the aperiodic epochs of the intermittency. Aperiodic epochs of intermittency happen at irregular intervals of time. Reproduced with permission from V. R. Unni and R. I. Sujith, “Flame dynamics during intermittency in a turbulent combustor,” *Proc. Combust. Inst.* **36**, 3791–3798 (2017). Copyright 2017 Elsevier.

During the bistable state corresponding to the subcritical bifurcation, referred to as a metastable state by Clavin *et al.*,³¹ the noise pushes the system back and forth the unstable branch of the subcritical Hopf bifurcation, causing the system to exhibit bursting behavior. Various models with additional details were developed later in order to study such bursting behavior.³² The time series of pressure fluctuations in a combustor derived from one such model³² incorporating the effect of stochastic fluctuations due to turbulence on thermoacoustic oscillations is shown in Fig. 4. We can notice that the pressure fluctuations indeed resemble the pressure fluctuations during intermittency as shown in Fig. 1 although the authors themselves do not say anything about intermittency.

More recently, Noiray and Schuermans³⁵ used a noise driven Van der Pol oscillator to model thermoacoustic oscillations in a turbulent combustor. This study provided a framework to identify the linear growth rates of a system using the pressure fluctuations measured during limit cycle oscillations in a turbulent thermoacoustic system. They viewed the fluctuations in the limit cycle amplitude during thermoacoustic instability in a turbulent combustor as a response of an oscillator perturbed by strong stochastic forcing due to turbulence and estimated the linear growth rate of the oscillator from the fluctuating amplitudes of the limit cycle

oscillations. We can note that this model also exhibits an intermittency-like behavior characterized by bursts of high amplitude periodic pressure oscillations.

In an alternate approach, Nair and Sujith³⁶ captured the intermittency route to thermoacoustic instability in a bluff body stabilized dump combustor using a kicked oscillator model. Their model was developed by incorporating modifications to the model for thermoacoustic instability suggested by Matveev and Culick.³⁷ The fundamental assumption in this model is that thermoacoustic instability (in many systems) is hydrodynamically coupled. That is, the periodic heat release rate during thermoacoustic instability is the result of periodic formation of coherent vortices that causes oscillations in the flame and drives the acoustic field. Furthermore, the shedding of vortices and their periodic impingement on the walls of the combustor or flame stabilization mechanisms cause a temporally localized high heat release rate. During thermoacoustic instability, these bursts of heat release happen at appropriate phases of the acoustic pressure fluctuations within the combustor. Considering this phenomenology, the thermoacoustic system is modeled as a kicked oscillator. The kicks, which are in feedback with pressure fluctuations, correspond to the energy added to the acoustic oscillator by the sudden localized intense heat release. However, the model

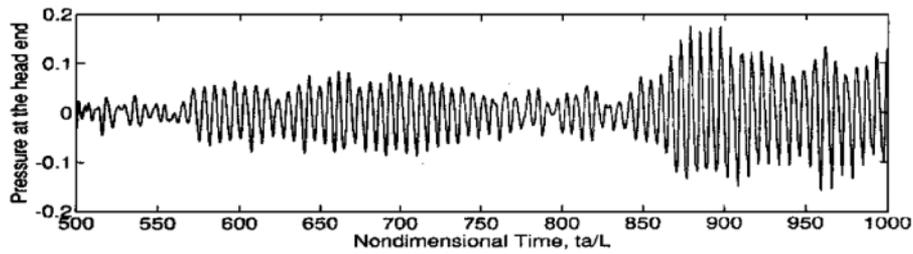


FIG. 4. Time series of unsteady pressure corresponding to thermoacoustic oscillation, resulting from the influence of stochastic fluctuations on a deterministic model as described by Burnley and Culick.³² Reproduced with permission from V. Burnley and F. C. Culick, "Influence of random excitations on acoustic instabilities in combustion chambers," *AIAA J.* **38**, 1403–1410 (2000).

developed by Matveev and Culick⁸⁷ does not exhibit intermittency. Nair and Sujith,⁸⁶ modified this model by incorporating the effect of turbulence on the acoustic oscillator as stochastic fluctuations in the velocity, which, in turn, introduced stochasticity in the timing of kicks. This modified model exhibited intermittency prior to the onset of thermoacoustic instability.

Nair *et al.*⁶² and Tony *et al.*³⁴ showed that the aperiodic pressure fluctuations in a combustor that occur during the stable operation, i.e., combustion noise, are deterministic in nature. However, the model suggested by Nair and Sujith⁸⁶ included some stochastic elements to capture intermittency. Shesadri *et al.*⁸⁸ introduced an improved deterministic model by eliminating the need for stochastic fluctuations to produce intermittency. This was achieved by introducing an appropriate feedback between the kicking times and the pressure and heat release rate oscillations. Modified versions of this model capture various synchronization states (discussed later in Sec. IV) observed in bluff body combustors⁸⁹ and can also be used to predict the amplitude of oscillations during thermoacoustic instability in a turbulent combustor.⁹⁰ These aspects are described in Sec. III D.

D. Kicked oscillator model

A simplified configuration of the experimental system is considered to construct the model (Fig. 5). The length of the

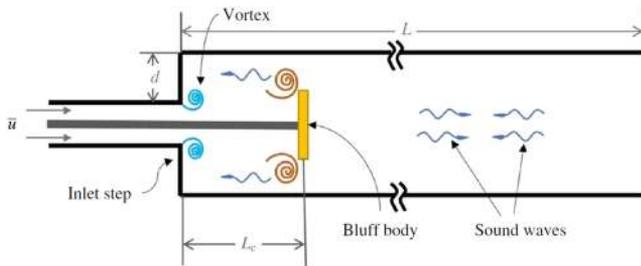


FIG. 5. Schematic diagram of a backward facing step combustor with a bluff body to stabilize the flame. The low order, kicked oscillator models for a combustor exhibiting thermoacoustic instability described in the work of Seshadri *et al.*⁸⁸ and Pawar *et al.*⁸⁹ are based on this schematic diagram. Reproduced with permission from Pawar *et al.*, "Thermoacoustic instability as mutual synchronization between the acoustic field of the confinement and turbulent reactive flow," *J. Fluid Mech.* **827**, 664–693 (2017). Copyright 2017 Cambridge University Press.

combustor is L , and the step at the inlet of the combustor has a height d . A fundamental simplifying assumption in this model is that the combustion of the reactants is localized at a distance L_c from the inlet. Vortices that entrain unburnt reactants are shed from the inlet as a result of the build-up of circulation (Γ) at the backward facing step. Γ is given by

$$\frac{d\Gamma}{dt} = \frac{u_{sep}^2}{2} + k \sum_j p'_{L_c}(t_j) \delta(t - t_j - \tau_a) + \sigma_1 N(0, 1). \quad (2)$$

Here, $u_{sep} = \bar{u} + u'(L_s, t)$. \bar{u} is the mean velocity, $u'(L_s, t)$ is the acoustic velocity at the backward facing step, and p'_{L_c} is the acoustic pressure at the point of combustion. Furthermore, t_j is the time when the j th vortex reaches the location where the localized combustion happens. τ_a is the delay in the response of the heat release rate. $N(0, 1)$ is the Gaussian white noise.

As time progresses, the circulation built up at the step exceeds a critical value, $\Gamma_{cr} = \Gamma_0 u_{sep}$. This leads to shedding of a vortex from the step, and consequently the circulation at the step is reset to zero.⁸⁷ Note that $\Gamma_0 = d/2 S_i$, where S_i is the Strouhal number.⁸⁷ In order to account for the imperfect nature of the inlet boundary condition and the influence of turbulence in the flow, we assume that the inlet velocity is perturbed by noise and correspondingly $\Gamma_{cr} = \Gamma_0 u_{sep} + \sigma_2 N(0, 1)$. Here, noise terms for both $d\Gamma/dt$ and $\Gamma_{critical}$ are the same but the strength of the noise scales with the standard deviations σ_1 and σ_2 , respectively. The alternate increase and decrease in the circulation indicate its oscillatory nature. As a vortex is shed from the inlet, it convects downstream with a velocity proportional to the mean velocity⁸⁷ as captured by the following equation:

$$dx_j/dt = \alpha \bar{u} + u'(x, t). \quad (3)$$

Here, α is the fraction of the mean velocity with which the vortex propagates downstream. Upon reaching the location of combustion (L_c), the vortices burn instantaneously producing a localized heat release \dot{q}' given as

$$\dot{q}' = \beta \sum_j \Gamma_j \delta(x - L_c) \delta(t - t_j). \quad (4)$$

This unsteady heat release, in turn, adds energy to the acoustic field affecting the acoustic pressure (p') and the acoustic velocity (u') fluctuations. We can decompose the acoustic pressure and velocity fluctuations into oscillations in the natural modes of the combustor

as

$$p'/\bar{p} = \sum_{n=1}^M \frac{\dot{\eta}_n(t)}{\omega_n} \cos(k_n x) \quad (5)$$

and

$$u'/(\bar{p}/\bar{\rho}c) = \sum_{n=1}^M \eta_n(t) \sin(k_n x). \quad (6)$$

Here, η_n and $\dot{\eta}_n$ are the time varying components of the M natural modes of the combustor, and $k_n = \omega_n/c$ is their corresponding wave numbers. The energy equation corresponding to these natural

acoustic modes of the combustor can then be written as

$$\ddot{\eta}_n + \zeta_n \dot{\eta}_n + \omega_n^2 \eta_n = b \omega_n \cos(k_n L_c) \sum_j \Gamma_j \delta(t - t_j). \quad (7)$$

Here, the damping coefficient for the n th mode is ζ_n , which is modeled as $\zeta_n = (2n - 1)^n \zeta_1$, where $\zeta_1 = 29 \text{ s}^{-1}$. The detailed list of parameters of the model is given in the work of Pawar *et al.*⁸⁹ This model is able to capture the transition from combustion noise to thermoacoustic instability through intermittency as shown in Fig. 6.

IV. SYNCHRONIZATION LEADING TO SELF-ORGANIZATION

Thermoacoustic instability is the result of interaction between hydrodynamic, combustion, and acoustic subsystems of a combustor. The mutual interaction between these subsystems leads to self-organization and the emergence of a pattern in the system. As the pattern evolves, it further affects the coupling between the various subsystems in the flow field. Even though the dynamics of a thermoacoustic system are constituted through such complex interactions between multiple processes happening at different scales, they are often modeled merely as a forced nonlinear oscillator. In the flame transfer function approach, the flame is forced with velocity oscillations having different frequencies, and the response of the flame is evaluated as a function of the forcing frequency. While such an approach to modeling may be effective during thermoacoustic instability, it does not capture the dynamics of the system during intermittency. To address this lacuna, one needs to possibly model the thermoacoustic system as a system of coupled oscillators where both acoustics and unsteady flame are the two coupled oscillators and the oscillators are mutually coupled. The flame transfer function approach does not attribute inherent dynamics to the flame; instead, the flame responds to the acoustic velocity fluctuations with a gain and a phase. However, the flame is a nonlinear oscillator by itself that has its own dynamics. This nonlinear oscillator can interact nonlinearly with the acoustic oscillator. A synchronization based model for the same thermoacoustic system⁹¹ was able to capture these different dynamical states. This model, developed by Weng *et al.*,⁹¹ considered both unsteady flame and the acoustic field as mutually coupled oscillators that undergo synchronization when the coupling strength is varied.

A characterization of the coupling between the acoustic and heat release rate oscillations during the transition to self-excited thermoacoustic instability in turbulent combustors was first done by Pawar *et al.*⁸⁹ They studied the transition to thermoacoustic instability in a turbulent dump combustor with a bluff body as the flame stabilization mechanism. Simultaneous measurements of acoustic pressure fluctuations (p' , measured using a piezoelectric transducer) and unsteady heat release rate (\dot{q}' , represented by CH^* chemiluminescence, which is measured using a photomultiplier tube outfitted with a CH^* filter) were used to establish that the onset of thermoacoustic instability is a phenomenon of mutual synchronization between the acoustic field and the unsteady flame.

Synchronization refers to the phenomenon of matching of rhythms of coupled oscillators. This universal phenomenon was first discovered by Huygens in the seventeenth century while he was observing the locking of oscillations of two pendulum clocks hung over a wall.⁹² Following that, synchronization has been

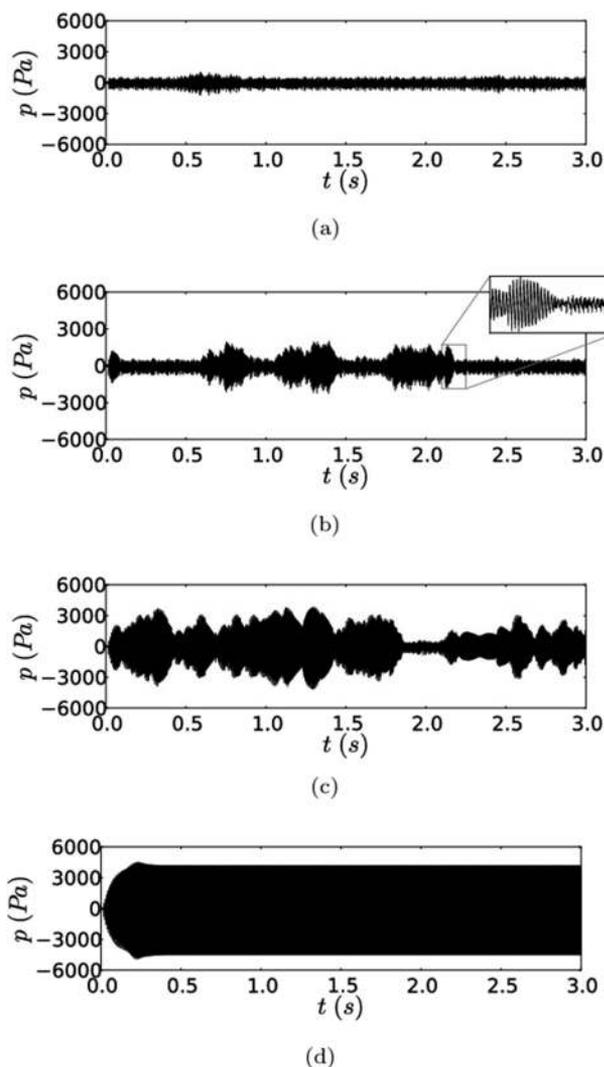


FIG. 6. Acoustic pressure signal variation at $x = 0.09 \text{ m}$ showing (a) chaos at $\bar{u} = 9.5 \text{ m/s}$, (b) intermittency at $\bar{u} = 10.4 \text{ m/s}$, (c) intermittency at $\bar{u} = 10.65 \text{ m/s}$, and (d) limit cycle oscillations at $\bar{u} = 10.8 \text{ m/s}$. The signals were obtained using the model. Reproduced with permission from A. Seshadri, V. Nair, and R. I. Sujith, "A reduced-order deterministic model describing an intermittency route to combustion instability," *Combust. Theory Modell.* **20**, 441–456 (2016). Copyright 2016 Taylor & Francis.

observed in a multitude of systems, such as chemical systems,⁹³ biological systems,⁹⁴ ecosystems,⁹⁵ communication systems,⁹⁶ and many engineering systems.^{97,98} The type of coupling and coupling strength between oscillators determine the characteristics of synchronization.

Pawar *et al.*⁸⁹ studied the synchronization between the unsteady heat release rate (\dot{q}') and the acoustic field (represented by the acoustic pressure, p') of the combustor. They showed that the transition to thermoacoustic instability in a turbulent combustor occurs from a desynchronized (DS) state corresponding to combustion noise to a generalized synchronized (GS) state corresponding to strong thermoacoustic instability via an intermittently phase synchronized (IPS) state corresponding to intermittency and a phase synchronized (PS) state corresponding to weak thermoacoustic instability.

They classified the different states of synchronization between p' and \dot{q}' signals using a statistical measure of synchronization based on recurrence plots known as the probability of recurrence $[P(\tau)]$. $P(\tau)$ is the probability with which the phase space trajectory returns to the neighborhood of a given point in the phase space after a time lag τ .⁹⁹ Figure 7 shows the time series corresponding to pressure fluctuations and heat release rate fluctuations at different states of synchronization. During the desynchronized state, p' and \dot{q}' are not synchronized. Correspondingly, the plot of $P(\tau)$ in Fig. 8(a) as a

function of time lag (τ) shows no correspondence between the peaks of $[P(\tau)]$ for p' and \dot{q}' . During the PS state, p' and \dot{q}' are phase synchronized, and we can observe a perfect locking of the positions of the peaks of $P(\tau)$ for both these signals [see Fig. 8(b)]. However, the absence of locking of the heights of the peaks of $P(\tau)$ suggests that the signals are phase synchronized. During the GS state, the signals have both synchronized phases and amplitudes. Furthermore, during the occurrence of GS [see Fig. 8(c)], both the positions and amplitude of peaks of $P(\tau)$ for both p' and \dot{q}' are locked. In the parlance of synchronization, two signals are called generalized synchronized, if one signal can be expressed as a function of the other.⁹² Pawar *et al.*⁸⁹ showed that during GS, \dot{q}' can be written as a function of p' . In the case of IPS, the signals are desynchronized during the aperiodic epochs of intermittency and phase synchronized during the periodic epochs. Correspondingly, the peaks of $P(\tau)$ are locked during the periodic epochs and not locked during the aperiodic epochs (Fig. 9).

Furthermore, Pawar *et al.*⁸⁹ was also able to show synchronization of p' and \dot{q}' at the onset of thermoacoustic instability in a modified version of the model developed by Seshadri *et al.*⁸⁸ (in addition, this model is described in Sec. III C). They also showed both from experiments and their model that during PS, the cycle to cycle variation in kicking times is greater than that for GS; this was attributed as the reason for strong synchronization during GS, as

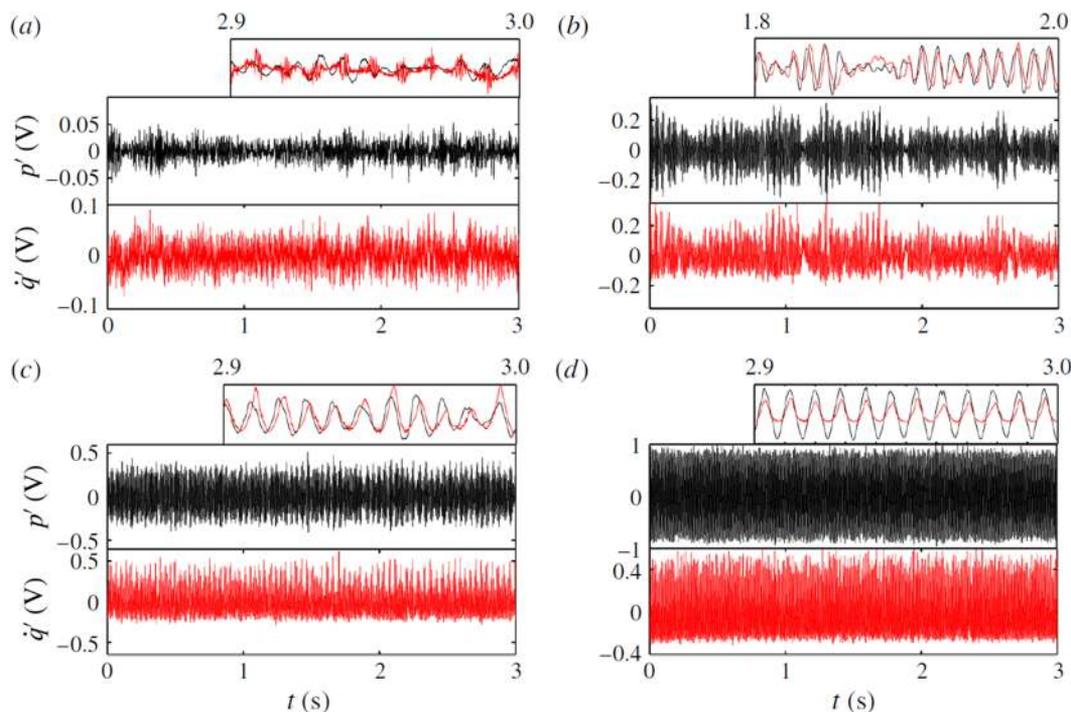


FIG. 7. Acoustic pressure (p' —black curve) and heat release rate (\dot{q}' —red curve) fluctuations at different states of synchronization. (a) Desynchronized state corresponding to combustion noise ($\bar{u} = 9.4$ m/s), (b) intermittent phase synchronized state corresponding to intermittency ($\bar{u} = 11.9$ m/s), (c) phase synchronized state corresponding to weakly periodic limit cycle oscillations ($\bar{u} = 13.2$ m/s), and (d) generalized synchronized state corresponding to the strongly periodic limit cycle state of thermoacoustic oscillations ($\bar{u} = 17.2$ m/s). Magnified views of the corresponding signals are shown in the insets above each panel. Reproduced with permission from Pawar *et al.*, “Thermoacoustic instability as mutual synchronization between the acoustic field of the confinement and turbulent reactive flow,” *J. Fluid Mech.* **827**, 664–693 (2017). Copyright 2017 Cambridge University Press.

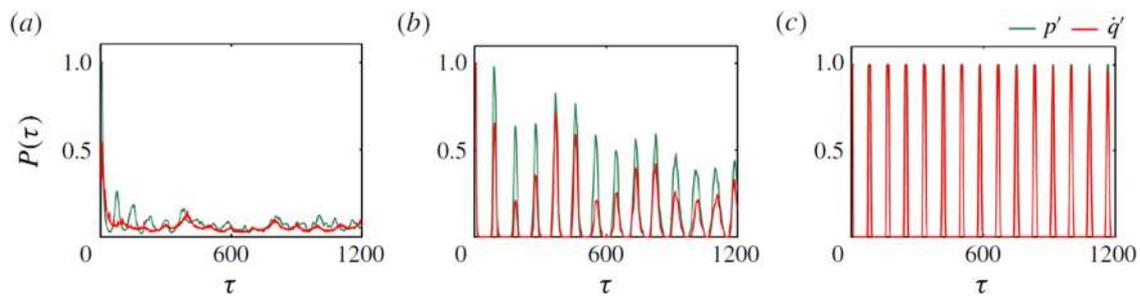


FIG. 8. Probability of recurrence, $P(\tau)$, for both p' and q' during (a) combustion noise (DS), (b) weak thermoacoustic instability (PS), and (c) strong thermoacoustic instability (GS). Reproduced with permission from Pawar *et al.*, "Thermoacoustic instability as mutual synchronization between the acoustic field of the confinement and turbulent reactive flow," *J. Fluid Mech.* **827**, 664–693 (2017). Copyright 2017 Cambridge University Press.

compared to PS. This was also captured in the model described in Sec. III C.

A turbulent combustor is a spatially extended system. Hence, the synchronization that happens in the temporal domain during thermoacoustic instability must also have a spatial aspect associated with it. Mondal *et al.*⁴⁷ studied this synchrony in the spatiotemporal dynamics of a turbulent combustor as thermoacoustic instability was approached. Their studies were performed on the same combustor as that was used by Pawar *et al.*⁸⁹ High speed CH^* chemiluminescence images representing the instantaneous local heat release rate [$\dot{q}'(x, y, t)$] were obtained simultaneously with acoustic pressure fluctuations [$p'(t)$]. In order to study the synchronization of the field of heat release rate with the acoustic fluctuations, the instantaneous phase difference between $p'(t)$ and $\dot{q}'(x, y, t)$ was calculated along a regular grid in the reaction field. The instantaneous phasors representing the phase difference between $p'(t)$ and $\dot{q}'(x, y, t)$ are

shown in Fig. 10. During the occurrence of combustion noise, the phasors are desynchronized and aperiodic. During thermoacoustic instability, the phasors are synchronized. The average phase difference between the phasors remains constant as they exhibit periodic behavior. During intermittency, the phasors exhibit coexistence of periodic synchrony with aperiodic asynchrony. Such a state of synchronization where a part of a network of oscillators is synchronized and the other part is desynchronized is known as a chimera state.¹⁰⁰ Such mixed synchronization states in a coupled network of oscillators were first identified by Kuramoto and Battogtokh,¹⁰¹ and named as chimera states by Abrams and Strogatz¹⁰⁰ after the Greek mythological monster *Chimera* that is constituted by parts of multiple animals, often pictured as a lion, with the head of a goat attached to its back, and a tail that ends with a snake's head. In thermoacoustic systems, Mondal *et al.*⁴⁷ also noted that regions of synchrony and asynchrony change in time indicating that the state corresponds to a

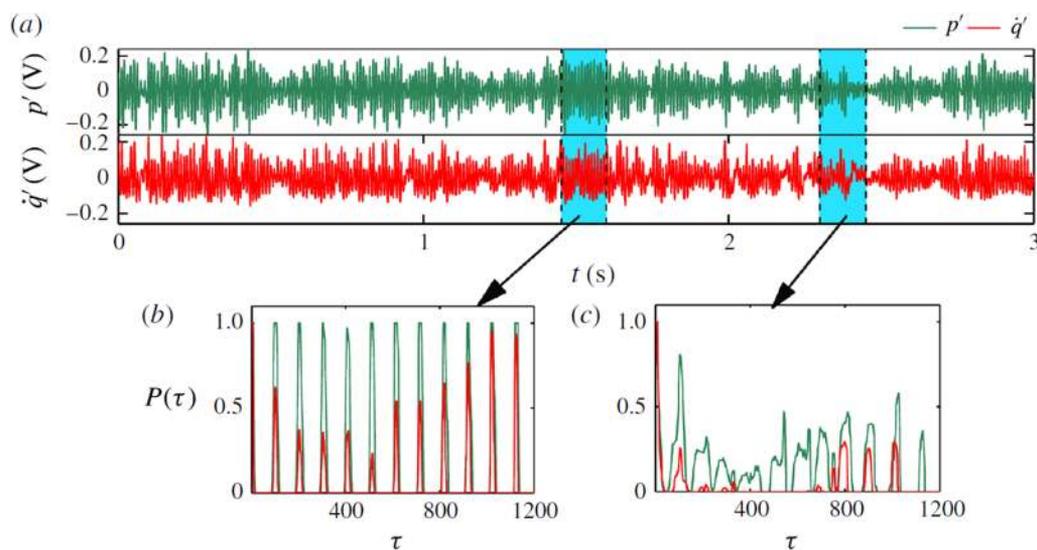


FIG. 9. Probability of recurrence, $P(\tau)$, for both p' and q' during intermittency. (a) Time series of p' and q' (DS), (b) $P(\tau)$ for both p' and q' during periodic epochs of intermittency, and (c) $P(\tau)$ for both p' and q' during aperiodic epochs of intermittency. Reproduced with permission from Pawar *et al.*, "Thermoacoustic instability as mutual synchronization between the acoustic field of the confinement and turbulent reactive flow," *J. Fluid Mech.* **827**, 664–693 (2017). Copyright 2017 Cambridge University Press.

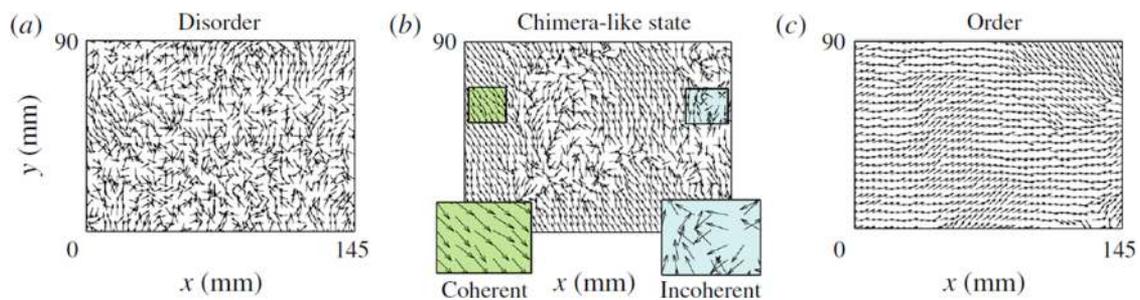


FIG. 10. Instantaneous phasors representing the phase difference between $\dot{q}'(x, y, t)$ and $p'(t)$. (a) Combustion noise—phasors are desynchronized. (b) Intermittency—patches of synchronized phasors coexisting with patches of asynchronous phasors. (c) Thermoacoustic instability—the phasors are synchronized. Reproduced with permission from S. Mondal, V. R. Unni, and R. I. Sujith, “Onset of thermoacoustic instability in turbulent combustors: An emergence of synchronized periodicity through formation of chimera-like states,” *J. Fluid Mech.* **811**, 659–681 (2017). Copyright 2017 Cambridge University Press.

breathing chimera. Chimera states were later also reported in other combustor configurations, such as swirl stabilized combustors,¹⁰² liquid fuel combustors,¹⁰³ and rocket motors.¹⁰⁴

Note that we call the dynamical state discovered by Mondal *et al.*⁴⁷ as the chimera state, whereas Mondal *et al.*⁴⁷ referred to this state as a chimera-like state. When Mondal *et al.*⁴⁷ published their article, chimera states referred to the dynamical state of mixed synchrony and asynchrony in an ensemble of identical oscillators. Mondal *et al.*⁴⁷ recognized that the local heat release oscillators that constitute the reaction field in their system are non-identical oscillators. Over the years, the definition of the chimera state has been extended to include mixed synchrony and asynchrony in ensemble of non-identical oscillators.¹⁰⁵ Hence, we refer to the chimera-like states discovered by Mondal *et al.*⁴⁷ simply as chimera states.

In essence, the onset of thermoacoustic instability is a transition from a disordered state to an ordered state. How do we describe disorder and order in a time series? As we will see in Sec. V, the disordered signal observed during the stable operation has fractal and multifractal attributes. In contrast, ordered signals seen during thermoacoustic instability are Euclidean in nature.

V. MULTIFRACTALITY IN THERMOACOUSTIC OSCILLATIONS AND ITS LOSS AT THE ONSET OF THERMOACOUSTIC INSTABILITY

Fractals are self-similar geometric objects whose geometric properties, such as area and perimeter, are the functions of the scale at which they are measured.¹⁰⁶ For example, consider a fractal curve, whose total length is being measured using a ruler of size S . N is the number of times the ruler needs to be translated along the length of the curve in order to measure the total length of the fractal. The length of the curve is then estimated as NS . If the curve is Euclidean and the length scale of the scale of measurement was s instead of S , then the number of translations n required to measure the length of the same curve using a ruler of length s is given as $n = NS/s$, whereas if the curve is fractal, $n = N(S/s)^\alpha$, where α is the fractal dimension. In the case of a time series, the fractal dimension is measured in terms of the Hurst exponent (H). The Hurst exponent is related to the fractal dimension as $\alpha = 2 - H$. The Hurst exponent can be estimated using detrended fluctuation analysis.

An even more complex topology is a multifractal for which the local scaling or fractal dimension varies in space. Many turbulent flows have multifractal characteristics born out of the multiplicative nature of turbulence energy cascade.¹⁰⁷ The multifractal scaling of a turbulent flow could, in turn, result in the multifractal nature of state variables of the system. Multifractality in a system is often represented by a spectrum of fractal dimensions known as the multifractal spectrum. An example of a multifractal spectrum is the singularity spectrum. For a multifractal function $F(x)$ defined in a vector space of x , the singularity spectrum $f(\alpha)$ is the fractal dimension of the set of all points in the support of $F(x)$, $\{x\}_\alpha$ for which the Hölder exponent is α . Here, the Hölder exponent α satisfies $\|F(x) - F(x + dx)\| \propto dx^\alpha$ for small dx .

Multifractal characteristics of pressure fluctuations in a turbulent combustor were first reported by Gotoda *et al.*¹⁰⁸ They showed that the phase space corresponding to the pressure oscillations prior to flame blowout exhibited multifractal characteristics. Later, Nair and Sujith²⁹ showed that pressure fluctuations during stable operation of the combustor, otherwise known as combustion noise, also have multifractal signature. They characterized the fractal behavior of the system by constructing a multifractal spectrum corresponding to the pressure fluctuations inside the combustor using multifractal detrended fluctuation analysis (MFDFA).¹⁰⁹ A well illustrated guide to MFDFA is given in the work of Ihlen.¹¹⁰

At the onset of thermoacoustic instability, the pressure oscillations become periodic and sinusoidal. Sinusoidal waves are not fractals, and hence at the onset of thermoacoustic instability, the pressure fluctuations lose their multifractal characteristics. The multifractal spectrum corresponding to the time series of unsteady pressure during the state of combustion noise from a bluff body combustor [Fig. 11(a)] and a swirl combustor [Fig. 11(b)] is presented in Fig. 11. In both cases, the multifractal spectrum has a finite width during the occurrence of combustion noise, indicating that the time series of pressure has multifractal characteristics. In contrast, during the occurrence of thermoacoustic instability, the time series of pressure fluctuations exhibits sinusoidal oscillations, which inherently is not a fractal. As a result, the multifractal spectrum shrinks to essentially a point.

Even though the multifractal characteristics of pressure time series are lost during thermoacoustic instability, the flow field inside

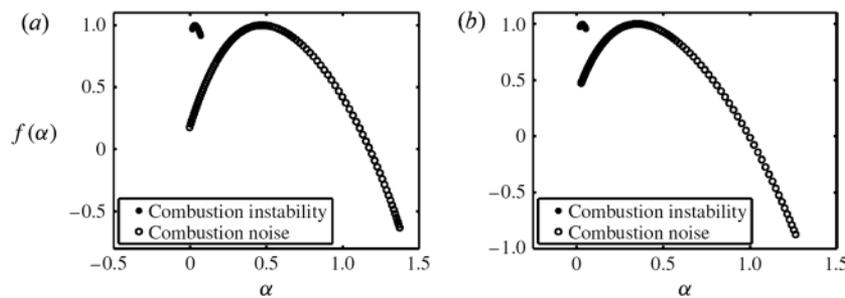


FIG. 11. Multifractal spectrum corresponding to combustion noise and thermoacoustic instability for (a) a bluff body combustor and (b) a swirl combustor. In both cases, the multifractal spectrum has a finite width during combustion noise, and the spectrum shrinks to a point during thermoacoustic instability. Adapted with permission from V. Nair and R. I. Sujith, "Multifractality in combustion noise: Predicting an impending combustion instability," *J. Fluid Mech.* **747**, 635–655 (2014). Copyright 2014 Cambridge University Press.

the combustor during thermoacoustic instability still remains turbulent and hence multifractal. Raghunathan *et al.*¹¹¹ investigated the fractal characteristics of the flame topology during thermoacoustic instability. They observed that the periodic pressure oscillation during thermoacoustic instability is the result of the periodic oscillation of the multifractal spectrum corresponding to the flame topology, which, in turn, is caused by the periodic emergence of coherent vortices during thermoacoustic instability. This study thus showed that the multitude of scales that are present in the system are still relevant during thermoacoustic instability even though there is an overriding behavior that connects the flow features with different temporal and spatial scales causing an emergence of a dominant scale of motion.

The presence of complex scaling behavior of pressure fluctuations and multifractal characteristics of the flow field in a combustor alludes to the complexity in the underlying processes. In Sec. **1D**, we discussed how a thermoacoustic system can be perceived as a complex system. Complex network theory provides a natural framework to analyze such complex systems. In Sec. **VI**, we will describe the use of complex networks to characterize the dynamics of thermoacoustic systems.

VI. COMPLEX NETWORKS TO CHARACTERIZE THERMOACOUSTIC INTERACTIONS

Networks are common in both natural and man-made world. The vast network of neurons that constitute a brain and the social media networks are examples of such networks.⁴¹ We consider each entity of such networks as a node. Neurons are the nodes in the neural network of the brain, and each individual account in the social media is a node in the social media network. Each node of a network is connected to some or all the other nodes of the network. The connection between any two nodes is called an edge. A system where the interaction between a large number of constituents of the system results in the emergence of a new property, which cannot be observed at the level of the individual components, is known as a complex system. Many natural and man-made networks are complex systems. In the case of the brain, the emergent properties are intelligence, consciousness, emotions, etc. In social media networks, emergence of consensus, political echochambers, etc., are observed. Complex network analysis is an appropriate framework to study such complex systems.

In Secs. **I–V**, we discussed how a turbulent combustor can be considered as a complex system. We can characterize this complex system using complex networks. In this approach, the complex system is represented using a large scale network with heterogeneous connectivity. The connectivity patterns in a complex network determine the dynamics of the complex system. A plethora of network properties, which quantify the topology of the network, can then be used to characterize the dynamics of the complex system. Naturally, the characteristics of the network and the interpretation of the results depend on the strategy of construction of the complex network.

Murugesan and Sujith¹¹² introduced complex network analysis to the study of thermoacoustic instability. They studied the patterns in the time series of unsteady pressure as a combustor approached thermoacoustic instability by constructing visibility networks corresponding to pressure fluctuations during the different stages of combustor operation. Visibility networks are constructed using the method suggested by Lacasa *et al.*^{113,114} Each local peak in the time series is considered as a node of the complex network. Two nodes are connected if the peaks can be connected with a straight line that does not intersect the curve representing the time series at any other point in between these peaks. The topology of the network constructed in this manner is visualized using Gephi,¹¹⁵ a tool for network visualization and analysis. Mathematically, the network is represented as a matrix (A), known as the adjacency matrix. Its elements $A_{ij} = 1$ if the i th node in the network is connected to the j th node. The diagonal elements of A , $A_{ii} = 0$ in order to avoid self-connections.

Visibility networks corresponding to combustion noise, intermittency, and thermoacoustic instability are shown in Fig. 12. The color of the nodes represents the degree of the node (k), which is the total number of connections of a node in the network. For the i th node, $k = \sum_{j=1}^N A_{ij}$, where N is the total number of nodes of the network. During the occurrence of combustion noise, owing to the variability in the amplitude of the local peaks in the pressure time series, the variability in k is high. Peaks with higher values often have higher visibility and hence correspond to nodes with higher k . On the other hand, peaks with lower values tend to have lower visibility and hence correspond to nodes with lower k . Murugesan and Sujith¹¹² showed that the percentage of nodes with degree k , $P(k)$ has a power law relation with k , which can be expressed as $P(k) = k^{-\gamma}$. Here, γ is the power law exponent and is positive.

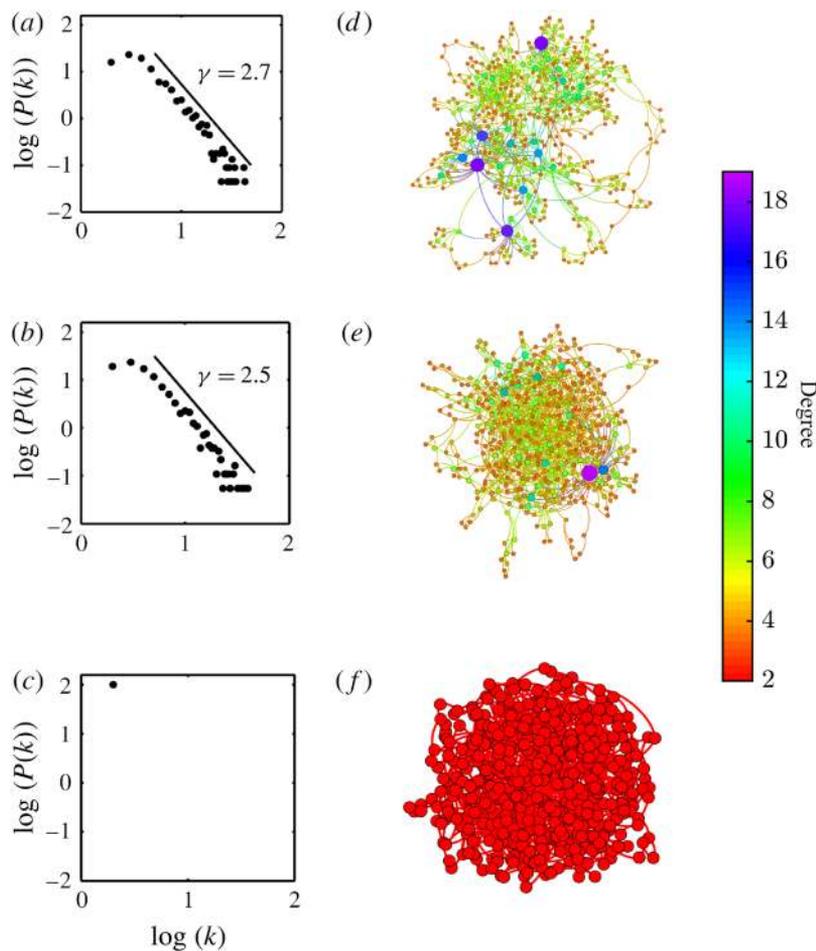


FIG. 12. Degree distribution and the visibility networks corresponding to combustion noise [(a) and (d)], intermittency [(b) and (e)], and thermoacoustic instability [(c) and (f)] constructed from the acoustic pressure measured from a combustor with a bluff body stabilized flame. The color of the node represents the degree of the node. Adapted with permission from M. Murugesan and R. I. Sujith, "Combustion noise is scale-free: Transition from scale-free to order at the onset of thermoacoustic instability," *J. Fluid Mech.* **772**, 225–245 (2015). Copyright 2015 Cambridge University Press.

They showed that for combustion noise in a bluff body stabilized combustor, $\gamma = 2.7$ and for a swirl stabilized combustor, $\gamma = 2.5$ [Fig. 12(a)]. The visualization of the corresponding network using Gephi¹¹⁵ shows the heterogeneity in connections in the network. The color and the size of a node correspond to the degree of the node.

During the occurrence of combustion noise, the degree is distributed across a wide range, indicating that combustion noise has a scale-free behavior. During thermoacoustic instability, the pressure fluctuations are sinusoidal, and hence, each local peak of the time series of pressure oscillations has visibility only to its immediate neighboring peaks. Hence, most nodes of the network tend to have $k = 2$. This corresponds to a single point in the plot of $\log[P(k)]$ vs $\log(k)$. In the visualization of the network, a clear change in the structures can be observed. During thermoacoustic instability, all nodes have $k = 2$. During intermittency, the network exhibits a mixed topology. Accordingly, $\gamma = 2.5$. In a more recent study, Gotoda *et al.*¹¹⁶ used a modified visibility network to characterize pressure fluctuations in a combustor close to flame blowout. They observed that the oscillations close to flame blowout also exhibit scale-free behavior. Later, Murayama *et al.*¹¹⁷ used statistical complexity of complex networks derived using visibility

criteria from the vorticity field of the reactive flow to obtain precursors to the onset of thermoacoustic instability. They showed that the vertex strength in the turbulence network and the community structure of the vorticity field describe the interaction of vortices during thermoacoustic instability.

Visibility networks can characterize the patterns in the time series of pressure fluctuations. However, characterization of the topology of the phase space and hence the dynamical nature of pressure fluctuations requires the analysis of the corresponding recurrence networks. Godavarthi *et al.*¹¹⁸ analyzed the recurrence networks corresponding to pressure fluctuations in a combustor. Recurrence networks are constructed from the recurrence matrix R described in Sec. III A. The elements of the corresponding adjacency matrix A for the recurrence network are

$$A_{ij} = R_{ij} - \delta_{ij}. \quad (8)$$

Here, δ_{ij} is the Kronecker delta, which is subtracted from R_{ij} in order to avoid self-connections in the recurrence network. When visualized using Gephi, recurrence networks preserve their topology. In order to compare the recurrence characteristics across different dynamic states, the size of the threshold (ϵ) is chosen appropriately

to construct R_{ij} . Here, ϵ is fixed. The recurrence networks for combustion noise, intermittency, and thermoacoustic instability are presented in Fig. 13. The color of the node in the network corresponds to the degree of the network. The network corresponding to combustion noise has an irregular structure. Furthermore, the degrees of the nodes are low during the occurrence of combustion noise. For thermoacoustic instability, the network has a circular topology, and the degree of each node is high. During the transition regime exhibiting intermittency, the network has a topology that has characteristics of both the network for combustion noise and that for thermoacoustic instability. Godavarthi *et al.*¹¹⁸ also showed that for an appropriate range of ϵ , the recurrence network corresponding to combustion noise has scale-free nature and that the scale-free behavior is lost during thermoacoustic instability.

In a more recent study, Godavarthi *et al.*¹¹⁹ used recurrence network analysis and recurrence quantification analysis to investigate the coupling between the unsteady heat release (\dot{q}') from the turbulent flame and pressure fluctuations. Using recurrence measures, such as correlation of probability of recurrence (CPR), joint probability of recurrence (JPR), determinism (DET), and recurrence rate (RR) of the joint recurrence matrix, they studied the synchronization transitions in a turbulent thermoacoustic system. Definitions for these measures are given in the work of Godavarthi *et al.*¹¹⁹ They showed that CPR and DET can be used to capture the occurrence of phase synchronization between \dot{q}' and p' during weak

thermoacoustic instability, and JPR and RR can be used to identify the occurrence of generalized synchronization during strong thermoacoustic instability. Interestingly, they also discovered an asymmetry in the bidirectional coupling between \dot{q}' and p' , where \dot{q}' is observed to exert a stronger influence on p' than the other way around. In another study, by estimating transfer entropy from a turbulent network constructed using velocity field and pressure fluctuations in a model rocket combustor, Hashimoto *et al.*¹⁰⁴ discovered a directional coupling between pressure fluctuations and local heat release rate fluctuations.

Okuno *et al.*¹²⁰ constructed complex networks consisting of cycle networks and phase space networks from the time series of pressure fluctuations in a model gas turbine combustor. They showed the presence of pseudo-periodicity and high-dimensionality in the dynamics of thermoacoustic instability. Furthermore, they showed the presence of power law distribution and small-world-like nature in the networks constructed from pressure time series.

Krishnan *et al.*¹²¹ investigated the spatiotemporal dynamics of acoustic power sources during the intermittency route to thermoacoustic instability using complex network theory. They studied the spatiotemporal dynamics of acoustic power sources in the reactive flow field of a combustor by constructing time-varying spatial networks corresponding to acoustic power production. Their study showed that small fragments of acoustic power sources, observed during combustion noise, nucleate, coalesce, and grow in size to form large clusters at the onset of thermoacoustic instability. Such growth of small clusters of acoustic power sources was found to occur during the growth of pressure oscillations during intermittency. In contrast, it was observed that during the decay of pressure oscillations during intermittency, such large clusters of acoustic power sources disintegrate into small ones.

VII. STRATEGIES FOR MITIGATION OF THERMOACOUSTIC INSTABILITIES

In gas turbine engines, often, acoustic dampers are used to suppress thermoacoustic instability.⁶ Acoustic dampers are tuned to absorb acoustic waves of a certain range of frequencies, thus suppressing the onset of thermoacoustic instabilities. There have been various other methodologies, such as fuel staging, injection of micro-jets, redesigning flame holding mechanisms, injectors, and nozzles, employed to mitigate thermoacoustic instability in different systems.⁶ All such strategies involve costly characterization and trial and error experiments to arrive at an optimal design. Recently, with the integration of renewable energy technologies that produce power at an unsteady rate into the power grid, gas turbine combustors that produce power need to operate over a wide range of power output levels in order to balance the grid power capacity. Thus, we need solutions to mitigate thermoacoustic instability for a wide range of operating conditions of gas turbine engines, which is designed to operate under different power levels using different fuels. This calls for new methods that are versatile, intelligent, and less costly. In this section, we focus on recent developments in three different approaches for mitigation of thermoacoustic instability that are based on complex system theory: (a) development of precursors that identify the proximity of a system to thermoacoustic instability, (b) methodologies to estimate the amplitude of thermoacoustic

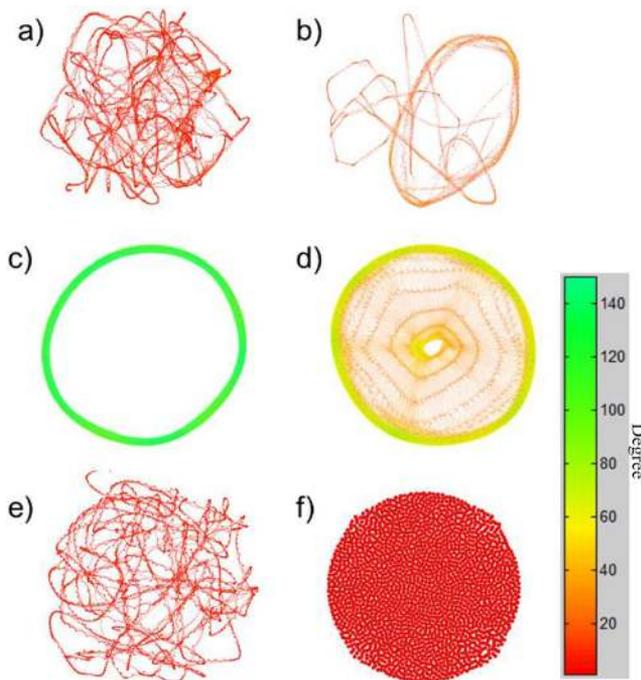


FIG. 13. Recurrence networks corresponding to combustion noise [(a) and (e)], intermittency [(b) and (d)], and thermoacoustic instability (c). (f) corresponds to a network of random noise. The color of the node represents the degree of the node. Image reproduced with permission from Godavarthi *et al.*, "Recurrence networks to study dynamical transitions in a turbulent combustor," *Chaos* 27, 063113 (2017). Copyright 2017 AIP Publishing LLC.

instability using data acquired far from the onset of thermoacoustic instability, and (c) smart passive and active control strategies for mitigating thermoacoustic instability.

A. Early warning systems for thermoacoustic instability

Nair *et al.*⁶² and Tony *et al.*³⁴ showed that combustion noise has chaotic characteristics, and this chaoticity is lost at the onset of thermoacoustic instability. Nair *et al.*⁶² further introduced the 0–1 test as a measure of the proximity of the combustor to an impending instability. They showed that this measure shows a smooth variation upon approaching thermoacoustic instability, enabling thresholds to be set for operational boundaries. In another study, Nair and Sujith²⁹ used the Hurst exponent H to characterize the onset of thermoacoustic instability. During the occurrence of combustion noise, the time series of pressure fluctuations showed fractal behavior, and hence, H is of the order of 0.2–0.3. As the system approaches the onset of thermoacoustic instability, where the pressure fluctuations are periodic, the corresponding H approaches zero. Since the variation in H is significant and occurs well ahead of the variation of traditional measures in indicating the onset of instability, it serves as a robust precursor to the onset of thermoacoustic instability.

We saw in Secs. I–VI that as thermoacoustic instability is approached, turbulent combustors exhibit intermittent oscillations. This makes the approach to thermoacoustic instability in turbulent systems rather gradual than the abrupt transitions observed in laminar combustors. As a consequence, measures from recurrence quantification analysis that quantify intermittency can possibly act as precursors to the onset of thermoacoustic instability.

Nair *et al.*^{35,122,123} and Gotoda *et al.*⁸⁰ used recurrence quantification analysis to quantitatively characterize the phase space topology corresponding to the pressure fluctuations in a turbulent combustor, thereby providing precursors to the onset of an impending thermoacoustic instability. First, the phase space corresponding to the pressure fluctuations inside the combustor was constructed following Takens' embedding theorem.^{35,124} Recurrence plots that represent the topology of the phase space were constructed as discussed earlier in Sec. III A. Then, the patterns in the recurrence plot were statistically quantified using various measures, such as recurrence rate (RR), trapping time (τ_0), and Shannon entropy (s).

Figure 14 shows the variation of RR , s , and τ_0 as a bluff body [Figs. 14(a)–14(c)] and a swirl [Figs. 14(d)–14(f)] combustor approach thermoacoustic instability. The particular meaning associated with each of these recurrence quantities is dependent on the manner in which the corresponding recurrence plot is constructed

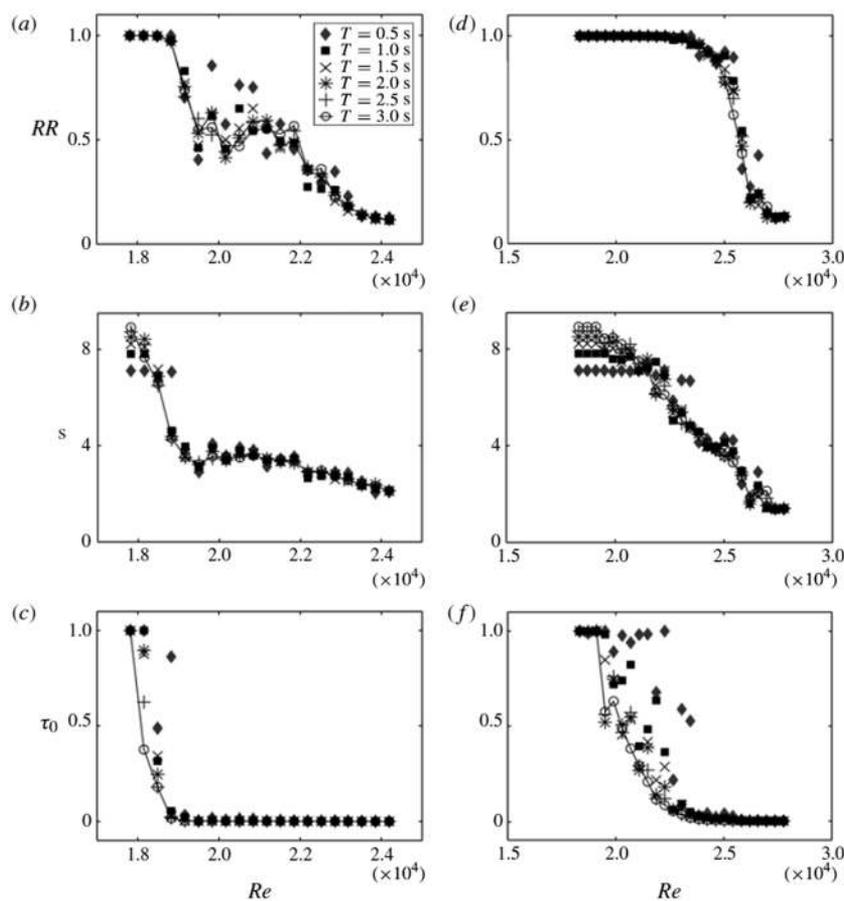


FIG. 14. Variation of RR , s , and τ_0 as thermoacoustic instability is approached in a swirl stabilized combustor [(a)–(c)] and bluff body combustor [(d)–(f)]. Different recurrence measures have different levels of sensitivity for detecting the onset of thermoacoustic instability. Here, T is the length of the time series used to estimate the recurrence measure. Reproduced with permission from V. Nair, G. Thampi, and R. I. Sujith, “Intermittency route to thermoacoustic instability in turbulent combustors,” *J. Fluid Mech.* **756**, 470–487 (2014). Copyright 2014 Cambridge University Press.

and the definition of the ϵ -neighborhood used for recurrence quantification analysis. In Fig. 14, we observe that thermoacoustic instability is approached as the Reynolds number (Re) of the flow is increased. A system's tendency to repeat its states is characterized by RR . In Figs. 14(a) and 14(d), RR reduces as thermoacoustic instability is approached. s measures the entropy of recurrences in the system. As thermoacoustic instability is approached, the phase space and hence the recurrence plot become more ordered, and hence, s reduces [Figs. 14(b) and 14(e)]. τ_0 characterizes the duration for which the system is trapped in the same part of the phase space, and it reduces as thermoacoustic instability is approached [Figs. 14(c) and 14(f)]. One can see that as thermoacoustic instability is approached, each recurrence parameter varies with a different level of sensitivity and a different level of robustness. Hence, one can use multiple recurrence measures in conjunction with each other to construct a reliable and versatile warning system for the onset of the thermoacoustic instability that prevents false positive and false negative warnings.

In a similar manner, parameters that represent the topology of recurrence networks can also be used as precursors to the onset of thermoacoustic instability. Godavarthi *et al.*¹¹⁸ showed that parameters derived from a recurrence network serve as precursors to dynamic transitions in a turbulent combustor. In another study, Godavarthi *et al.*¹¹⁹ showed that one can use cross-recurrence analysis to characterize the coupling between the unsteady pressure and fluctuating heat release rate, thereby detecting the onset of thermoacoustic instability.

The onset of thermoacoustic instability can also be detected by quantifying patterns in the time series of pressure fluctuations. Murugesan *et al.*^{125,126} used visibility networks for characterizing patterns in the time series and developed precursors for thermoacoustic instability. Unni *et al.*^{127,128} used symbolic time series analysis for unraveling hidden patterns in the time series. Furthermore, they used a parameter known as an anomaly measure that quantitatively compares patterns in the time series of a state variable at any operational condition to the pattern of the time series during thermoacoustic instability. The anomaly measure, in turn, serves as a precursor to the onset of thermoacoustic instability. Note that all the precursors mentioned above can also detect other dynamic transitions in turbulent combustors, such as blowout and flashback.^{108,118,127,129,130} Furthermore, they have also been used as precursors to detect the onset of oscillatory instability in other systems that exhibit emergence of such instabilities induced by turbulent flows. Various instability detection systems that employ these methodologies have been patented recently.^{123,125,127,131}

B. Estimating the amplitude at the onset of thermoacoustic instability

Another important aspect in devising a mitigation strategy is the ability to predict the amplitude of limit cycle oscillations during thermoacoustic instability in order to appropriately design the control mechanisms and estimate the additional load on the structure of the engine due to the pressure oscillations. For example, the onset of a low amplitude thermoacoustic instability may not pose any problem to the operation.

Traditionally, the amplitude of limit cycle oscillation is estimated using flame transfer functions/flame describing functions that

characterize the linear/nonlinear response of a flame to perturbations. While success has been reported in predicting the amplitude of oscillations in this manner, such an analysis requires costly experiments to characterize the response of a flame to different types of external forcing in the range of amplitudes and frequencies. Furthermore, designing actuators that can produce high amplitude oscillations comparable to the amplitude of limit cycle oscillations is very difficult. In addition, it is not desirable to excite such high amplitudes in high pressure rigs or in gas turbine combustors.

Recently, Seshadri *et al.*⁹⁰ introduced a novel methodology based on intermittency statistics for predicting the amplitude of pressure oscillations during thermoacoustic instability. For combustors where thermoacoustic instability is associated with periodic vortex formation in the reactive flow, they considered the acoustic field of the thermoacoustic system as a kicked oscillator. During every cycle of vortex formation, the vortex causes a roll-up of the flame front encapsulating the unburned reactants as it rolls up. When the vortex impinges on the walls of the combustor or flame holding mechanisms, the encapsulated unburned reactants undergo reactions causing a localized and near instantaneous heat release. Such "kicks" of heat release, in turn, add energy to the acoustic field. As described in Sec. III C, such a model can capture the transition to thermoacoustic instability via intermittency as observed in turbulent combustors.

The model used for estimation of amplitude is a much simplified version of the model described in Sec. III C. A damped simple harmonic oscillator of frequency ω and damping ξ is considered. It is kicked with a strength B at time instants t_j , otherwise known as kicking times,

$$\ddot{p}' + \xi \dot{p}' + \omega^2 p' = B \sum_j \delta(t - t_j). \quad (9)$$

The distribution of t_j determines the characteristics of the time series, $p'(t)$ (herein, representing the pressure fluctuations in a combustor). For randomly distributed t_j , $p'(t)$ is random and has low amplitude resembling combustion noise. For t_j equally spaced in time such that $t_j - t_{j-1} = 2\pi/\omega$ for all j , $p'(t)$ is periodic and has maximum amplitude, resembling limit cycle oscillations during thermoacoustic instability. One can then construct $p'(t)$ with the characteristics of intermittency by choosing $t_j - t_{j-1} = 2\pi/\omega$ for some js , and $t_j - t_{j-1}$ is a random number for the remaining js . Depending on the ratio between the number of js for which $t_j - t_{j-1}$ is a random number and the number of js for which $t_j - t_{j-1} = 2\pi/\omega$, $p'(t)$ would either be predominantly aperiodic or periodic. We call this ratio the aperiodic probability (P_a), and it roughly indicates the fraction of the time for which an intermittent signal exhibits aperiodicity.

In thermoacoustic systems with turbulent flows, the aperiodic part of the oscillations during intermittency has a lower amplitude when compared to the periodic part. Hence, from experimental data, P_a can be estimated as the ratio of the duration of $p'(t)$ where the local envelope of $p'(t)$ is below a set threshold (T_{ap}) and the total duration of the intermittent signal, $p'(t)$ (T_{tot}). The local envelope of $p'(t)$ is obtained through the Hilbert transform of $p'(t)$, $H(p'(t))$. Using the kicked oscillator model described above, we obtain synthetic time series $p'_s(t)$ that have the same P_a as that of $p'(t)$, as a

solution to Eq. (9) by selecting t_j such that

$$t_j = t_{j-1} + (1 - C(P_a))2\pi/\omega + C(P_a)\sigma 2\pi/\omega |N(0, 1)|. \quad (10)$$

Here, C is a biased coin toss in which one occurs with probability P_a and zero occurs with a probability of $1 - P_a$. t_0 is zero. σ is the variance of Gaussian white noise $N(0, 1)$. σ was chosen as five by Sheshadri *et al.*⁹⁰ for combustion noise and one otherwise.

For an intermittent signal measured from a combustor, both P_a and P_{rms} are estimated. Now, parameters of the kicked oscillator model are optimized to create an intermittent signal with the same statistics (i.e., P_a and P_{rms}) as the signal from experiments. Here, the parameters of the kicked oscillator model are ω , ξ , and B . ω and ξ can be obtained from experiments.⁹⁰ The only remaining variable B is identified by optimizing P_{rms} for a given P_a . Once the parameters of the model are optimized, the amplitude represented in terms of root mean square of periodic oscillations in the thermoacoustic system is estimated as the amplitude of $p'(t)$ for $P_a = 0$ obtained from the optimized model.

C. Passive control strategies

Passive control strategies are often favored for control of thermoacoustic instability since they are cheaper and long lasting. Acoustic dampers are used to absorb the acoustic energy produced due to unsteady combustion and thus avoid the growth of the amplitude of pressure oscillations inside the combustor. Designing an appropriate damper requires the knowledge of the linear growth rates of acoustic pressure inside a thermoacoustic system. However, direct measurement of the linear growth rate is difficult since the time scales associated with the variation in the engine operating conditions are considerably larger than the time scales associated with the exponential growth of acoustic oscillations. This implies that one needs to measure the linear growth rate of a system while it exhibits limit cycle oscillations. Noiray and Schuermans⁸⁵ developed a methodology to measure the linear growth rates of a thermoacoustic system during the occurrence of thermoacoustic instability. They modeled the effect of turbulence as stochastic forcing by turbulent flow and developed stochastic differential equations describing thermoacoustic instability. The stochastic forcing by turbulence makes limit cycle oscillations have “random” amplitude modulation. They showed that by measuring these modulations in amplitude and using the stochastic differential equations describing thermoacoustic instabilities, one can estimate the unknown deterministic quantities in the stochastic differential equation, namely, the linear growth rate.

We saw in Sec. III B that thermoacoustic instability is the result of emergence of a standing wave pattern in confined reactive flow resulting from the interaction between various subsystems (acoustics, hydrodynamics, and combustion) influenced by the emerging pattern. Thus, affecting changes to any of the subsystems will result in changes in other subsystems and in the emergent pattern. For example, by introducing optimized dampers, one can influence the acoustic subsystem in such a way as to cause suppression of thermoacoustic instability. George *et al.*¹³² showed that on increasing turbulence intensity in a combustor with a swirl stabilized flame by introducing turbulence generators upstream of the combustion chamber, the coherence in the local acoustic power production is reduced. For an increase in turbulence intensity, this resulted in the

reduction of the amplitude of pressure oscillations during thermoacoustic instability. However, they observed that an increase in turbulence intensity advanced the onset of thermoacoustic instability in the parameter space.

1. Smart passive control

Complex networks can be used to characterize the spatiotemporal dynamics of a thermoacoustic system. Unni *et al.*¹³³ characterized the flow dynamics at different operational regimes of a combustor by analyzing the topology of a complex network derived from the velocity field. The velocity field of the turbulent combustor (with a bluff body stabilized combustor) was obtained from particle image velocimetry (PIV). Each grid point in the velocity field was considered as a node of the complex network. Two nodes of the network, i and j , were connected if the Pearson correlation (PC_{ij}) of the speeds of flow between those nodes was above a critical threshold,

$$PC_{ij} = \frac{\sum_{t=1}^n (V_i(t) - \bar{V}_i)(V_j(t) - \bar{V}_j)}{\sqrt{\sum_{t=1}^n (V_i(t) - \bar{V}_i)^2} \sqrt{\sum_{t=1}^n (V_j(t) - \bar{V}_j)^2}}. \quad (11)$$

Here, $V_i(t)$ is the magnitude of velocity at the i th node at time instant t . The threshold correlation coefficient PC_{thr} was chosen as 0.25 to ensure the maximum variation in the link density ρ for the constructed network across different dynamic states of the combustor. ρ is defined as

$$\rho = \frac{2E}{N(N-1)}, \quad (12)$$

where E is the actual number of links in the network, and $N(N-1)/2$ is the maximum number of possible links of a given undirected network with N nodes.

For each dynamic state, a complex network is constructed in this manner. Similar to the previous cases of network construction, here also the network is represented by an adjacency matrix A whose element A_{ij} is one if nodes i and j are connected and zero otherwise. In addition, self-connections are avoided by ensuring that the diagonal elements of A_{ij} are zero. The topology of the networks is then characterized by estimating various centrality measures for each node of A , such as degree (k_i), betweenness centrality (b_i), closeness centrality (C_i), and clustering coefficient (CC_i). Definition for each of these measures is given in the work of Unni *et al.*¹³³

Relative strengths of k_i , b_i , C_i , and CC_i were then studied to identify the critical nodes of the network responsible for the establishment of global connectivity in the network and hence the emergence of thermoacoustic instability. A high value of k_i indicates that the node is connected to many other nodes. A large value of b_i suggests that the node is critical for the global connectivity in the network due to the high number of shortest paths that pass through them. A high value of C_i is indicative of the node being closely connected to many other nodes in the system. Thus, nodes with high k_i , b_i , and C_i are the backbones of the connectivity in the system. CC_i indicates the interconnectivity between the neighbors of a node. A small value of CC_i implies that a relatively less number of neighbors of the node are interconnected, and hence, a small change in the connectivity of the node could affect a large change in the topology of the network. Thus, Unni *et al.*¹³³ suggested that the nodes for which k_i , b_i , and C_i are high and CC_i is low correspond to critical regions in the flow. Furthermore, they proposed that the critical region identified in such network analysis might serve as the optimal location

where a control action to inhibit thermoacoustic instability needs to be directed at.

This methodology of identifying critical regions responsible for the onset of thermoacoustic instability was further refined by Krishnan *et al.*¹³⁴ by using weighted networks instead of unweighted networks. They estimated the strength of a node (s), weighted local clustering coefficient (\bar{C}) and weighted closeness centrality (\bar{c}) in order to identify the critical region in the reaction field. Definitions for s , \bar{C} , and \bar{c} are provided in the work of Krishnan *et al.*¹³⁴ In the critical region, s , \bar{C} , and \bar{c} are high. They identified a region inside the bluff body combustor as the critical region, and the flow in this region was perturbed by injecting microjets. As a result, thermoacoustic instability was suppressed above a critical value of momentum flux of the microjet. They showed that the critical region identified through network analysis is the most effective location in suppressing thermoacoustic instability through microjet injection. The critical locations identified by the network analysis where microjet injection was performed are shown in Fig. 15(a). In Fig. 15(b), we can see that with the introduction of the microjets, the critical locations no longer remain to be hubs in the network. As a result, thermoacoustic instability is suppressed.

D. Control strategies based on synchronization

1. Open-loop forcing

Open-loop control has been proposed and demonstrated as one of the strategies to mitigate thermoacoustic instability over the years. In this method, the combustor flow field is perturbed at off-resonance frequencies through external sources, such as actuators or speakers.¹³⁵ This, in turn, breaks the positive coupling developed between the acoustic field and the heat release rate fluctuations during the state of thermoacoustic instability, thus quenching these instabilities. In the previous studies on open-loop control,^{136–138} the implementation of acoustic forcing was selected based on experimental experiences. The recent progress in application of synchronization theory to thermoacoustic systems has led to a systematic

approach^{139–143} that relies on the framework of forced synchronization¹⁴⁴ to mitigate such instabilities.

When the self-excited natural oscillations in a system are externally forced at different values of amplitude and frequency of forcing,⁹² in the context of forced synchronization theory, the response of the forced system is characterized in terms of Arnold tongue. Arnold tongue is a plot between the critical values of amplitude and frequency of the forcing where forced synchronization occurs in the system. At the onset of forced synchronization, the temporal variation of the instantaneous phase difference between the natural and forcing signals becomes bounded (phase locked), and the system oscillates only at the forcing frequency. Forced synchronization of oscillators usually occurs through two routes—(i) locking route and (ii) suppression route.¹⁴⁴

In the locking route, when the amplitude of forcing is increased gradually for a fixed forcing frequency, the natural frequency moves toward the forcing frequency and gets locked with the forcing at a critical value of the forcing amplitude. Conversely, in the suppression route, the natural frequency peak in the spectrum diminishes gradually without exhibiting any movement toward the forcing frequency and displays a complete suppression during the occurrence of forced synchronization. The states of the coupled dynamics of a forced system are further classified as phase drifting, phase trapping, and phase locking. Phase drifting indicates the desynchronization behavior of the oscillator with the forcing signal due to the unbounded variation of the relative phase between the signals. In contrast, during phase trapping, although the relative phase is bounded, it exhibits oscillatory behavior in time.

Using the framework of forced synchronization, Balusamy *et al.*¹⁴² showed the presence of different states, such as phase drifting, phase trapping, and phase locking in the forced response of the acoustic pressure oscillations developed during thermoacoustic instability in a swirl stabilized turbulent combustor. They also showed the existence of behaviors, such as frequency pulling and pushing in the forced response of this system for different

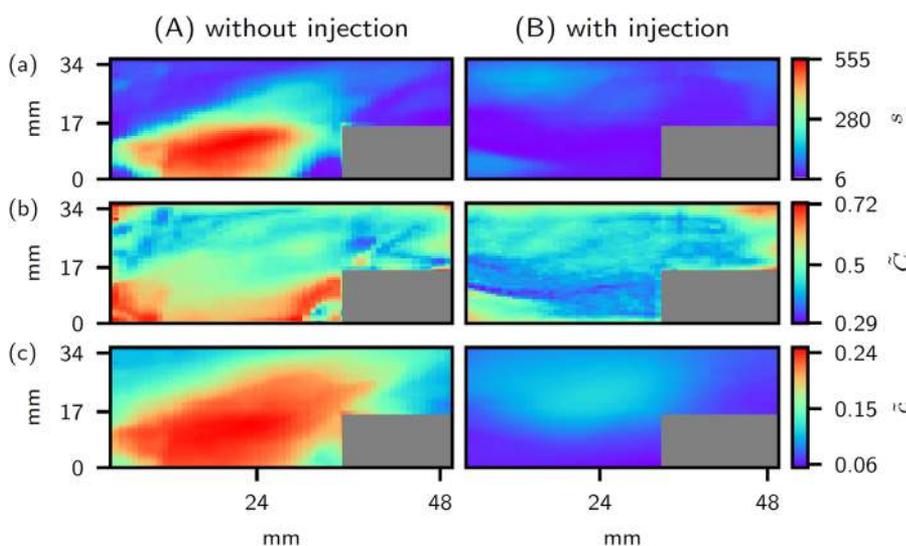


FIG. 15. Network properties (s , \bar{C} , and \bar{c}) estimated during (a) thermoacoustic instability and (b) during suppression of thermoacoustic instability achieved with micro-jet injection. Note that the micro-jet is injected targeting the critical regions represented by locations with high value of network properties [red region in (a)]. We observe that the region on top of the bluff body shaft no longer remains as the critical region when microjets are introduced and hence thermoacoustic oscillations are suppressed. Reproduced with permission from Krishnan *et al.*, "Mitigation of oscillatory instability in turbulent reactive flows: A novel approach using complex networks," *Europhys. Lett.* **128**, 14003 (2019). Copyright 2019 Cambridge University Press.

conditions of periodic forcing. Kashinath *et al.*¹⁴¹ extended the analysis of forced synchronization to quasiperiodic and chaotic oscillations in a model of ducted laminar premixed flame. They also showed the presence of different routes to forced synchronization, such as locking and suppression in their model. Subsequently, Guan *et al.*¹⁴⁰ and Mondal *et al.*¹³⁹ showed the experimental evidence of these routes in a ducted laminar premixed flame burner and an electrically heated Rijke tube, respectively. Furthermore, these studies proposed that the physical mechanism behind the mitigation of thermoacoustic instability through open-loop forcing is asynchronous quenching that occurs when forcing frequency is sufficiently far from the natural frequency of thermoacoustic instability. During asynchronous quenching, the instantaneous phase angle between the acoustic and heat release rate fluctuations oscillates around 90° , leading to quenching of thermoacoustic oscillations due to the presence of alternate cycles of acoustic damping and driving in the system. Recently, Roy *et al.*¹⁴³ simultaneously studied the forced synchronization characteristics of both the acoustic pressure and the heat release rate fluctuations during thermoacoustic instability in a ducted premixed laminar flame combustor. They showed that the phenomenon of asynchronous quenching happens in the region of forcing where the underlying flow field possesses a preferred mode of the oscillations. Furthermore, Mondal *et al.*¹³⁹ observed a phenomenon called synchronance, i.e., synchronization leading to resonant amplification, in the forced Rijke tube system when forcing is imposed near the natural frequency of the acoustic oscillations.

Gopakumar *et al.*¹⁴⁵ showed that in a swirl combustor, by introducing a rotating swirler and by appropriately choosing the rotational speed of the swirler, one can suppress the onset of thermoacoustic instability.^{146,147} Furthermore, Gopakumar *et al.*¹⁴⁸ showed that the mitigation of thermoacoustic instability due to rotating swirler happens through an intermittency route. They were able to further develop a phenomenological model based on synchronization of Kuramoto oscillators that was able to mimic the dynamic behavior observed in experiments.¹⁴⁹

Thus, significant advances have been made in the open-loop control of laminar thermoacoustic systems through its implementation in the framework of forced synchronization. However, implementation of open-loop control to complex turbulent combustors would be challenging and needs further attention.

2. Amplitude death

When oscillators are coupled to each other, they exhibit interesting phenomena, such as phase locking and synchronization, phase flip bifurcation, and oscillation quenching. Weak coupling causes their interaction to be limited to their phases, causing phase locking or synchronization. In contrast, strong coupling can affect the amplitude in addition to the phase and can even lead to complete cessation of oscillations. Such oscillation quenching has two manifestations: amplitude death (AD) and oscillation death (OD). In AD, the oscillations of the individual oscillators cease, and all oscillators reach the same steady state of the system. In contrast, during OD, individual oscillators occupy altered steady states, which are different from the original steady state of the system. Mitigating thermoacoustic instabilities using AD or OD seems to be an attractive proposition, as this does not involve any complications of actuators or electromechanical feedback.

Recently, Thomas *et al.*¹⁵⁰ investigated AD in thermoacoustic systems using a mathematical model of coupled horizontal Rijke tubes. They investigated the effect of time delay and dissipative couplings and showed that AD can be achieved in such systems. They showed that time delay coupling of sufficient strength, and appropriate delay can lead to AD. In contrast, for dissipative coupling to lead to AD, there needs to be sufficient detuning, i.e., difference in the natural frequencies of the two oscillators. They also showed that simultaneous application of the two couplings can more easily lead to AD, i.e., when the lesser coupling strength is lesser.

Practical thermoacoustic systems are subjected to noisy or turbulent fluctuations. Modeling these fluctuations as additive white noise, Thomas *et al.*¹⁵¹ investigated if AD can be achieved in the presence of these fluctuations. They showed that although a complete cessation of oscillations or AD is not possible in the presence of noise, a significant reduction in the amplitude of coupled limit cycle oscillations can be attained by the application of strong coupling. Dange *et al.*¹⁵² demonstrated AD in experiments involving two horizontal Rijke tubes. They coupled the Rijke tubes using a single connecting tube whose length and diameter were varied. They postulate that the length and diameter of the coupling tube indicate the time delay and the coupling strength, respectively. At specific coupling conditions, they showed that the coupled Rijke tubes exhibited amplitude death even though under the same operating conditions with no coupling, both Rijke tubes exhibited thermoacoustic instability. This work on AD was extended to turbulent combustors by Jegal *et al.*¹⁵³ in an experimental study of mutual synchronization between two coupled combustors. Combustors housed lean-premixed swirl stabilized turbulent flame and are subjected to either symmetric or asymmetric inlet boundary conditions. When two combustors oscillating at different natural frequencies were coupled, they mutually synchronized with each other, exhibiting global oscillations at a common frequency. Jegal *et al.*¹⁵³ observed that such a coupling can induce strong oscillations, even when each combustor is individually stable in isolation if they share symmetric boundary conditions. Furthermore, for certain asymmetric inlet boundary conditions, the coupling between the combustors resulted in amplitude death, even though each combustor in isolation exhibited large amplitude limit cycle oscillations.

VIII. OSCILLATORY INSTABILITIES IN OTHER FLUID SYSTEMS

Oscillatory instabilities are common in fluid flows. The methodologies and techniques from complex system theory that we employed to study thermoacoustic instabilities can also be used to study other flow induced oscillatory instabilities in fluid dynamic systems. Two such oscillatory instabilities are aeroacoustic instabilities and aeroelastic instabilities. In Secs. VIII A and VIII B, we give brief descriptions of the recent advances to study aeroacoustic and aeroelastic instability using the complex system approach.

A. Aeroacoustic systems

Turbulent flow passing through a confinement causes pressure fluctuations. These fluctuations are called sound if the pressure fluctuations propagate or pseudo-sound if the pressure fluctuations are non-propagating.¹⁵⁴ Such fluctuations are characterized by a multiplicity of time scales associated with the turbulent flow and the

acoustic field of the confinement. These fluctuations can undergo self-organization, when one of the local hydrodynamic time scales matches an acoustic time scale, resulting in large amplitude self-sustained periodic oscillations. Examples of self-sustained aeroacoustic oscillations include whistling in pipes (pipe tones or Pfeifentöne),¹⁵⁵ edge tones,¹⁵⁶ cavity noise,¹⁵⁷ howling of ejectors,¹⁵⁸ and screech in jets with shocks.¹⁵⁹

Flow through an orifice in a duct can under the right conditions generate strong coherent sound; this phenomenon is called whistling. When air passes through the orifice, shear layer separation occurs at the upstream edge, shedding vortices. This separated shear layer can interact with the downstream edge, and the resulting acoustic waves propagate upstream and interact with the shear layer at the leading edge. If the phase of the pressure fluctuations are favorable to the phase of shear layer oscillations and the vortex shedding that occurs upstream, the vortex shedding process is amplified. This feedback mechanism,¹⁶⁰ where an acoustic disturbance that propagates upstream from the point of vortex impingement, induces vorticity fluctuations at the origin of the shear layer. These vorticity fluctuations, in turn, generate and amplify the acoustic field; this amplification, if sufficiently strong, can lead to the establishment of pipe tones.

From a complex system perspective, Nair and Sujith³⁸ showed that this aeroacoustic instability behaves in a very similar way to thermoacoustic instabilities. During stable operation, the pressure fluctuations are aperiodic. They transition to periodic oscillations via intermittency, i.e., a state where there are epochs of periodic bursts dispersed amid epochs of aperiodicity, just as in the case of thermoacoustic instability. As is the case with thermoacoustic systems, the pressure signal during stable operation is characterized by a broad multifractal spectrum; the multifractal spectrum collapses at the onset of aeroacoustic instability. Further measures, such as the Hurst exponent, Shannon entropy, average passage time, and 0–1 test measure, vary smoothly as we approach an impending instability, well ahead of the rise in pressure amplitude, thus providing us with precursors.³⁸ CFD simulations of aeroacoustic phenomena using LES were able to capture features, such as intermittency and multifractality.¹⁶¹

B. Aeroelastic systems

Aeroelastic instability is the result of transfer of energy from oscillatory flow to a flexible structure immersed in the flow and results in limit cycle oscillations of the structure. Korbahti *et al.*¹⁶² discovered that the onset of limit cycle oscillations in an aeroelastic system with turbulent flow happens through a regime of intermittency. This is similar to that observed prior to the onset of thermoacoustic instability. Later, other studies also reported such intermittent oscillations prior to the onset of aeroelastic instabilities.^{39,163,164} It was also observed that the characteristics of intermittent oscillations depend on the time scales of fluctuations in the flow relative to the time scales of the aeroelastic system.¹⁶⁵ Multifractal characteristics of aeroelastic oscillations were used as precursors to flutter by Venkatramani *et al.*¹⁶⁶

Recently, Raaj *et al.*¹⁶⁴ analyzed transition to aeroelastic instability in the framework of synchronization theory. They characterized the transition to limit cycle oscillations via intermittency based on the synchronization of pitch and plunge dynamics of an airfoil.

In a model for aeroelastic instability, they also showed that noise with different time scales results in different types of intermittency. Noise with time scales much larger than the time scales of the limit cycle oscillations resulted in on–off type of intermittency. In contrast, if time scales of noise were smaller than the time scales of the limit cycle oscillations, intermittency resembled that prior to thermoacoustic instability observed in turbulent combustors.

Thus, the main findings described in Secs. I–VII based on the application of complex system theory to thermoacoustic systems indeed hold good for aeroacoustic and aeroelastic systems, and possibly for any oscillatory instability arising in turbulent flows.

IX. EMERGING CHALLENGES AND THE WAY FORWARD

Until recently, thermoacoustic instability was associated with only two states: fixed points and limit cycles. The discovery of states, such as intermittency, quasiperiodicity, and chaos, in thermoacoustic systems with laminar flow ushered in dynamical system theory into the study of thermoacoustics. The complex system approach followed soon and helped make rapid strides in understanding the onset of thermoacoustic instabilities in turbulent combustors. The present article focuses on the latter.

The stable state of a thermoacoustic system was shown to be high dimensional chaos, contaminated by white and colored noise. The transition from stability to instability occurs through a state of intermittency, where epochs of periodicity are interspaced between epochs of aperiodicity in an apparently random manner. Studies unraveled the flow physics behind intermittency. While different flow systems have different physical mechanisms that cause intermittency, the intermittency route to oscillatory instability was itself found to be a common feature in all systems examined so far including hydrodynamic instabilities in systems with density stratification,^{71,72} aeroacoustic³⁸ and aeroelastic instabilities,³⁹ and compressor surge⁴⁰ that feature turbulent flow.

During stable operation, the pressure fluctuations display multifractality, which indeed reflects the existence of multiple spatial and temporal scales in the turbulent reacting flow. In contrast, thermoacoustic instability is characterized by a single or at most a few discrete scales. Thus, multifractality is lost at the onset of thermoacoustic instability. As we go through intermittency en route to thermoacoustic instability, the loss of multifractality is gradual.

Furthermore, complex networks, such as visibility graphs and recurrence networks, were used to study transitions in thermoacoustic systems. Scale-free behavior was observed during the occurrence of combustion noise. This scale-free behavior is lost, and the network transitions to a regular network at the onset of thermoacoustic instability.

Traditionally, we implicitly adopt a signal plus noise paradigm. The irregular fluctuations that we see in the measurements are a direct consequence of the inherent complexity of the thermoacoustics of turbulent reacting flow systems. Separation of these measurements into a signal and noise is like throwing the baby out with the bathwater.

Thus, rapid strides were made in the application of time series analysis to study thermoacoustic transitions. These advances were

used to detect transitions in thermoacoustic systems—not just thermoacoustic instability but also blowout. Furthermore, we find that the above summarized discoveries hold true even for oscillatory instabilities that do not involve combustion—e.g., hydrodynamic instability, aeroacoustic instability, aeroelastic instability, and compressor surge. Recent studies even show that there exists universal scaling behavior across different systems that exhibit transition to oscillatory instability via intermittency.^{167,168} Such scaling laws can be exploited to predict amplitude at the onset of the oscillatory instability by performing measurements at conditions far from the onset. Measures, such as Hurst exponents, intermittency statistics obtained using recurrence quantification analysis, and measures from complex network theory, were used to obtain precursors that provide early warning to the onset of instabilities, well ahead of the rise in pressure amplitudes. It is interesting to note that the same measures work as precursors to blowout.¹³¹ Furthermore, we can now predict the amplitude at the onset of instability, by using time series data obtained during stable operation or during intermittency. These developments are now being translated to industrial gas turbine combustors. The next decade may see AI based precursors or hybrid precursors with AI combined with precursors from complex system theory gaining ground.¹⁶⁹

Multifidelity LES computations have reached an advanced stage. They are currently validated with experiments by comparing the rms value of acoustic pressure, which is really a gross measure. Two simulations with completely different time series can give the same rms value for acoustic pressure. In contrast, measures, such as Hurst exponents and multifractal spectrum, are much tighter measures that can be used to enable stricter validation of the simulations with experimental data.¹⁶¹

Phenomenological models that predict intermittency are now available. More work needs to be done to model bursts in thermoacoustic systems as a slow–fast system. Moreover, the models do not predict the multifractal characteristics. Further progress needs to be made to develop amplitude equations from the partial differential equations. Data driven discovery of governing equations is another cutting edge approach that may pay rich dividends in thermoacoustics.

A beginning has been made to study the spatiotemporal dynamics of thermoacoustic systems using the frameworks of (1) synchronization theory, (2) complex networks, (3) fractal and multifractal analysis, and (4) pattern formation. The study of the spatiotemporal dynamics is indeed the next frontier.

Complex systems are known to display pockets of order amid disorder. When we examine the time series during the occurrence of intermittency, we find bursts of high amplitude periodic oscillations, which correspond to pockets of order, scattered amid epochs of low amplitude aperiodic fluctuations, which correspond to pockets of disorder. Spatiotemporal analysis using synchronization theory reveals that during the occurrence of intermittency, we see chimera states, where pockets of order co-exist with disorder. During the emergence of order, large coherent vortices emerge from the collective interaction of small vortices.

Synchronization theory provides the framework for open-loop control. Recent studies have shown that forcing a thermoacoustic system at frequencies far from the instability frequency can mitigate the oscillations due to asynchronous quenching. Furthermore,

the concept of amplitude death where two self-excited oscillators can be coupled to quench the oscillations has been implemented in thermoacoustic systems.

Time varying spatial networks have been constructed to characterize the onset of thermoacoustic instability through the intermittency route. We see that clusters of acoustic driving occur through a dynamic nucleation and agglomeration process. Spatial networks have been used to identify critical regions in the reactive flow field that are vulnerable to disturbances. Passive control of thermoacoustic instability can be achieved by disrupting such regions by mildly altering the flow field. This strategy enables us to develop smart passive control. Such analysis can be performed even at a design stage using LES simulations. The next step is to construct multilayer networks, to unravel the connection between the formation of the large coherent structures and the emergence of regions of positive acoustic power. Modeling thermoacoustic instability as a phase transition in a network could be a possible approach that could yield rich dividends. A statistical mechanics approach to solving thermoacoustic instability may be possible through complex networks.

Turbulent flow has multifractal characteristics. At the onset of thermoacoustic instability, the multifractal spectrum corresponding to the flame topology is seen to oscillate periodically, and this leads to the emergence of periodicity in time series. Analysis of local fractal dimensions can be used to detect critical regions in the flow field, which can be disturbed to implement passive control. Lagrangian coherent structures can also be used to identify structures that produce sound.¹⁷⁰ The role of coherent structures in combustion control has been studied in the past.¹⁷¹ Further studies utilizing the complex system approach are needed to identify the hidden connectivities between coherent structures and exploit it for control of combustion dynamics.

One issue that has not been addressed so far using complex system theory is the onset of thermoacoustic instability in systems with preheat. In such systems, the wake of the flame holder is unstable. In contrast, in systems without preheat, the wake is hydrodynamically stable.

Improving our understanding of thermoacoustic instabilities and controlling them will require the application of the latest advances in complex system theory to this challenging and exciting field. To achieve this, groups that do laser diagnostics and groups that do LES need to work together with researchers who work with complex system theory, to be able to transcend the boundaries between engineering, physics, and mathematics. We also face a new frontier in translating the lessons learnt from the complex system approach into design and engineering practices.

The real test for complex system theory is its ability to provide something useful to the designer to mitigate thermoacoustic instability and to the engineer on the field to evade it. Complex system theory itself needs to be advanced to accommodate the complexities of turbulent flow; this is indeed an exciting prospect.

ACKNOWLEDGMENTS

We would like to thank Vishal Acharya, Ben Emerson, Andrea Di Via, Hiroshi Gotoda, Larry Li, Tim Lieuwen, Achintya Mukhopadhyaya, and Vineeth Nair, for their comments on the original manuscript. R.I.S. thanks the Department of Science and Technology (DST), the Office of Naval Research Global (ONRG),

and Air Force Office of Scientific Research (AFOSR) for their financial support and the Swarnajayanti and JC Bose fellowships from the Science and Engineering Research Board of the Department of Science and Technology. R.I.S. thanks Ramesh Kolar (ONRG) and Venkateswaran Sankaran (AFOSR) for their support. V.R.U. thanks the University of California, San Diego, for the post-doctoral fellowship.

DATA AVAILABILITY

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

REFERENCES

- ¹M. P. Juniper and R. I. Sujith, "Sensitivity and nonlinearity of thermoacoustic oscillations," *Annu. Rev. Fluid Mech.* **50**, 661–689 (2018).
- ²R. I. Sujith, M. P. Juniper, and P. J. Schmid, "Non-normality and nonlinearity in thermoacoustic instabilities," *Int. J. Spray Combust. Dyn.* **8**, 119–146 (2016).
- ³R. Biggs, "Rocketdyne–F-1 Saturn first stage engine," in *Remembering the Giants: Apollo Rocket Propulsion Development*, edited by S. C. Fisher and S. A. Rahman (NASA, 2009), pp. 15–26.
- ⁴L. P. Bloomfield, *Accidental Encounters with History (and Some Lessons Learned)* (Hot House Press, 2005).
- ⁵G. P. Sutton, "History of liquid-propellant rocket engines in Russia, formerly the Soviet Union," *J. Propul. Power* **19**, 1008–1037 (2003).
- ⁶T. C. Liewu and V. Yang, *Combustion Instabilities in Gas Turbine Engines: Operational Experience, Fundamental Mechanisms, and Modeling* (American Institute of Aeronautics and Astronautics, 2005).
- ⁷T. Poinso and D. Veynante, *Theoretical and Numerical Combustion* (RT Edwards, Inc., 2005).
- ⁸J. G. Lee, K. Kim, and D. Santavicca, "Measurement of equivalence ratio fluctuation and its effect on heat release during unstable combustion," *Proc. Combust. Inst.* **28**, 415–421 (2000).
- ⁹S. Ducruix, T. Schuller, D. Durox, and S. Candel, "Combustion dynamics and instabilities: Elementary coupling and driving mechanisms," *J. Propul. Power* **19**, 722–734 (2003).
- ¹⁰T. C. Liewu, *Unsteady Combustor Physics* (Cambridge University Press, 2012).
- ¹¹J. J. Keller, "Thermoacoustic oscillations in combustion chambers of gas turbines," *AIAA J.* **33**, 2280–2287 (1995).
- ¹²A. Huber and W. Polifke, "Dynamics of practical premixed flames, Part II: Identification and interpretation of CFD data," *Int. J. Spray Combust. Dyn.* **1**, 229–249 (2009).
- ¹³S. H. Strogatz, *Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry, and Engineering* (CRC Press, 2018).
- ¹⁴T. C. Liewu, "Experimental investigation of limit-cycle oscillations in an unstable gas turbine combustor," *J. Propul. Power* **18**, 61–67 (2002).
- ¹⁵K. Balasubramanian and R. I. Sujith, "Thermoacoustic instability in a Rijke tube: Non-normality and nonlinearity," *Phys. Fluids* **20**, 044103 (2008).
- ¹⁶K. Balasubramanian and R. I. Sujith, "Non-normality and nonlinearity in combustion-acoustic interaction in diffusion flames," *J. Fluid Mech.* **594**, 29–57 (2008).
- ¹⁷M. P. Juniper, "Triggering in the horizontal Rijke tube: Non-normality, transient growth and bypass transition," *J. Fluid Mech.* **667**, 272–308 (2011).
- ¹⁸C. C. Jahnke and F. E. Culick, "Application of dynamical systems theory to nonlinear combustion instabilities," *J. Propul. Power* **10**, 508–517 (1994).
- ¹⁹J. D. Sterling, "Nonlinear analysis and modelling of combustion instabilities in a laboratory combustor," *Combust. Sci. Technol.* **89**, 167–179 (1993).
- ²⁰S. Lei and A. Turan, "Nonlinear/chaotic modeling and control of combustion instabilities," *Int. J. Bifurcation Chaos* **20**, 1245–1254 (2010).
- ²¹A. Fichera, C. Losenno, and A. Pagano, "Experimental analysis of thermoacoustic combustion instability," *Appl. Energy* **70**, 179–191 (2001).
- ²²R. Keanini, K. Yu, and J. Daily, "Evidence of a strange attractor in ramjet combustion," in *27th Aerospace Sciences Meeting* (AIAA, 1989), p. 624.
- ²³L. Kabiraj and R. I. Sujith, "Nonlinear self-excited thermoacoustic oscillations: Intermittency and flame blowout," *J. Fluid Mech.* **713**, 376–397 (2012).
- ²⁴L. Kabiraj, A. Saurabh, P. Wahi, and R. I. Sujith, "Route to chaos for combustion instability in ducted laminar premixed flames," *Chaos* **22**, 023129 (2012).
- ²⁵L. Kabiraj, R. I. Sujith, and P. Wahi, "Bifurcations of self-excited ducted laminar premixed flames," *J. Eng. Gas Turbines Power* **134**, 031502 (2012).
- ²⁶D. Premraj, S. A. Pawar, L. Kabiraj, and R. Sujith, "Strange nonchaos in self-excited singing flames," *Europhys. Lett.* **128**, 54005 (2020).
- ²⁷K. Kashinath, I. C. Waugh, and M. P. Juniper, "Nonlinear self-excited thermoacoustic oscillations of a ducted premixed flame: Bifurcations and routes to chaos," *J. Fluid Mech.* **761**, 399–430 (2014).
- ²⁸Y. Guan, V. Gupta, and L. K. B. Li, "Intermittency route to self-excited chaotic thermoacoustic oscillations," *J. Fluid Mech.* **894**, R3 (2020).
- ²⁹V. Nair and R. I. Sujith, "Multifractality in combustion noise: Predicting an impending combustion instability," *J. Fluid Mech.* **747**, 635–655 (2014).
- ³⁰V. R. Unni and R. I. Sujith, "Multifractal characteristics of combustor dynamics close to lean blowout," *J. Fluid Mech.* **784**, 30–50 (2015).
- ³¹P. Clavin, J. Kim, and F. Williams, "Turbulence-induced noise effects on high-frequency combustion instabilities," *Combust. Sci. Technol.* **96**, 61–84 (1994).
- ³²V. Burnley and F. C. Culick, "Influence of random excitations on acoustic instabilities in combustion chambers," *AIAA J.* **38**, 1403–1410 (2000).
- ³³T. C. Liewu and A. Banaszuk, "Background noise effects on combustor stability," *J. Propul. Power* **21**, 25–31 (2005).
- ³⁴J. Tony, E. Gopalakrishnan, E. Sreelekha, and R. I. Sujith, "Detecting deterministic nature of pressure measurements from a turbulent combustor," *Phys. Rev. E* **92**, 062902 (2015).
- ³⁵V. Nair, G. Thampi, and R. I. Sujith, "Intermittency route to thermoacoustic instability in turbulent combustors," *J. Fluid Mech.* **756**, 470–487 (2014).
- ³⁶V. R. Unni and R. I. Sujith, "Flame dynamics during intermittency in a turbulent combustor," *Proc. Combust. Inst.* **36**, 3791–3798 (2017).
- ³⁷N. B. George, V. R. Unni, M. Raghunathan, and R. I. Sujith, "Pattern formation during transition from combustion noise to thermoacoustic instability via intermittency," *J. Fluid Mech.* **849**, 615–644 (2018).
- ³⁸V. Nair and R. I. Sujith, "Precursors to self-sustained oscillations in aeroacoustic systems," *Int. J. Aeroacoust.* **15**, 312–323 (2016).
- ³⁹J. Venkatramani, V. Nair, R. I. Sujith, S. Gupta, and S. Sarkar, "Precursors to flutter instability by an intermittency route: A model free approach," *J. Fluids Struct.* **61**, 376–391 (2016).
- ⁴⁰B. Kerres, V. Nair, A. Cronhjort, and M. Mihaescu, "Analysis of the turbocharger compressor surge margin using a Hurst-exponent-based criterion," *SAE Int. J. Engines* **9**, 1795–1806 (2016).
- ⁴¹A.-L. Barabási, "The network takeover," *Nat. Phys.* **8**, 14 (2011).
- ⁴²K. Smith, H. Brighton, and S. Kirby, "Complex systems in language evolution: The cultural emergence of compositional structure," *Adv. Complex Syst.* **6**, 537–558 (2003).
- ⁴³N. Johnson, *Simply Complexity: A Clear Guide to Complexity Theory* (Oneworld Publications, 2009).
- ⁴⁴T. J. Poinso, A. C. Trouve, D. P. Veynante, S. M. Candel, and E. J. Esposito, "Vortex-driven acoustically coupled combustion instabilities," *J. Fluid Mech.* **177**, 265–292 (1987).
- ⁴⁵K. Schadow, E. Gutmark, T. Parr, D. Parr, K. Wilson, and J. Crump, "Large-scale coherent structures as drivers of combustion instability," *Combust. Sci. Technol.* **64**, 167–186 (1989).
- ⁴⁶V. Nair and R. I. Sujith, "Intermittency as a transition state in combustor dynamics: An explanation for flame dynamics near lean blowout," *Combust. Sci. Technol.* **187**, 1821–1835 (2015).
- ⁴⁷S. Mondal, V. R. Unni, and R. I. Sujith, "Onset of thermoacoustic instability in turbulent combustors: An emergence of synchronized periodicity through formation of chimera-like states," *J. Fluid Mech.* **811**, 659–681 (2017).
- ⁴⁸H. Haken, "Synergetics," *Phys. Bull.* **28**, 412 (1977).
- ⁴⁹I. Prigogine and E. N. Hiebert, "From being to becoming: Time and complexity in the physical sciences," *Phys. Today* **35**(1), 69 (1982).

- ⁵⁰I. Prigogine, "Time, structure and fluctuations," in *Nobel Lectures in Chemistry 1971–1980* (World Scientific Publishing Co., 1977), pp. 263–285.
- ⁵¹A. Di Vita, "Maximum or minimum entropy production? How to select a necessary criterion of stability for a dissipative fluid or plasma," *Phys. Rev. E* **81**, 041137 (2010).
- ⁵²A. Di Vita, "On Rayleigh's criterion of thermo-acoustics," Ph.D. thesis (Università di Genova, 2016).
- ⁵³A. P. Dowling and Y. Mahmoudi, "Combustion noise," *Proc. Combust. Inst.* **35**, 65–100 (2015).
- ⁵⁴S. R. Chakravarthy, R. Sivakumar, and O. J. Shreenivasan, "Vortex-acoustic lock-on in bluff-body and backward-facing step combustors," *Sadhana* **32**, 145–154 (2007).
- ⁵⁵W. C. Strahle, "On combustion generated noise," *J. Fluid Mech.* **49**, 399–414 (1971).
- ⁵⁶W. C. Strahle, "Some results in combustion generated noise," *J. Sound Vib.* **23**, 113–125 (1972).
- ⁵⁷W. C. Strahle, "Refraction, convection, and diffusion flame effects in combustion-generated noise," *Symp. (Int.) Combust.* **14**, 527–535 (1973).
- ⁵⁸M. Lighthill, "On sound generated aerodynamically I. General theory," *Proc. R. Soc. London, Ser. A* **211**, 564 (1952).
- ⁵⁹H. Hassan, "Scaling of combustion-generated noise," *J. Fluid Mech.* **66**, 445–453 (1974).
- ⁶⁰S. Candel, D. Durox, S. Ducruix, A.-L. Birbaud, N. Noiray, and T. Schuller, "Flame dynamics and combustion noise: Progress and challenges," *Int. J. Aeroacoust.* **8**, 1–56 (2009).
- ⁶¹R. Rajaram and T. Liewen, "Acoustic radiation from turbulent premixed flames," *J. Fluid Mech.* **637**, 357–385 (2009).
- ⁶²V. Nair, G. Thampi, S. Karuppusamy, S. Gopalan, and R. I. Sujith, "Loss of chaos in combustion noise as a precursor of impending combustion instability," *Int. J. Spray Combust. Dyn.* **5**, 273–290 (2013).
- ⁶³D. T. Kaplan and L. Glass, "Direct test for determinism in a time series," *Phys. Rev. Lett.* **68**, 427 (1992).
- ⁶⁴E. Karlis, Y. Hardalupas, and A. Taylor, "Experimental characterization of intermittency of thermoacoustic instability in a swirl stabilized combustor," in 8th European Combustion Meeting, Dubrovnik, Croatia, 18–21 April 2017.
- ⁶⁵S. A. Pawar, R. Vishnu, M. Vadivukkarasan, M. Panchagnula, and R. I. Sujith, "Intermittency route to combustion instability in a laboratory spray combustor," *J. Eng. Gas Turbines Power* **138**, 041505 (2016).
- ⁶⁶P. Kasthuri, V. R. Unni, and R. I. Sujith, "Bursting and mixed mode oscillations during the transition to limit cycle oscillations in a matrix burner," *Chaos* **29**, 043117 (2019).
- ⁶⁷S. Manikandan and R. Sujith, "Rate dependent transition to thermoacoustic instability via intermittency in a turbulent afterburner," *Exp. Therm. Fluid Sci.* **114**, 110046 (2020).
- ⁶⁸P. Kasthuri, I. Pavithran, S. A. Pawar, R. I. Sujith, R. Gejji, and W. Anderson, "Dynamical systems approach to study thermoacoustic transitions in a liquid rocket combustor," *Chaos* **29**, 103115 (2019).
- ⁶⁹U. Sen, T. Gangopadhyay, C. Bhattacharya, A. Mukhopadhyay, and S. Sen, "Dynamic characterization of a ducted inverse diffusion flame using recurrence analysis," *Combust. Sci. Technol.* **190**, 32–56 (2018).
- ⁷⁰M. Murugesan, B. Singaravelu, A. K. Kushwaha, and S. Mariappan, "Onset of flame-intrinsic thermoacoustic instabilities in partially premixed turbulent combustors," *Int. J. Spray Combust. Dyn.* **10**, 171–184 (2018).
- ⁷¹S. Suresha, R. I. Sujith, B. Emerson, and T. Liewen, "Nonlinear dynamics and intermittency in a turbulent reacting wake with density ratio as bifurcation parameter," *Phys. Rev. E* **94**, 042206 (2016).
- ⁷²B. Emerson, J. O'Connor, M. Juniper, and T. Liewen, "Density ratio effects on reacting bluff-body flow field characteristics," *J. Fluid Mech.* **706**, 219–250 (2012).
- ⁷³Y. Pomeau and P. Manneville, "Intermittent transition to turbulence in dissipative dynamical systems," *Commun. Math. Phys.* **74**, 189–197 (1980).
- ⁷⁴D. Zhilenko and O. Krivosova, "Experimental observation of chaos-chaos intermittency types in spherical Couette flow," *Dokl. Phys.* **59**, 45–48 (2014).
- ⁷⁵V. Anishchenko, A. Neiman, and L. Chua, "Chaos-chaos intermittency and 1/f noise in Chua's circuit," *Int. J. Bifurcation Chaos* **4**, 99–107 (1994).
- ⁷⁶S. Elaskar and E. Del Río, *New Advances on Chaotic Intermittency and Its Applications* (Springer, 2017).
- ⁷⁷C. L. Webber, Jr. and N. Marwan, *Recurrence Quantification Analysis* (Springer, 2015).
- ⁷⁸K. Klimaszewska and J. J. Żebrowski, "Detection of the type of intermittency using characteristic patterns in recurrence plots," *Phys. Rev. E* **80**, 026214 (2009).
- ⁷⁹S. A. Pawar and R. I. Sujith, *Droplets and Sprays* (Springer, 2018), pp. 403–430.
- ⁸⁰H. Gotoda, Y. Shinoda, M. Kobayashi, Y. Okuno, and S. Tachibana, "Detection and control of combustion instability based on the concept of dynamical system theory," *Phys. Rev. E* **89**, 022910 (2014).
- ⁸¹R. Sampath and S. R. Chakravarthy, "Investigation of intermittent oscillations in a premixed dump combustor using time-resolved particle image velocimetry," *Combust. Flame* **172**, 309–325 (2016).
- ⁸²S. A. Pawar, M. V. Panchagnula, and R. I. Sujith, "Phase synchronization and collective interaction of multiple flamelets in a laboratory scale spray combustor," *Proc. Combust. Inst.* **37**, 5121–5128 (2019).
- ⁸³J. G. Hong, K. C. Oh, U. D. Lee, and H. D. Shin, "Generation of low-frequency alternative flame behaviors in a lean premixed combustor," *Energy Fuels* **22**, 3016–3021 (2008).
- ⁸⁴C. M. Arndt, A. M. Steinberg, I. G. Boxx, W. Meier, M. Aigner, and C. D. Carter, "Flow-field and flame dynamics of a gas turbine model combustor during transition between thermo-acoustically stable and unstable states," in *ASME Turbo Expo 2010: Power for Land, Sea, and Air* (American Society of Mechanical Engineers, 2010), pp. 677–687.
- ⁸⁵N. Noiray and B. Schuermans, "Deterministic quantities characterizing noise driven Hopf bifurcations in gas turbine combustors," *Int. J. Non-Linear Mech.* **50**, 152–163 (2013).
- ⁸⁶V. Nair and R. I. Sujith, "A reduced-order model for the onset of combustion instability: Physical mechanisms for intermittency and precursors," *Proc. Combust. Inst.* **35**, 3193–3200 (2015).
- ⁸⁷K. I. Matveev and F. Culick, "A model for combustion instability involving vortex shedding," *Combust. Sci. Technol.* **175**, 1059–1083 (2003).
- ⁸⁸A. Seshadri, V. Nair, and R. I. Sujith, "A reduced-order deterministic model describing an intermittency route to combustion instability," *Combust. Theory Modell.* **20**, 441–456 (2016).
- ⁸⁹S. A. Pawar, A. Seshadri, V. R. Unni, and R. I. Sujith, "Thermoacoustic instability as mutual synchronization between the acoustic field of the confinement and turbulent reactive flow," *J. Fluid Mech.* **827**, 664–693 (2017).
- ⁹⁰A. Seshadri, I. Pavithran, V. R. Unni, and R. I. Sujith, "Predicting the amplitude of limit-cycle oscillations in thermoacoustic systems with vortex shedding," *AIAA J.* **56**, 3507–3514 (2018).
- ⁹¹Y. Weng, V. R. Unni, R. I. Sujith, and A. Saha, "Synchronization framework for modeling transition to thermoacoustic instability in laminar combustors," *Nonlin. Dyn.* (published online 2020); [arXiv:2005.06186](https://arxiv.org/abs/2005.06186) [nlin.AO].
- ⁹²A. Pikovsky, M. Rosenblum, and J. Kurths, *Synchronization: A Universal Concept in Nonlinear Sciences* (Cambridge University Press, 2003), Vol. 12.
- ⁹³I. Schreiber and M. Marek, "Transition to chaos via two-torus in coupled reaction-diffusion cells," *Phys. Lett. A* **91**, 263–266 (1982).
- ⁹⁴L. Glass, "Synchronization and rhythmic processes in physiology," *Nature* **410**, 277 (2001).
- ⁹⁵B. Blasius, A. Huppert, and L. Stone, "Complex dynamics and phase synchronization in spatially extended ecological systems," *Nature* **399**, 354 (1999).
- ⁹⁶L. Kocarev and U. Parlitz, "General approach for chaotic synchronization with applications to communication," *Phys. Rev. Lett.* **74**, 5028 (1995).
- ⁹⁷J. Heagy, T. Carroll, and L. Pecora, "Synchronous chaos in coupled oscillator systems," *Phys. Rev. E* **50**, 1874 (1994).
- ⁹⁸R. Roy and K. S. Thornburg, Jr., "Experimental synchronization of chaotic lasers," *Phys. Rev. Lett.* **72**, 2009 (1994).
- ⁹⁹M. C. Romano, M. Thiel, J. Kurths, I. Z. Kiss, and J. Hudson, "Detection of synchronization for non-phase-coherent and non-stationary data," *Europhys. Lett.* **71**, 466 (2005).
- ¹⁰⁰D. M. Abrams and S. H. Strogatz, "Chimera states for coupled oscillators," *Phys. Rev. Lett.* **93**, 174102 (2004).

- ¹⁰¹Y. Kuramoto and D. Battogtokh, "Coexistence of coherence and incoherence in nonlocally coupled phase oscillators," in *Nonlinear Phenomena in Complex Systems* (Elsevier, 2002).
- ¹⁰²S. A. Pawar, S. Mondal, N. B. George, and R. I. Sujith, "Temporal and spatiotemporal analyses of synchronization transition in a swirl-stabilized combustor," *AIAA J.* **57**, 836–847 (2018).
- ¹⁰³Y. Guan, L. K. Li, B. Ahn, and K. T. Kim, "Chaos, synchronization, and desynchronization in a liquid-fueled diffusion-flame combustor with an intrinsic hydrodynamic mode," *Chaos* **29**, 053124 (2019).
- ¹⁰⁴T. Hashimoto, H. Shibuya, H. Gotoda, Y. Ohmichi, and S. Matsuyama, "Spatiotemporal dynamics and early detection of thermoacoustic combustion instability in a model rocket combustor," *Phys. Rev. E* **99**, 032208 (2019).
- ¹⁰⁵J. Halatek and E. Frey, "Rethinking pattern formation in reaction-diffusion systems," *Nat. Phys.* **14**, 507 (2018).
- ¹⁰⁶B. B. Mandelbrot, *The Fractal Geometry of Nature* (W. H. Freeman, New York, 1983), Vol. 173.
- ¹⁰⁷C. Meneveau and K. Sreenivasan, "The multifractal nature of turbulent energy dissipation," *J. Fluid Mech.* **224**, 429–484 (1991).
- ¹⁰⁸H. Gotoda, M. Amano, T. Miyano, T. Ikawa, K. Maki, and S. Tachibana, "Characterization of complexities in combustion instability in a lean premixed gas-turbine model combustor," *Chaos* **22**, 043128 (2012).
- ¹⁰⁹J. W. Kantelhardt, S. A. Zschiegner, E. Koscielny-Bunde, S. Havlin, A. Bunde, and H. E. Stanley, "Multifractal detrended fluctuation analysis of nonstationary time series," *Physica A* **316**, 87–114 (2002).
- ¹¹⁰E. A. F. Ihlen, "Introduction to multifractal detrended fluctuation analysis in matlab," *Front. Physiol.* **3**, 141 (2012).
- ¹¹¹M. Raghunathan, N. B. George, V. R. Unni, P. R. Midhun, K. V. Reeja, and R. I. Sujith, "Multifractal analysis of flame dynamics during transition to thermoacoustic instability in a turbulent combustor," *J. Fluid Mech.* **888**, A14 (2020).
- ¹¹²M. Murugesan and R. I. Sujith, "Combustion noise is scale-free: Transition from scale-free to order at the onset of thermoacoustic instability," *J. Fluid Mech.* **772**, 225–245 (2015).
- ¹¹³L. Lacasa, B. Luque, F. Ballesteros, J. Luque, and J. C. Nuño, "From time series to complex networks: The visibility graph," *Proc. Natl. Acad. Sci. U. S. A.* **105**, 4972–4975 (2008).
- ¹¹⁴L. Lacasa, B. Luque, J. Luque, and J. C. Nuño, "The visibility graph: A new method for estimating the Hurst exponent of fractional Brownian motion," *Europhys. Lett.* **86**, 30001 (2009).
- ¹¹⁵M. Bastian, S. Heymann, and M. Jacomy, "Gephi: An open source software for exploring and manipulating networks," in *Third International AAAI Conference on Weblogs and Social Media*, Paris, France, 17–20 May 2009.
- ¹¹⁶H. Gotoda, H. Kinugawa, R. Tsujimoto, S. Domen, and Y. Okuno, "Characterization of combustion dynamics, detection, and prevention of an unstable combustion state based on a complex-network theory," *Phys. Rev. Appl.* **7**, 044027 (2017).
- ¹¹⁷S. Murayama, H. Kinugawa, I. T. Tokuda, and H. Gotoda, "Characterization and detection of thermoacoustic combustion oscillations based on statistical complexity and complex-network theory," *Phys. Rev. E* **97**, 022223 (2018).
- ¹¹⁸V. Godavarthi, V. Unni, E. Gopalakrishnan, and R. I. Sujith, "Recurrence networks to study dynamical transitions in a turbulent combustor," *Chaos* **27**, 063113 (2017).
- ¹¹⁹V. Godavarthi, S. A. Pawar, V. R. Unni, R. I. Sujith, N. Marwan, and J. Kurths, "Coupled interaction between unsteady flame dynamics and acoustic field in a turbulent combustor," *Chaos* **28**, 113111 (2018).
- ¹²⁰Y. Okuno, M. Small, and H. Gotoda, "Dynamics of self-excited thermoacoustic instability in a combustion system: Pseudo-periodic and high-dimensional nature," *Chaos* **25**, 043107 (2015).
- ¹²¹A. Krishnan, R. Sujith, N. Marwan, and J. Kurths, "On the emergence of large clusters of acoustic power sources at the onset of thermoacoustic instability in a turbulent combustor," *J. Fluid Mech.* **874**, 455–482 (2019).
- ¹²²V. Nair, G. Thampi, and R. I. Sujith, "Engineering precursors to forewarn the onset of an impending combustion instability," in *ASME Turbo Expo 2014: Turbine Technical Conference and Exposition* (American Society of Mechanical Engineers, 2014), pp. GT 2014–26020.
- ¹²³V. Nair, G. Thampi, S. Karuppusamy, S. Gopalan, and R. I. Sujith, "System and method for predetermining the onset of impending oscillatory instabilities in practical devices," U.S. patent 9804054 (31 October 2017).
- ¹²⁴H. Abarbanel, *Analysis of Observed Chaotic Data* (Springer Science & Business Media, 2012).
- ¹²⁵M. Murugesan, V. R. Unni, V. Nair, and R. I. Sujith, "Devices and methods for early prediction of impending instabilities of a system," U.S. patent 10337414B2 (2 July 2019).
- ¹²⁶M. Murugesan and R. I. Sujith, "Detecting the onset of an impending thermoacoustic instability using complex networks," *J. Propul. Power* **32**, 707–712 (2016).
- ¹²⁷V. R. Unni, V. Nair, R. I. Sujith, and A. Mukhopadhyay, "System and method for controlling oscillatory instabilities in a device," U.S. patent 10095247B2 (9 October 2018).
- ¹²⁸V. R. Unni, A. Mukhopadhyay, and R. I. Sujith, "Online detection of impending instability in a combustion system using tools from symbolic time series analysis," *Int. J. Spray Combust. Dyn.* **7**, 243–255 (2015).
- ¹²⁹L. Christodoulou, L. Kabiraj, A. Saurabh, and N. Karimi, "Characterizing the signature of flame flashback precursor through recurrence analysis," *Chaos* **26**, 013110 (2016).
- ¹³⁰V. R. Unni and R. I. Sujith, "Precursors to blowout in a turbulent combustor based on recurrence quantification," in *52nd AIAA/SAE/ASEE Joint Propulsion Conference* (AIAA, 2016).
- ¹³¹V. R. Unni, V. Nair, and R. I. Sujith, "System and method for detecting precursors to control blowout in combustion systems," U.S. patent 10267519B2 (23 April 2019).
- ¹³²N. B. George, V. R. Unni, M. Raghunathan, and R. I. Sujith, "Suppression of thermoacoustic instability in a swirl-stabilized combustor by inducing blockage in the inlet flow stream," in *2018 AIAA Aerospace Sciences Meeting (Kissimmee, Florida)* (AIAA, 2018), p. 0396.
- ¹³³V. R. Unni, A. Krishnan, R. Manikandan, N. B. George, R. I. Sujith, N. Marwan, and J. Kurths, "On the emergence of critical regions at the onset of thermoacoustic instability in a turbulent combustor," *Chaos* **28**, 063125 (2018).
- ¹³⁴A. Krishnan, R. Manikandan, P. R. Midhun, K. V. Reeja, V. R. Unni, R. I. Sujith, N. Marwan, and J. Kurths, "Mitigation of oscillatory instability in turbulent reactive flows: A novel approach using complex networks," *Europhys. Lett.* **128**, 14003 (2019).
- ¹³⁵K. McManus, T. Poinso, and S. M. Candel, "A review of active control of combustion instabilities," *Prog. Energy Combust. Sci.* **19**, 1–29 (1993).
- ¹³⁶T. Lieuwen and Y. Neumeier, "Nonlinear pressure-heat release transfer function measurements in a premixed combustor," *Proc. Combust. Inst.* **29**, 99–105 (2002).
- ¹³⁷E. Lubarsky, D. Shcherbik, B. Zinn, K. McManus, T. Fric, and S. Srinivasan, "Active control of combustion oscillations by non-coherent fuel flow modulation," in *9th AIAA/CEAS Aeroacoustics Conference and Exhibit, Hilton Head, South Carolina* (AIAA, 2003).
- ¹³⁸B. Bellows, A. Hreiz, and T. Lieuwen, "Nonlinear interactions between forced and self-excited acoustic oscillations in premixed combustor," *J. Propul. Power* **24**, 628–631 (2008).
- ¹³⁹S. Mondal, S. A. Pawar, and R. I. Sujith, "Forced synchronization and asynchronous quenching of periodic oscillations in a thermoacoustic system," *J. Fluid Mech.* **864**, 73–96 (2019).
- ¹⁴⁰Y. Guan, V. Gupta, K. Kashinath, and L. K. Li, "Open-loop control of periodic thermoacoustic oscillations: Experiments and low-order modelling in a synchronization framework," *Proc. Combust. Inst.* **37**, 5315–5323 (2019).
- ¹⁴¹K. Kashinath, L. K. Li, and M. P. Juniper, "Forced synchronization of periodic and aperiodic thermoacoustic oscillations: Lock-in, bifurcations and open-loop control," *J. Fluid Mech.* **838**, 690–714 (2018).
- ¹⁴²S. Balusamy, L. K. Li, Z. Han, M. P. Juniper, and S. Hochgreb, "Nonlinear dynamics of a self-excited thermoacoustic system subjected to acoustic forcing," *Proc. Combust. Inst.* **35**, 3229–3236 (2015).
- ¹⁴³A. Roy, S. Mondal, S. A. Pawar, and R. I. Sujith, "On the mechanism of open-loop control of thermoacoustic instability in a laminar premixed combustor," *J. Fluid Mech.* **884**, A2 (2020).

- ¹⁴⁴A. Balanov, N. Janson, D. Postnov, and O. Sosnovtseva, *Synchronization: From Simple to Complex* (Springer Science & Business Media, 2008).
- ¹⁴⁵R. Gopakumar, S. Mondal, R. Paul, S. Mahesh, and S. Chaudhuri, "Mitigating instability by actuating the swirler in a combustor," *Combust. Flame* **165**, 361–363 (2016).
- ¹⁴⁶S. Chaudhuri, R. Gopakumar, S. Mondal, R. Paul, and S. Mahesh, "Mitigating instability by actuating the swirler in a combustor," U.S. patent application 15/753,117 (19 July 2018).
- ¹⁴⁷S. Mahesh, R. Gopakumar, B. V. Rahul, A. K. Dutta, S. Mondal, and S. Chaudhuri, "Instability control by actuating the swirler in a lean premixed combustor," *J. Propul. Power* **34**, 708–719 (2018).
- ¹⁴⁸R. Gopakumar, R. B. Vishwanath, J. Singh, A. Dutta, and S. Chaudhuri, "On the dynamics of instability mitigation by actuating swirler motion in a lean premixed turbulent combustor," in *ASME 2017 Gas Turbine India Conference* (American Society of Mechanical Engineers, 2017), p. V001T04A008.
- ¹⁴⁹A. K. Dutta, G. Ramachandran, and S. Chaudhuri, "Investigating thermoacoustic instability mitigation dynamics with a Kuramoto model for flamelet oscillators," *Phys. Rev. E* **99**, 032215 (2019).
- ¹⁵⁰N. Thomas, S. Mondal, S. A. Pawar, and R. I. Sujith, "Effect of time-delay and dissipative coupling on amplitude death in coupled thermoacoustic oscillators," *Chaos* **28**, 033119 (2018).
- ¹⁵¹N. Thomas, S. Mondal, S. A. Pawar, and R. I. Sujith, "Effect of noise amplification during the transition to amplitude death in coupled thermoacoustic oscillators," *Chaos* **28**, 093116 (2018).
- ¹⁵²S. Dange, K. Manoj, S. Banerjee, S. A. Pawar, S. Mondal, and R. I. Sujith, "Oscillation quenching and phase-flip bifurcation in coupled thermoacoustic systems," *Chaos* **29**, 093135 (2019).
- ¹⁵³H. Jegal, K. Moon, J. Gu, L. K. Li, and K. T. Kim, "Mutual synchronization of two lean-premixed gas turbine combustors: Phase locking and amplitude death," *Combust. Flame* **206**, 424–437 (2019).
- ¹⁵⁴A. P. Dowling and J. E. Ffowcs Williams, *Sound and Sources of Sound* (Hawood, 1983).
- ¹⁵⁵A. Anderson, "Structure and velocity of the periodic vortex-ring flow pattern of a primary Pfeifenton (pipe tone) jet," *J. Acoust. Soc. Am.* **27**, 1048–1053 (1955).
- ¹⁵⁶A. Powell, "On the edgetone," *J. Acoust. Soc. Am.* **33**, 395–409 (1961).
- ¹⁵⁷D. Rockwell and E. Naudascher, "Self-sustaining oscillations of flow past cavities," *J. Fluids Eng.* **100**, 152–165 (1978).
- ¹⁵⁸D. Middleton, "Theoretical and experimental investigations into the acoustic output from ejector flows," *J. Sound Vib.* **11**, 447–473 (1970).
- ¹⁵⁹J. Panda, "An experimental investigation of screech noise generation," *J. Fluid Mech.* **378**, 71–96 (1999).
- ¹⁶⁰D. Rockwell and E. Naudascher, "Self-sustained oscillations of impinging free shear layers," *Annu. Rev. Fluid Mech.* **11**, 67–94 (1979).
- ¹⁶¹M. Kamin, J. Mathew, and R. I. Sujith, "A numerical study of an acoustic-hydrodynamic system exhibiting an intermittent prelude to instability," *Int. J. Aeroacoust.* **18**, 536–553 (2019).
- ¹⁶²B. Korbahti, E. Kagambage, T. Andrianne, N. A. Razak, and G. Dimitriadis, "Subcritical, nontypical and period-doubling bifurcations of a delta wing in a low speed wind tunnel," *J. Fluids Struct.* **27**, 408–426 (2011).
- ¹⁶³D. C. Poirel and S. J. Price, "Structurally nonlinear fluttering airfoil in turbulent flow," *AIAA J.* **39**(10), 1960–1968 (2001).
- ¹⁶⁴A. Raaj, J. Venkatramani, and S. Mondal, "Synchronization of pitch and plunge motions during intermittency route to aeroelastic flutter," *Chaos* **29**, 043129 (2019).
- ¹⁶⁵J. Venkatramani, S. K. Kumar, S. Sarkar, and S. Gupta, "Physical mechanism of intermittency route to aeroelastic flutter," *J. Fluids Struct.* **75**, 9–26 (2017).
- ¹⁶⁶J. Venkatramani, V. Nair, R. I. Sujith, S. Gupta, and S. Sarkar, "Multi-fractality in aeroelastic response as a precursor to flutter," *J. Sound Vib.* **386**, 390–406 (2017).
- ¹⁶⁷I. Pavithran, V. R. Unni, A. J. Varghese, R. Sujith, A. Saha, N. Marwan, and J. Kurths, "Universality in the emergence of oscillatory instabilities in turbulent flows," *Europhys. Lett.* **129**, 24004 (2020).
- ¹⁶⁸I. Pavithran, V. R. Unni, A. J. Varghese, D. Premraj, R. Sujith, C. Vijayan, A. Saha, N. Marwan, and J. Kurths, "Universality in spectral condensation," [arXiv:2004.10585](https://arxiv.org/abs/2004.10585) (2020).
- ¹⁶⁹T. Kobayashi, S. Murayama, T. Hachijo, and H. Gotoda, "Early detection of thermoacoustic combustion instability using a methodology combining complex networks and machine learning," *Phys. Rev. Appl.* **11**, 064034 (2019).
- ¹⁷⁰C. Premchand, N. B. George, M. Raghunathan, V. R. Unni, R. I. Sujith, and V. Nair, "Lagrangian analysis of intermittent sound sources in the flow-field of a bluff-body stabilized combustor," *Phys. Fluids* **31**, 025115 (2019).
- ¹⁷¹C. Paschereit, W. Weisenstein, and E. Gutmark, "Role of coherent structures in acoustic combustion control," in *29th AIAA, Fluid Dynamics Conference* (AIAA, 1998), p. 2433.