Area Occupancy-Based Adaptive Density Estimation for Mixed Road Traffic

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ABSTRACT Managing congestion in mixed traffic conditions, characterized by heterogeneous and lane-less traffic, is a challenging task. Traditionally density, defined as the number of vehicles in a road stretch, is used to quantify congestion. However, direct measurement of density is difficult and hence is usually estimated from other variables. In this paper, a relationship is derived between traffic density and area occupancy, a variable that can incorporate heterogeneity and lane-less movement. Using the derived density-area occupancy relation, a non-continuum macroscopic single state linear time varying model was developed. Estimation of density was done by using the Kalman filtering technique and corroborated with simulated density. The need for dynamic estimation is motivated by evaluating the performance of two static estimation schemes in the presence of uncertainties. Performance was tested for different traffic scenarios such as congestion and non-recurrent traffic incidents. Further, to improve the estimation accuracy in scenarios involving transitions in traffic conditions, an adaptive estimator was developed. It was found that the adaptive estimator provided the best estimation accuracy.

INDEX TERMS Adaptive Kalman filter, area occupancy, heterogeneous traffic, lane-less traffic, traffic density estimation.

I. INTRODUCTION AND BACKGROUND

Mitigating traffic congestion to enhance the performance of transportation systems is a major challenge for traffic engineers globally. Increasing road traffic causes high traffic demand and the exceeding demand over road capacity results in traffic congestion. One of the major factors that contribute to congestion is the lack of infrastructure to meet demand. Considering the limitations in the expansion and addition of existing infrastructure, effective management of the available infrastructure is the most viable option for reducing traffic congestion. Among the various traffic control mechanisms, traffic signals are the most widely used ones. However, inadequately timed traffic signals are major recurring sources of traffic congestion, which can be attributed to poor allocation of green time, poor coordination between adjacent intersections, and inability to respond to real-time situations. This necessitates the need for developing more effective methods for traffic signal control. With the existing infrastructure, one among the most effective ways to control traffic is to assign the signal dynamically depending on the current state of traffic [1]. However, dynamic traffic control requires the availability of real-time traffic states, whose direct measurement poses a problem. In this context, the research problem can be separated into two stages. Dynamic estimation of the traffic states is to be carried out first to gather the information required for the control system. Based on this, a control scheme can be subsequently developed. This paper focuses on the former for a heterogeneous and lane-less traffic condition.

The term heterogeneous refers to traffic composed of vehicles of different classes. If the nature of traffic is such that vehicles occupy any space in the roadway without following the lanes, it is referred as lane-less traffic. Combined, this traffic condition is termed as mixed traffic in the present study. In addition to varying vehicle dimensions, other factors such as varying vehicle speeds, frequent intersections, and side roads pose additional challenges. In these circumstances, developing a model representing the traffic condition and obtaining the current traffic state for control is difficult. For characterizing such a traffic, certain spatial and/or temporal variables are needed. Variables like speed, flow, space

The associate editor coordinating the review of this manuscript and approving it for publication was Muhammad Awais Javed.

headway, time headway and/or traffic density, characterize a typical traffic stream. Out of these, a good indicator of the level of congestion is traffic density [2], defined as the number of vehicles occupying a length of roadway [3]. As density is a variable related over space, its direct measurement is difficult. One way of overcoming this is to estimate it from directly measurable variables using suitable models for dynamic density estimation.

Complex traffic systems can be represented using traffic flow models that characterize and predict future traffic behavior by deriving mathematical relationships between appropriate variables. They can be classified into microscopic and macroscopic traffic models by the level of detail with which they characterize the behavior of the traffic. Each vehicle behavior is characterized in microscopic models, and hence are intensive in terms of data and computational complexity. On the other hand, macroscopic models consider traffic stream in total and capture the aggregate nature of traffic. Such models are suitable for representing real-time traffic characteristics and are pursued in this study. Macroscopic models can be categorized into either continuum or non-continuum models.

Macroscopic traffic flow models are mostly based on continuum models [4]–[6]. However, the above models are not without deficiencies as suggested by many authors [7], [8]. One of the major criticisms is that even in congested states, the number of vehicles in a road section does not justify the continuum assumption. Conversely, non-continuum models consider traffic as an assembly of dynamic systems with each representative vehicle as a component system. Considering the above factors, a macroscopic non-continuum approach is adopted to estimate density (a discrete variable representing the number of vehicles in a section) under mixed traffic conditions.

Reported studies in this area for heterogeneous conditions are limited and discussed below. Wong *et al.* extended the Lighthill, Whitham and Richards (LWR) model by considering heterogeneous drivers to formulate a macroscopic multi-class model [9]. Another extension of LWR model incorporating various vehicle classes was proposed by Logghe and Immers [10]. A dynamic model was developed by Tang *et al.* from the relationship between the microscopic and macroscopic variables for heterogeneous traffic [11]. Arasan and Kosh [12] and Venkatesan *et al.* [13] developed microscopic simulation model for heterogeneous traffic. The principles of Cellular Automata were applied to model heterogeneous traffic in [14] and [15]. Use of non-continuum macroscopic models for heterogeneous conditions based on the conservation law for vehicles and the fundamental relation of traffic flow was reported in the study by Anand *et al.* [16]. Models based on vehicle conservation and a hypothesized constitutive equation was reported by Padiath *et al.* [17] and Thankappan *et al.* [18]. Since the proportion of different vehicles in heterogeneous traffic may vary with time, a steady-state model may not capture the traffic characteristics appropriately. However, in majority of

the above studies, steady-state traffic stream models have been developed empirically under heterogeneous traffic conditions. Most of these studies have incorporated heterogeneity of the traffic system by converting various classes of vehicles into equivalent passenger car unit (PCU). However, under mixed traffic, determination of exact PCU value for each type of vehicle is difficult [19]. To address these limitations, a dynamic model that captures lane indiscipline and heterogeneity with various classes of vehicles needs to be developed.

From a historical stand point, the estimation and analysis of traffic density is of significant interest in the field of traffic research. The most common method for measuring traffic density in early days was the photographic technique [3]. However, it is time consuming and its implementation in real-time is challenging. Later, three approaches were reported in the early 1960's, namely input-output technique, using fundamental speed-flow relationship and from percent occupancy [3]. These static estimation schemes have their own limitations in calculating density, especially under mixed traffic conditions. These schemes will not work well in the presence of dynamic disturbances and hence, dynamic estimation of traffic density is a better choice.

The Kalman filter [20] is an efficient recursive algorithm for the estimation of states of dynamic systems from noisy measurements. In a recent survey conducted by Seo *et al.* [21], it was reported that for non-continuum model-based approach, the Kalman Filter and its extended versions are widely used among the well-known implementations. Application of Kalman filter for the estimation of traffic density can be traced back from the early 1970's. In later years, investigations in the field of estimation of traffic density based on Kalman filter increased and a few recent studies under mixed traffic conditions are discussed below. In [22], Singh and Li presented estimation of traffic density for multi-lane roadways using a Markov model approach. A hybrid extended Kalman filtering approach for traffic density estimation along signalized arterial was reported by Di *et al.* in [23]. For the estimation of traffic density under heterogeneous traffic, a non-continuum macroscopic model, based on conservation of vehicles and a hypothesized constitutive equation was developed by Thankappan *et al.* [18]. A data fusion approach using both location-based and spatial data sources was explained by Anand *et al.* in [24] and a nonlinear model, based on the conservation principle and the fundamental traffic flow was presented in [25] by Dhivyabharathi *et al.* In [26], Bekiaris-liberis *et al.* developed linear parameter-varying models derived from the conservation equation with mixed connected and conventional vehicles. Fulari *et al.* [27] developed a data fusion-based estimation scheme by using the dynamical systems approach. Nantes *et al.* developed a model-based methodology to build a real-time traffic prediction model for arterial corridors using data from multiple sources in [28]. A strip-based approach based on

density-occupancy relation was proposed by George *et al.* in [29].

From the above literature review, it can be observed that a more generic model needs to be developed that captures lane indiscipline and heterogeneity. Identifying an appropriate measurable variable sensitive to the dynamic changes in traffic shall help in formulating the same. In heterogeneous traffic, varying vehicle dimensions contribute significantly to lack of lane discipline. Vehicles of varying lengths differ in the time occupied on a particular length of road. Considering the absolute road width irrespective of number of lanes, it may be assumed that, the number of vehicles across the width of the road can vary according to the width of the vehicle. A variable that takes care of both these aspects may be better suited for representing the heterogeneous and lane-less nature of the traffic. Hence, this paper uses area occupancy [30], a modified measure of time occupancy, as the output variable. When a small section of road is considered, area occupancy can express the amount of time a vehicle having a particular dimension occupies a given area of that section, whose values depend on the composition of traffic and the speed of the vehicles. Thus, area occupancy can capture the heterogeneity and lane-less nature of the traffic.

The contributions of this paper to the existing literature in the field of heterogeneous traffic control research are therefore the following:

- Area occupancy, a variable that can capture heterogeneity and lane indiscipline, was used as measurement to estimate traffic density.
- Relation between density and area occupancy was derived from basic traffic flow modeling concepts.
- Estimation of density using a dynamic traffic flow equation as the state equation and the derived density-area occupancy relation as the output equation was implemented.
- Adaptive Kalman filtering technique was used to estimate traffic density using the above equations, to incorporate the high variability in the traffic condition under consideration.

The outline of the paper is as follows. Section II presents the model formulation that can effectively capture the mixed traffic characteristics. The details of the extraction of data and evaluation of the estimation scheme by simulating a selected study stretch are discussed in Section III. Section IV encompasses the estimation scheme using conventional Kalman filter and quantitative comparisons between the proposed approach and the strip-based study presented in [29]. A sensitivity study was further done for varying initial density conditions. The proposed dynamic estimation scheme was also compared against static estimation schemes for accuracy, data requirement and computational complexity. In Section V, the adaptive capabilities of the estimator for different traffic scenarios such as congestion and non-recurrent traffic incidents were explored. Two adaptive schemes were developed, one with a window of fixed size and other with a window of increasing size, and their performance was evaluated.

II. MODEL FORMULATION

A non-continuum macroscopic model represented in state-space form was developed to characterize the aggregate behavior of vehicles. Two sets of equations, namely state equation and output equation or measurement equation, were derived. The variable relative flow, defined as the net difference between the rate of vehicles moving in and moving out of the section, was the input variable, with traffic density as the state variable.

In the measurement model, area occupancy, a variable that can take into account varying vehicle dimensions, was used as the measure for capturing heterogeneity. Area occupancy is the proportion of time the set of observed vehicles occupy the chosen stretch of a roadway [30]. It captures how long a vehicle of a particular size is moving on a section of road. Measured over time, it can capture heterogeneity since the time taken to travel the same distance with the same speed will vary according to the vehicle lengths. The entire width of the road is considered as a single unit as it takes into account the area of the road section occupied by the vehicle. This helps in addressing the lane indiscipline condition due to the varying vehicle width.

The state equation that gives the relationship between the state variable and the input variable was formulated by applying conservation law for vehicles inside the section. A typical mid-block road section (bounded by two intersections) was considered as the study stretch. The rate at which vehicles move into the section s was taken as $q_{en,s(i,i+1)}$ and the rate at which vehicles move out of the section as $q_{ex,s(i,i+1)}$ in the interval between the ith and $(i + 1)^{th}$ instants of time. Considering the vehicle count within the section at i^{th} instant of time to be $N_{s(i)}$, the vehicle count inside the section at $(i + 1)^{th}$ instant of time, by vehicle conservation is

$$N_{s(i+1)} = N_{s(i)} + h(q_{en,s(i,i+1)} - q_{ex,s(i,i+1)}), \qquad (1)$$

where h is the time interval between i^{th} and $(i + 1)^{th}$ instants of time and $(q_{en,s(i,i+1)} - q_{ex,s(i,i+1)})$ is the relative flow. Corresponding density, $\rho_{s(i+1)}$ can be written from (1), which forms the state equation, as

$$\rho_{s(i+1)} = \rho_{s(i)} + \frac{h}{L}(q_{en,s(i,i+1)} - q_{ex,s(i,i+1)}), \qquad (2)$$

where L is the section length.

The percent area occupancy, expressed as the proportion of time the chosen section of the road is occupied by vehicles [31], is given by

$$\alpha_{(i)} = \frac{\sum_{k=0}^{N_d} a_k t_{k(i)}}{Ah} 100, \qquad (3)$$

where $t_{k(i)}$ and a_k represents k^{th} vehicle's detection zone occupancy time and occupied area of the detection zone in the interval h respectively, N_d is the number of vehicles passing over the detector in the interval h, and A is the area of the detection zone. The output equation relating area occupancy to density was derived by multiplying and dividing (3)

by N_d as

$$\alpha_{(i)} = \frac{\sum_{k=0}^{N_d} a_k t_{k(i)}}{A N_d} \frac{N_d}{h} 100, \tag{4}$$

where N_d/h is the flow rate, represented as $q_{d(i-1,i)}$ in vehicle per hour. From fundamental relations of traffic flow,

$$q_{d(i-1,i)} = \rho_{d(i)} U_{sms(i)}, \tag{5}$$

where $U_{sms(i)}$ is the space mean speed.

Assuming the density inside the detector as the representative of density of the section, the final equation relating area occupancy and density can be represented as

$$\alpha_{(i)} = \frac{\sum_{k=0}^{N_d} a_k t_{k(i)}}{A N_d} U_{sms(i)} \rho_{s(i)} 100. \tag{6}$$

The state equation given by (2) and the output equation given by (6) together represent the state-space model for the traffic system under study. The stochastic nature of the traffic was included in the model by adding process disturbance $w_{(i)}$ in state equation and measurement noise $v_{(i)}$ in measurement equation, as given by

$$x_{(i+1)} = a x_{(i)} + b u_{(i)} + w_{(i)},$$
$$y_{(i)} = c_{(i)} x_{(i)} + v_{(i)}, \tag{7}$$

where $x_{(i)}$ the state variable is density, $u_{(i)}$ the input variable is relative flow, and $y_{(i)}$ the output variable is area occupancy. Here, the variables $w_{(i)}$ and $v_{(i)}$ were considered as normally distributed zero mean independent, white noise signals with finite covariance q and r respectively. This represents the traffic system as a single input single output linear time varying system with the state parameter a, equal to 1, input parameter b, equal to $\frac{h}{L}$ and output parameter $c_{(i)}$, varying with time given as

$$c_{(i)} = \frac{\sum_{k=0}^{N_d} a_k t_{k(i)}}{A N_d} U_{sms(i)} 100. \tag{8}$$

III. DATA EXTRACTION
The proposed work is illustrated as a block diagram in Fig. 1. The relative flow, area occupancy and the variables for obtaining the time varying output parameter $c_{(i)}$ are the data requirements in the present study. An urban arterial road along Rajiv Gandhi Salai in the city of Chennai, India, was identified as the study stretch, which is a mid-block road section bordered by two intersections as shown in Fig. 2. It is a six-lane road having three lanes in each direction. For the purpose of the current study, three lanes in one direction having widths of 3.5 m each were selected. The length of the study stretch L is 1.73 km with 1 km between detector locations A and B, and 0.73 km between detector locations B and C as shown in Fig. 2. These were considered separately because of the presence of a signal near C, which makes traffic conditions in BC different from AB, which is beyond the influence of the signal. Figure 3 shows the routine scenario observed on the study stretch depicting the heterogeneous and lane-less nature of the traffic.
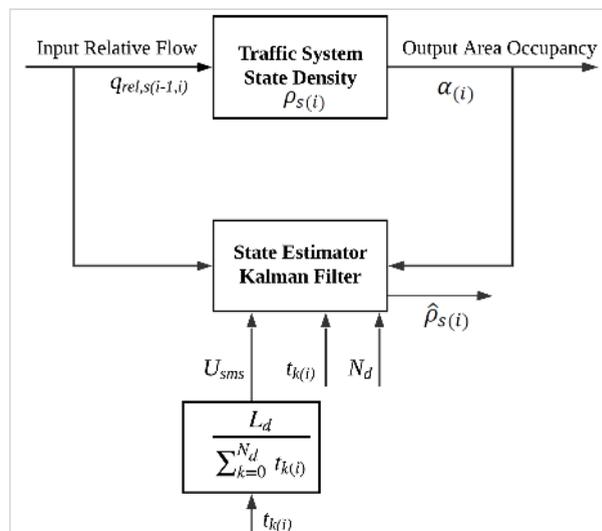

FIGURE 1. Block diagram of the proposed work.

The study stretch was simulated in a traffic simulation software, VISSIM [32] for generating the required data. VISSIM is mainly intended for simulating lane disciplined and homogeneous traffic observed on European roads. Hence, the default simulation parameters of the software may not be suitable for the mixed traffic scenario. However, it can be calibrated for the traffic under consideration, since it offers options for left side driving, incorporating different types of vehicles, varying lane widths, adjusting lateral distance between vehicles, same lane overtaking and staggered queuing at intersections [24]. A previous study [24] calibrated the above parameters to represent the traffic condition under study. With the calibrated model, the identified road stretch was simulated for different traffic scenarios. A warm-up period of 15 minutes was provided for each simulation. This study considered four different classes of vehicles namely three-wheelers, light motor vehicles, heavy motor vehicles and two-wheelers. The proportions of the various classes of vehicles obtained from field along with their standard dimensions [33] are listed in Table 1. Besides these, the inputs provided to VISSIM based on field values were traffic volume data for each 15 minute interval and signal control data. A fixed time signal with 32 s green and 4 s yellow, out of 100 s cycle time was used for controlling the signal at the intersection. The simulation was done by varying the input volume for different scenarios. For each case, the model was simulated four times with a random seed increment of 5 and an average value was taken. From simulation, the required data from upstream and downstream end of the study stretch, and the simulated density for corroborating the model were collected. Virtual detectors, each of 10 m length were placed at locations A, B and C as shown in Fig. 2. Area occupancy, and time varying parameter $c_{(i)}$, were obtained from this data by taking a weighted average based on flow through each detector. The variables $t_{k(i)}$ and N_d required for calculating area occupancy, and $c_{(i)}$ were taken class wise. The required
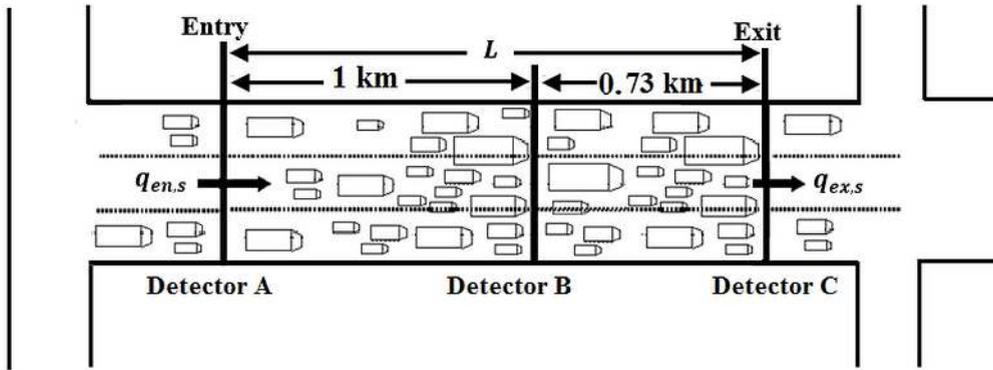
FIGURE 2. Study stretch of the proposed work.

FIGURE 3. Prevalent traffic on the study stretch.

TABLE 1. Proportion of vehicle and their dimensions under study.

Type	Length (m)	Width (m)	Proportion (%)
2-Wheeler	1.8	0.6	47.5
3-Wheeler	2.6	1.4	5.3
Light Motor Vehicle	4	1.6	44.6
Heavy Motor Vehicle	10.3	2.5	2.6

data were extracted from VISSIM for every 100 s, which is the time interval, h, used in this study.

IV. ESTIMATION SCHEME AND ITS EVALUATION

Four different density estimation schemes, two of which are static and the other two dynamic, were implemented and compared. The four estimation schemes were named as ES1 to ES4, with ES1 and ES2 being static schemes, and ES3 and ES4 being dynamic ones. ES4 is the proposed estimation scheme in this study. Details of these schemes are discussed below.

Static Estimation Schemes - ES1 and ES2

The first estimation scheme named ES1 is a standard approach based on occupancy, which is defined as the time the vehicle occupies the given roadway. The standard relationship between density and occupancy [3] given by (9) is used in this regard:

$$O_{(i)} = (L_v + L_d)\rho_{(i)}100. \qquad (9)$$

This equation was used to calculate density using the measured percent occupancy $O_{(i)}$ for a known average vehicle length L_v and detection zone length L_d. Based on the knowledge of proportion and length of vehicles given in Table 1, a weighted average vehicle length was calculated.

In the second estimation scheme ES2, input-output technique [3] was used to estimate density. In this technique, density is estimated with the knowledge of initial count of vehicles inside the selected road stretch and the number of vehicles moving in and out of the section over time.

Baseline Dynamic Estimation Scheme - ES3

In ES3, a non-continuum macroscopic single state linear model developed by George *et al.* [29] was used. Lane indiscipline was incorporated by dividing the study stretch into multiple parallel strips and density was estimated using Kalman filter. The output variable was time occupancy, which along with proportion-based weighted average vehicle length, incorporated heterogeneity.

Proposed Dynamic Estimation Scheme - ES4

In this scheme, the traffic system was modeled as a single state non-continuum macroscopic linear time varying model as given by (2) and (6). To take into account heterogeneity and lane indiscipline, area occupancy, a variable that considers varying vehicle dimensions, was used for measurement. Further, lane indiscipline was incorporated by treating whole of the road width as a single unit without considering traffic lanes. The system model described with a state equation and an output equation (7) was used in Kalman filtering technique to develop a model-based estimation scheme.

A. IMPLEMENTATION AND EVALUATION

The proposed scheme ES4 was implemented using Kalman filtering technique [20], which is based on the mathematical model describing the system characteristics, where statistical properties of the process error and the measurement noise are taken into consideration. The system model and the state estimate from the previous time interval are used to predict
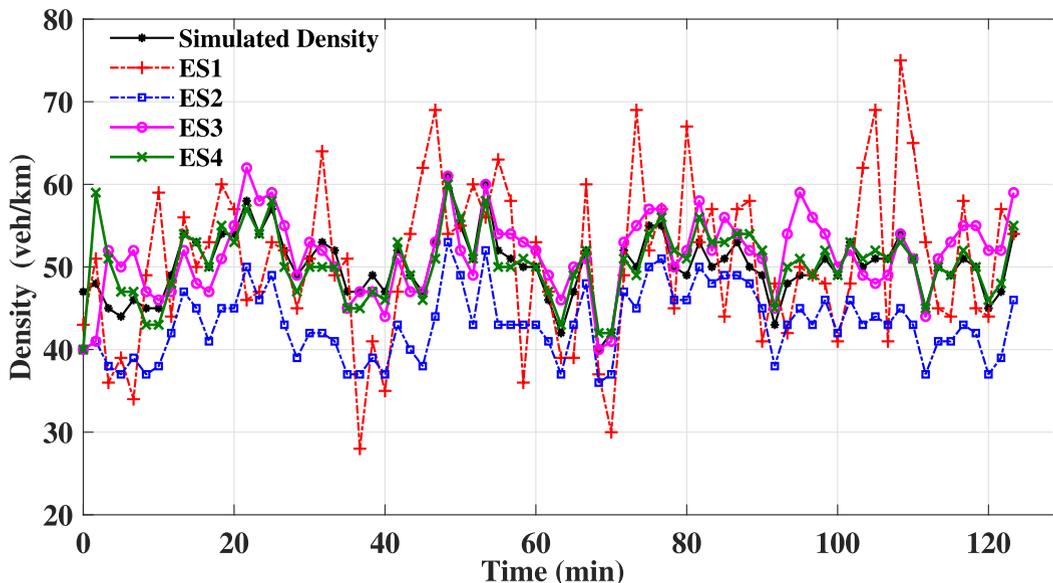
the state variables. Measurements are then used to obtain an updated state estimate. Let $\hat{x}^+_{(i+1)}$ denote the updated estimate of the state at the $(i + 1)^{th}$ instant and $\hat{x}^-_{(i+1)}$ denote the predicted estimate of the state variables at the $(i + 1)^{th}$ instant of time. Let $p^-_{(i+1)}$ and $p^+_{(i+1)}$ respectively denote the predicted and updated estimation error variance at the $(i + 1)^{th}$ instant of time. Then the steps in the implementation are as shown below.

i Prediction step or time update calculates the predicted state estimate and the predicted estimation error variance as

$$\hat{x}^-_{(i+1)} = a\hat{x}^+_{(i)} + bu_{(i)}, \quad (10)$$

$$p^-_{(i+1)} = ap^+_{(i)}a + q. \quad (11)$$

ii Correction step or measurement update calculates the Kalman gain, the updated state estimate and the updated estimation error variance as

$$K_{(i+1)} = p^-_{(i+1)}c_{(i+1)}\left[c_{(i+1)}p^-_{(i+1)}c_{(i+1)} + r\right]^{-1}, \quad (12)$$

$$\hat{x}^+_{(i+1)} = \hat{x}^-_{(i+1)} + K_{(i+1)}[z_{(i+1)} - c_{(i+1)}\hat{x}^-_{(i+1)}], \quad (13)$$

$$p^+_{(i+1)} = [1 - K_{(i+1)}c_{(i+1)}]p^-_{(i+1)}. \quad (14)$$

The proposed estimation scheme (ES4) was implemented with an assumed initial density value using the developed state-space model as in (2) and (6) with the above algorithm.

To evaluate the proposed model-based estimation scheme, the estimated density was compared with the values obtained from VISSIM simulation. The traffic density was estimated by all four estimation schemes using the same input data and initial density value. In real time situations, data extraction from loop detectors would include errors. A uniformly distributed random error in the interval [-2, 0] was added to the vehicle count to mimic these errors. Signal to noise

ratio of 15 was assumed for the error in occupancy and area occupancy. The error was assumed to be Gaussian white noise sequence with zero mean and standard deviation of σ, computed from the chosen signal to noise ratio. The estimation accuracy was quantified using Mean Absolute Percentage Error (MAPE) and Root Mean Square Error (RMSE) as given by

$$MAPE = \frac{1}{m}\sum_{i=1}^{m}\frac{\left|\rho_{est(i)} - \rho_{sim(i)}\right|}{\rho_{sim(i)}}100, \quad (15)$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{m}(\rho_{est(i)} - \rho_{sim(i)})^2}{m}}, \quad (16)$$

where $\rho_{est(i)}$ and $\rho_{sim(i)}$ are the estimated and simulated values of density in the ith instant of time respectively and m is the total number of observations.

Figure 4 shows a sample comparison between the estimated density obtained from all four estimation schemes and simulated values obtained from VISSIM. From Fig. 4, it can be observed that ES1 is not well suited for characterizing mixed traffic conditions, while the error in the initial condition persisted with time for ES2. Table 2 shows the corresponding MAPE and RMSE values. For all three days, data were generated for light traffic condition. It can be seen that the mean MAPE value is around 15 % for ES1 and ES2. Further improvement in accuracy may not be attainable with these schemes since static estimation schemes cannot account for uncertainties and noisy data. Moreover for ES1, the average vehicle length, L_v, must be known, which is difficult to determine in mixed traffic condition and for ES2, any error in the count will cumulate over time. This exercise demonstrated the need for a dynamic density estimation method.

In the results shown in Fig. 4, it can be observed that the scheme ES3 and proposed scheme ES4 converged to the

TABLE 2. MAPE and RMSE for various density estimation methods for three different days.

Days	ES1		ES2		ES3		ES4	
	MAPE (%)	RMSE (veh/km)	MAPE (%)	RMSE (veh/km)	MAPE (%)	RMSE (veh/km)	MAPE (%)	RMSE (veh/km)
1	15.08	12.84	17.29	12.65	5.19	4.52	3.95	3.63
2	13.42	10.86	13.6	8.55	6.22	5.04	3.42	3.7
3	13.95	8.61	13.94	7.31	5.43	3.4	2.83	2.09
Mean	14.15	10.77	14.94	9.5	5.61	4.32	3.4	3.14

TABLE 3. Comparison of static and dynamic estimation schemes.

Attributes	ES1	ES2	ES3	ES4
Model				
Nature of model	Linear discrete algebraic equation	Linear first order difference equation	Single input single output linear time invariant difference equation	Single input single output linear time varying difference equation
State variable	Not applicable	Not applicable	Density	Density
Input variable	Time occupancy	Relative flow	Relative flow	Relative flow
Output variable	Density	Density	Time occupancy	Area occupancy
Estimation				
Type	Static	Static	Dynamic	Dynamic
Compiling time (for one 120 minute data set)	0.85 ms	0.95 ms	3.26 ms	3.3 ms
Accuracy	Low	Low	High	Highest
Data required	Time occupancy	Relative flow	Relative flow, Time occupancy	Relative flow, SMS speed, class wise occupancy time and no. of vehicles passing over the detector

simulated density, unlike in estimation schemes ES1 and ES2, where the initial error propagated over time. In scheme ES3, the model had two dynamic parameters namely weighted average vehicle length and number of parallel paths, which were taken to be constant for a particular vehicle composition [29]. Though the estimated density from this method initially converged to that of the simulated values, it started diverging later.

ES4, on the other hand, shows consistent performance after convergence. The performance of the estimation schemes ES3 and ES4 were evaluated using MAPE and RMSE for three different days and is also presented in Table 2. It can be observed that the estimation performance significantly improved using the proposed dynamic estimation scheme (ES4). The mean of MAPE values was less than 4 % with RMSE of 3 veh/km for ES4, which indicates highly accurate performance of the estimator [34].

In Table 3, an overall comparison between the four schemes is presented. Data were processed using an Intel® Core™ i7 Processor. It can be seen that the computational complexity measured in terms of compiling time was better for static estimation schemes, while accuracy level was higher for dynamic estimation schemes. When data collection is

considered, ES4 requires class wise collection of the variables $t_{k(i)}$ and N_d. However, the performance was better for ES4 compared to the other three schemes.

B. SENSITIVITY STUDY FOR INITIAL DENSITY
The convergence of the proposed estimation scheme ES4 for varying initial condition of density is analyzed in this section. The estimation scheme was implemented for initial density varying from 0 veh/km to 100 veh/km. The estimated density was compared with the simulated density and the results are shown in Fig. 5. It can be seen that for all assessed initial conditions, the estimation scheme converged with convergence time less than 10 minutes. Subsequently, analyses of the proposed scheme to various traffic scenarios were done and are discussed in the next section.

V. ADAPTIVITY TO DIFFERENT TRAFFIC SCENARIOS
Traffic conditions in roads are quite unpredictable and vary due to both recurring and non-recurring conditions. For instance, an incident may occur causing blockage leading to increase in density. In addition, there will be variations in traffic from peak to off-peak. This necessitates checking whether the estimation scheme can detect such scenarios

TABLE 4. Test matrix for different scenario.

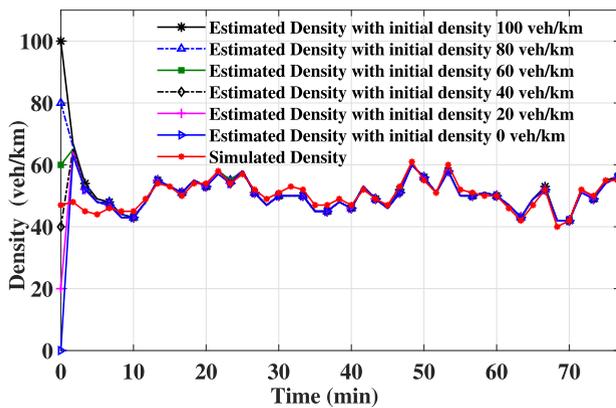| Sl. No.	Scenario	Cycle time (seconds)	Green time (seconds)	Input volume (veh/h)	Time period (h)	No. of lanes
1	From uncongested to congested and vice versa	100	32	2850-5130 5130-1520	2.5 h 2.5 h	3
2	Light traffic condition (uncongested)	100	32	2850-3420	2 h	3
3	Scenario where there is an incident	100	32	2850-3420	2 h	2 lanes blocked for 20 minutes
4	Heavy traffic condition (jam density)	100	32	5700-6840 0-1520	1.15 h 1.15 h	3

FIGURE 5. Convergence for different values of initial density.

TABLE 5. MAPE and RMSE for ES4 estimation scheme for different scenarios.

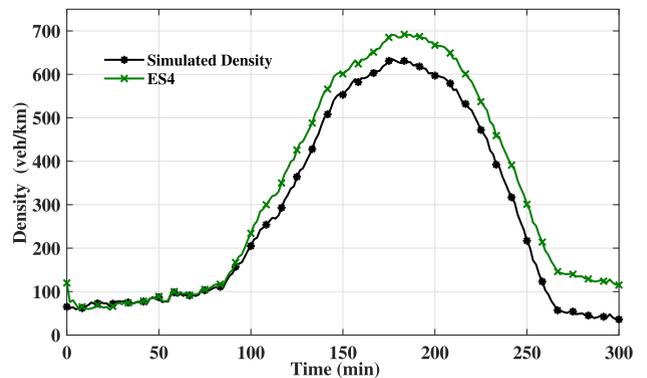| Sl. No.	Scenario	ES4 MAPE (%)	RMSE (veh/km)
1	From uncongested to congested and vice versa	44.52	57.1
2	Light traffic condition (uncongested)	3.95	3.6
3	Heavy traffic condition (jam density)	32.26	67.81
4	Scenario where there is an incident	6.71	8.23

FIGURE 6. Corroboration of ES4 scheme for scenario 1 (transition).

through appropriate changes in density. Hence, a test matrix was generated for different scenarios, listing the various inputs to VISSIM for simulating the network for data generation as given in Table 4. Different scenarios simulated varied from light traffic condition to heavy traffic, transition periods, jam density condition and an incident scenario where an incident was injected in the study stretch and assumed to block two lanes for 20 minutes. ES4 was used for the estimation of density for these scenarios and results obtained are shown in Table 5. It can be seen that for congested and transition scenarios, the performance is poor. Here, it has to be noted that fixed process covariance q and measurement noise covariance r were used in the Kalman filter. This result shows the necessity of an adaptive scheme. Results of scenario 1 are presented in Fig. 6 to highlight this point further. In this scenario, 5 hour data including a transition from light traffic to heavy traffic and vice versa is considered. Starting from the point of transition from the low-density regime, an increase in error could be observed for ES4. This may be due to the fact that the change in noise characteristics was not considered with changes in scenario. Hence, to take into account these varying conditions, an adaptive Kalman filter was developed, as discussed next.

In the Kalman filter, the process covariance q represents the uncertainty in the process and process model. In this study, process noise is likely to be small, since the process

model was developed using the vehicle conservation equation and the input given to the model is expected to be accurate. However, inaccuracy in both measurement and measurement equation can be expected to a certain amount. Hence, the filter was made adaptive with fixed q and varying r. In ES4, the process disturbance and measurement noise were assumed to be normally distributed zero mean independent, white noise signals. The filter was made adaptive by varying r and taking a non zero mean value for measurement noise. In order to evaluate the changing properties of the measurement noise and its covariance, the predicted state estimate was used to obtain the residue from the measurement equation in (7). The statistical properties of measurement noise were recalculated before the correction step. The residue was calculated for each time step as the difference between the actual measurement and the
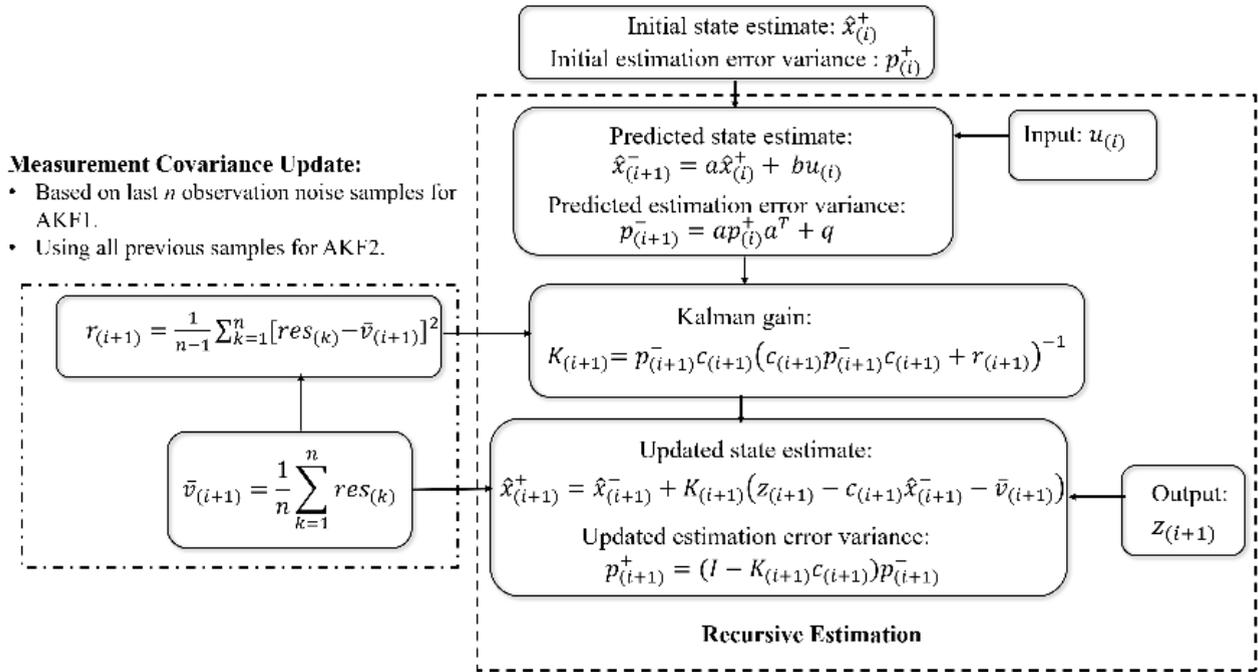
Measurement Covariance Update:
- Based on last *n* observation noise samples for AKF1.
- Using all previous samples for AKF2.

$$r_{(i+1)} = \frac{1}{n-1} \sum_{k=1}^{n} [res_{(k)} - \bar{v}_{(i+1)}]^2$$

$$\bar{v}_{(i+1)} = \frac{1}{n} \sum_{k=1}^{n} res_{(k)}$$

Initial state estimate: $\hat{x}_{(i)}^+$
Initial estimation error variance : $p_{(i)}^+$

Predicted state estimate:
$$\hat{x}_{(i+1)}^- = a\hat{x}_{(i)}^+ + bu_{(i)}$$
Predicted estimation error variance:
$$p_{(i+1)}^- = ap_{(i)}^+ a^T + q$$

Input: $u_{(i)}$

Kalman gain:
$$K_{(i+1)} = p_{(i+1)}^- c_{(i+1)} \left(c_{(i+1)} p_{(i+1)}^- c_{(i+1)} + r_{(i+1)} \right)^{-1}$$

Updated state estimate:
$$\hat{x}_{(i+1)}^+ = \hat{x}_{(i+1)}^- + K_{(i+1)} \left(z_{(i+1)} - c_{(i+1)}\hat{x}_{(i+1)}^- - \bar{v}_{(i+1)} \right)$$
Updated estimation error variance:
$$p_{(i+1)}^+ = (I - K_{(i+1)}c_{(i+1)})p_{(i+1)}^-$$

Output:
$z_{(i+1)}$

Recursive Estimation

FIGURE 7. Algorithm for adaptive Kalman filter schemes.

predicted value, given by

$$res_{(i+1)} = [z_{(i+1)} - c_{(i+1)}\hat{x}_{(i+1)}^-]. \tag{17}$$

The quantity $\bar{v}_{(i+1)}$ was calculated as the simple mean of these residuals and the measurement covariance $r_{(i+1)}$ as the variance of the residuals as

$$\bar{v}_{(i+1)} = \frac{1}{n} \sum_{k=1}^{n} res_{(k)},$$

$$r_{(i+1)} = \frac{1}{n-1} \sum_{k=1}^{n} [res_{(k)} - \bar{v}_{(i+1)}]^2, \tag{18}$$

where *n* is the number of time instants prior to $(i+1)$ used in the process.

In the correction step, the updated state estimate was modified as

$$\hat{x}_{(i+1)}^+ = \hat{x}_{(i+1)}^- + K_{(i+1)}[z_{(i+1)} - c_{(i+1)}\hat{x}_{(i+1)}^- - \bar{v}_{(i+1)}]. \tag{19}$$

Based on the choice of *n*, two algorithms were developed: adaptive Kalman filter with a window of fixed size (AKF1) and adaptive Kalman filter with a window of increasing size (AKF2). In the fixed window size filter AKF1, the number of samples (window size) was fixed and at any time step, the previous *n* number of samples were taken for calculating $\bar{v}_{(i+1)}$ and $r_{(i+1)}$. In AKF2, the size of the window increased at every time step and all previous samples were taken for calculating $\bar{v}_{(i+1)}$ and $r_{(i+1)}$. The proposed algorithms AKF1 and AKF2 for the estimation of traffic density can be summarized using the algorithm as in Fig. 7.

A comparison of the three estimation schemes ES4, AKF1 and AKF2 was done next and the results are

TABLE 6. MAPE and RMSE for various adaptive Kalman estimation schemes for different scenarios.

Sl. No.	Scenario	AKF1		AKF2	
		MAPE (%)	RMSE (veh/km)	MAPE (%)	RMSE (veh/km)
1	From uncongested to congested and vice versa	3.32	6.1	5.14	5.45
2	Light traffic condition (uncongested)	2.9	3.13	3.01	3.29
3	Heavy traffic condition (jam density)	3.01	16.66	6.91	21.73
4	Scenario where there is an incident	2.02	2.12	2.75	2.75

discussed here. Performance was evaluated in terms of MAPE and RMSE and the results obtained are shown in Table 5 and 6. The proposed adaptive algorithms AKF1 and AKF2 can be found to have better performance than ES4.

Further analysis of the performance of the methods was done to understand the adaptive nature of the Kalman filter. Figure 8 shows scenario 1, where it can be seen that all the three estimation schemes initially converged to the simulated density during low density regime. However, from the point of transition, ES4 diverged, as clearly observed in Fig. 6. Being a scheme with fixed noise statistics, having zero mean and fixed variance, it was not able to capture data characteristics for transition and change in traffic conditions.

In comparison, the adaptive schemes AKF1 and AKF2 that estimated the noise statistics for each time interval performed better as they captured the change in data characteristics.
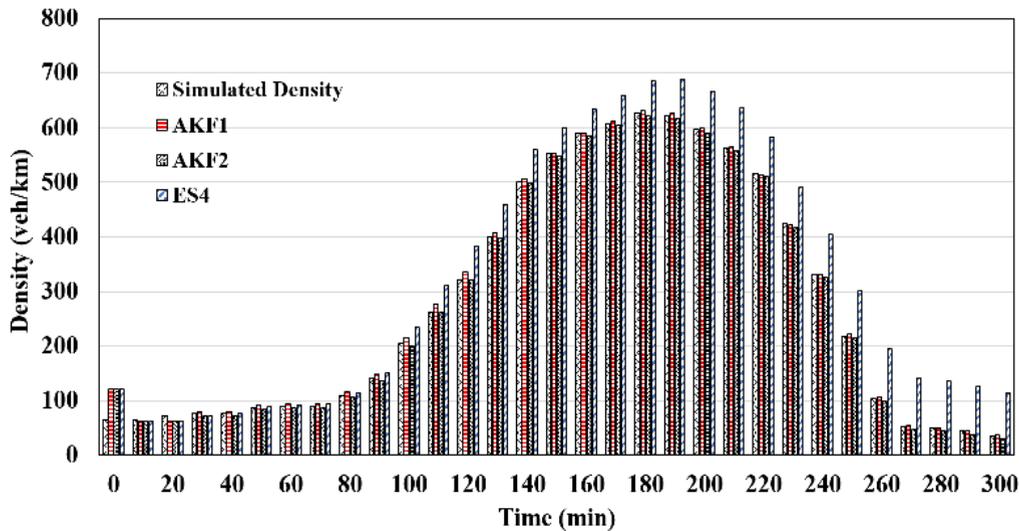
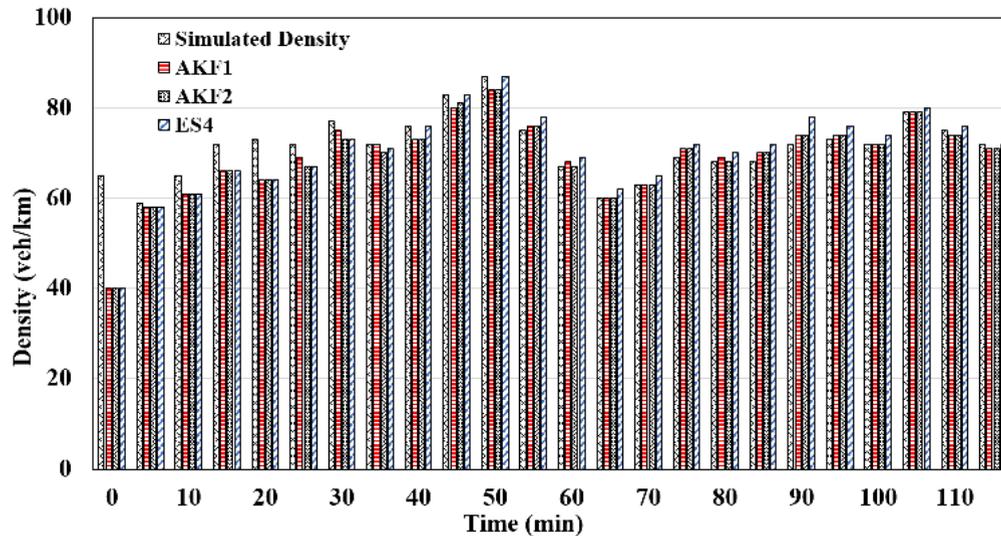**FIGURE 8.** Performance comparisons under scenario 1 (transition).

FIGURE 9. Performance comparisons under scenario 2 (light traffic).

Figure 9 shows scenario 2 characterized by light traffic conditions with less data variations, where all the schemes performed well. In scenario 3, scheme ES4 led to a bias where the density reached jam density during fully congested conditions, as observable from Fig. 10. In scenario 4, an incident was injected for a period of 20 minutes, causing the blockage of two lanes. This resulted in building up of a queue in between detectors B and C causing a change in traffic condition (increase in density). During queue build-up and release, detectors B and C were congested. The errors in the measurement model and measurement were well captured by the adaptive schemes as shown in Fig. 11. The scheme ES4 with fixed variance resulted in an offset after the incident was over.

Thus, for transition and congestion scenarios, when the error in measurement or measurement model can be higher, ES4 with fixed measurement noise statistics was not able to capture the variation. The adaptive schemes AKF1 and AKF2 that estimate the mean and variance of the measurement noise performed better in these cases. In AKF1, the variations were better captured, where mean and variance were estimated with the last n observed noise samples compared to AKF2 where all previous samples were used. Overall, it can be seen that AKF1 scheme estimated section density better, especially during the congestion period and transitions. Further, in AKF2, the window size increases with time, and hence AKF1 requires lower online data storage.
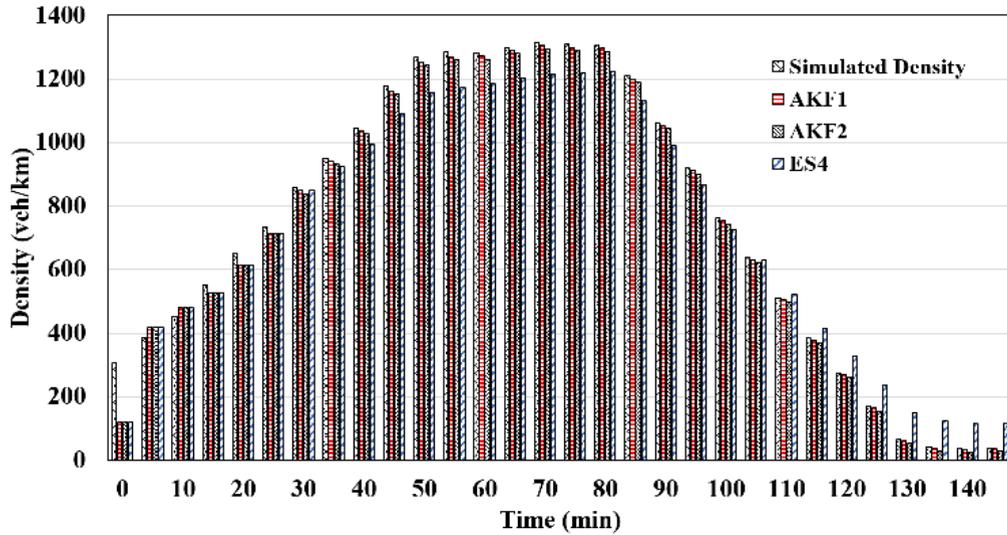
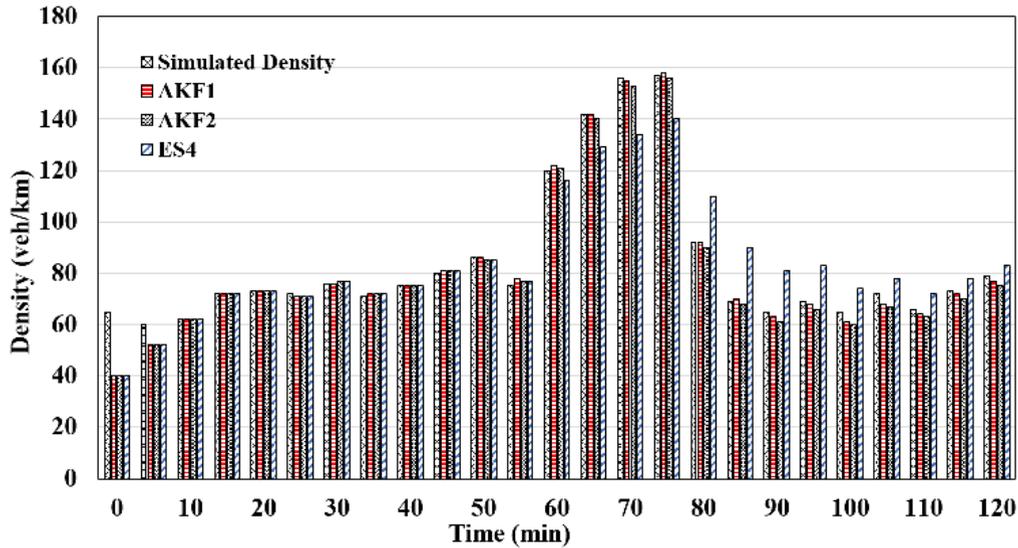**FIGURE 10.** Performance comparisons under scenario 3 (jam density).

FIGURE 11. Performance comparisons under scenario 4 (incident).

VI. SUMMARY AND CONCLUSION

Estimation of density is an essential component in the study of traffic congestion in mixed traffic conditions characterized by heterogeneous and lane-less traffic. In spite of being a good indicator of congestion, the on field measurement of density is rather demanding. Hence, this paper used area occupancy as a surrogate variable for traffic density estimation, which is capable of considering varying vehicle dimensions as encountered in heterogeneous and lane-less traffic. Model-based schemes for estimation of density under mixed traffic conditions were developed by applying the Kalman filter technique.

The key aspects of the research can be summarized as follows:

- The paper presents a method to estimate road traffic congestion using density as a measure under mixed traffic condition.

- The traffic system was characterized using a non-continuum macroscopic single state linear time varying model.
- The derived density-area occupancy relation was used to formulate the output equation that captured the heterogeneity and lane indiscipline.
- Corroboration of the estimation scheme with the simulated density showed an average Mean Absolute Percentage Error of less than 4 %, which illustrated an accurate estimate.
- Sensitivity analyses done for initial conditions showed the robustness of the scheme.
- When compared with an existing dynamic estimation scheme and two static estimation schemes, the proposed one showed better performance.
- Since the proposed scheme was unable to capture the variations in noise statistics for different scenarios,

adaptive Kalman filter with fixed window size (AKF1) and with increasing window size (AKF2) were developed.

- On performance analysis under various scenarios, AKF1 outperformed the earlier scheme and AKF2 with a Mean Absolute Percentage Error of less than 4 %.

Based on the results of the present study, it may be inferred that the adaptive Kalman filter estimation scheme based on area occupancy can accurately estimate density under mixed traffic conditions. The proposed scheme has the potential for real-time implementation in heterogeneous and lane-less traffic that shall contribute to the development of appropriate control schemes.

ACKNOWLEDGMENT

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