Biomed. Technik
26 (1981), 147–150

S. Radhakrishnan
D. N. Ghista
G. Jayaraman
K. M. Patil

Analysis for the Invivo Determination of the Non-Linear Elastic Parameters of an Artery

Die Bestimmung von nichtlinearen elastischen Parametern von Arterien in vivo

Key-words: Arteries, Pulse velocities, transversely isotropic, Strain energy density function

An expression for the pulse wave velocity in a non-linear elastic pressurized arterial tube is derived; in terms of the arterial constitutive (experimentally derived strain energy density function) parameters and the undeformed and deformed arterial diameters. The pulse wave velocity expression is then employed for developing the analysis for invivo determination of the arterial constitutive parameters from arterial imaging data. The analysis provides a method to non-invasively determine the values of the non-linear elastic parameters of the artery by using the invivo monitored values of its diameter and the pulse wave velocity. This information on the elastic parameters of an artery could characterize the state of the artery and hence a diagnostic indication of the atheromatous involvement of the blood vessel.

Schlüsselwörter: Arterie, Pulsgeschwindigkeit, transversale Isotropie, Strain energy density Funktion

Ein Ausdruck für die Pulswellengeschwindigkeit in einer nichtlinearen elastischen unter Druck stehenden Arterie wird abgeleitet, und zwar unter Verwendung von Parametern, die experimentell aus der strain energy density Funktion gewonnen wurden, sowie den nichtdeformierten und deformierten Arteriendurchmessern.
Der Ausdruck für die Pulswellengeschwindigkeit wird für die Bestimmung der arteriellen Parameter in vivo benutzt, wobei Daten aus Aufnahmen von Arterien zugrundegelegt werden.
Die Analyse stellt eine Methode dar zur nicht-invasiven Bestimmung der nichtlinearen elastischen Parametern von Arterien, wobei neben den in vivo-bestimmten Durchmessern die Pulswellengeschwindigkeit benutzt wird.
Diese Information über die elastischen Parameter von Arterien könnten dazu dienen, deren Zustand zu charakterisieren, und somit z. B. einen diagnostischen Hinweis auf eine Atheromatose zu liefern.

Nomenclature

A, k = arterial material elastic parameters
a_1, a_2 = undeformed external and internal radius
r_1, r_2 = deformed external and internal radius
c = pulse propagation velocity
h = thickness of blood vessel
I = strain invariant
K = integration constant
P = internal pressure
S = deformed cross-sectional area of vessel
W = strain energy density function
λ = axial extension ratio
ϱ_F = density of fluid medium

1 Introduction

The in vivo pulse wave propagation velocity can be obtained transcutaneously by monitoring the time delay »ΔT« in the propagation of pressure pulse wave between two sites. Hence, the measurement of pulse wave velocity (PWV) through an elastic tube or a blood vessel can provide a noninvasive determination of its elasticity, and hence a diagnostic indication of artheromatous involvement, with the aid of a rigorously representative model for PWV incorporating the anisotropy, geometrical and material non-linearities.

The original expressions for the pulse velocity in a thin isotropic elastic tube were derived by Young, Moses and Korteweg. In reality, the artery has thick walls and exhibit anisotropic behaviour, as shown by Fenn [3] and Attinger [1]. Initially, Mirsky [4] presented a Moens-Korteweg type relation, based on the exact three dimensional equations of infinitisimal elasticity for an orthotropic elastic tube. Later, he (Mirsky [5]) extended his work to finite deformation theory, since observations of arteries removed from the body reveal extractions of the order of 30 % to 40 %. In that analysis a formula was obtained for the pulse wave velocity in a cylindrical tube with an initial extension, in terms of the internal pressure and the tube geometry. He however, approximated the arterial medium by a Mooney material, which represents the behaviour of an incompressible material such as vulcanized rubber.

Simon et al. [8] determined experimentally the requisite in vitro arterial constitutive property, by measuring the finite deformation response of an axially constrained arterial tube segment. The constitutive property of this incompressible, non-linear elastic, orthotropic arterial tube, in a state of plane strain, was obtained in terms of the partial derivative of the strain energy density function; this function was different from that of the Mooney material assumption of Mirsky [5]. Simon et al. [8] obtained the strain energy density function W as an exponential function of the first strain invariant, whereas for a Mooney material W is a linear algebraic function of the strain invariants.

2 Scope of the present work

The present work is carried out to provide the analysis for a method to noninvasively determine the values of the nonlinear elastic parameters of an artery, by ultrasound monitoring of its diameter and hence its pulse wave velocity. To this end, the arterial elasticity is expressed by means of the strain energy density function of Simon et al. [8], and an expression for the pulse propagation velocity is derived in terms of the nonlinear elasticity parameters as well as the values of the diameters of the undeformed and deformed arterial tube. The representative arterial constitutive relation, adopted in our analysis, is that obtained by Simon et al. [8] as $\delta_W/\delta_I = Ae^{kI}$. The pulse wave velocity is a frequency dependent quantity, which approaches a constant value at moderate frequencies. Hence a quasi-static formulation for PWV is obtained by adopting its distensibility definition

$$c^2 = \frac{1}{\varrho_F}\frac{\Delta_P}{\Delta_S}\cdot S \qquad (1)$$

to derive its expression in terms of the W function parameters and its deformed and undeformed diameters. In the above expression, S is the arterial cross-sectional area prior to the arrival of the pressure pulse Δ_P, and Δ_S is the resulting change in S due to Δ_P.

3 Analysis

The analysis is based on the following assumptions:

i) the artery is a thick walled cylindrical tube.
ii) the artery undergoes finite deformation.
iii) the arterial medium is homogeneous, non-linear elastic incompressible transversely isotropic material.

The strain energy density function becomes (under these assumption)

$$W = W_1(I, K_1)$$

wherein the strain invariants I and K_1 are given as $I_1 = I = 3 + 2e_{ii}$; e_{ij} being the Lagrangian strain components, and $K_1 = (\lambda^2 - 1)/2$; λ being the axial extension ratio. For an artery held at a constant in vivo extension ratio, the quantity K_1 will be a constant, and hence W (the strain energy density function) will be a function of I_1 only.

(iv) The constitutive equation, proposed by Simon et al. [8], is adopted in our analysis

$$\frac{\delta W_1}{\delta I} = Ae^{kI} \qquad (2a)$$

wherein A and k (constants for a given extension ratio λ) represent the arterial elastic parameters and the strain invariant, I is defined in terms of the extension ratios as

$$I = I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2 \qquad (2b)$$

3.1 Stress Analysis

Using the above assumptions, the inplane stress τ_{ij} (i, j = 1, 2) per unit area of the prescribed finitely deformed cylindrical tube can be expressed as:

(radial stress) $$\tau'' = 2\bar{h} + \frac{2Q^2}{\lambda^2}\frac{\delta W_1}{\delta I} \qquad (3a)$$

(tangential stress) $$r^2\tau^{22} = 2\bar{h} + \frac{2}{Q^2}\frac{\delta W_1}{\delta I} \qquad (3b)$$

(inplane shear stress) $$\tau^{12} = 0 \qquad (3c)$$

In equations (3a) and (3b), $\bar{h}$ is known »hydrostatic pressure term« and (by invoking the incompressibility criterion)

$$\varrho^2 = r^2 Q^2(r) = \lambda (r^2 + K) \qquad (4)$$

where r = deformed radial coordinate,
r_1, a_1 = deformed and initial external radii,
and K = integration constant.

Pulse velocites in transversely isotropic cylindrical tubes.

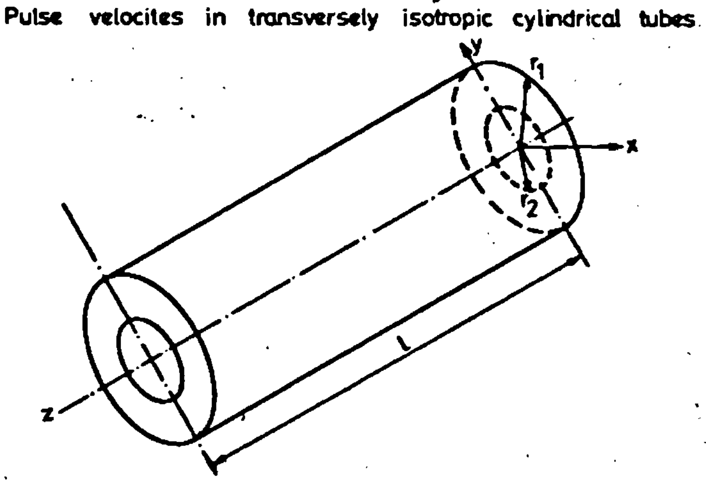

Fig. 1. View of an artery showing the coordinates used
L = Length of arterial segment; r_2 = Deformed internal radius of the tube; r_1 = Deformed external radius of the tube.
The expression for pulse wave velocity as given by the moens-korteweg relationship is $C^2 = [1/p_F]\,\Delta P/\Delta S.\ S.$ Where —— (1); C = Velocity of pulse; p_F = Density of fluid medium; S = Cross sectional area of tube; $\Delta P/\Delta S$ = Change in pressure with respect to change in area of cross section.

In the derivation of (4) it is assumed that a circular cylinder (Fig. 1) of length 1, and radii a_2 and a_1 in the undeformed state undergoes (i) a simple uniform extension of extension ratio λ parallel to the axis of the tube (ii) a uniform inflation in which its length remains constant and radii change to r_2 and r_1.

Any point (r, θ, z) in the deformed state was initially at the point (ϱ, θ, z/λ) where ϱ is a function of r. For the case of incompressibility condition assumed, the relation between ϱ and r is obtained by equating the volumes in the deformed and undeformed states so that,

$$2\pi r\,dr\,dz = 2\pi\varrho\,d\varrho\,dz/\lambda \qquad (5)$$

$$\therefore\ \frac{\varrho\,d\varrho}{dr} = \lambda r \qquad (6)$$

which on integration yields

$$\varrho^2 = \lambda(r^2 + K) = r^2 Q^2(r) \qquad (7)$$

where K is a constant.

Since when $r = r_2, \varrho = a_2$ and at $r = r_1, \varrho = a_1$ (8)

$$\lambda K = a_2^2 - \lambda r_2^2 = a_1^2 - \lambda r_1^2 \qquad (9)$$

$$\therefore\ K = \frac{a_2^2}{\lambda} - r_2^2 = \frac{a_1^2}{\lambda} - r_1^2 \qquad (10)$$

The stress equations of equilibrium are:

$$\frac{\delta \tau''}{\delta r} + \frac{1}{r}\tau'' - r\tau^{22} = 0 \qquad (11)$$

$$\frac{\delta \tau^{22}}{\delta \theta} = \frac{\delta \tau^{23}}{\delta \theta} = 0 \qquad (12)$$

On substituting (3a) and (3b) into (11) and (12), we get

$$\frac{d}{dr}\left[2\frac{\delta W_1}{\delta I}\frac{Q^2}{\lambda^2} + 2\bar{h}\right] = \frac{\delta W_1}{\delta I}\left(\frac{1}{Q^2} - \frac{Q^2}{\lambda^2}\right)\frac{1}{r} \qquad (13)$$

Integrating the above and solving for $2\bar{h}$, gives

$$2\bar{h} = -L(r) - 2\frac{\delta W_1}{\delta I}\frac{Q^2}{\lambda^2} + C \qquad (14)$$

where C is an integration constant,

$$L(r) = 2\int_{r_1}^{r} \frac{\delta W_1}{\delta I}\left(\frac{Q^2}{\lambda^2} - \frac{1}{Q^2}\right) dr/r \qquad (15)$$

and Q^2 is given by equation (4).

On combining equations (3a), (3b), (14) and (15), and imposing the boundary conditions

$$\tau''(r_2) = -P, \tau''(r_1) = 0,$$

we obtain

$$C = 0 \qquad (16)$$

and the internal pressure

$$P = L(r_2) = 2\int_{r_1}^{r_2} \frac{\delta W_1}{\delta I}\left(\frac{Q^2}{\lambda^2} - \frac{1}{Q^2}\right) dr/r \qquad (17)$$

where r_2 is the deformed internal radius.

On substituting in this equation for Q^2/λ from equation (4), for $\delta W_1/\delta I$ from equation (2) and for I from (2b) i.e.

$$I = \lambda_1^2 + \lambda_2^2 + \lambda_3^2$$

wherein $\lambda_1 = dr/d\varrho = Q/\lambda$; $\lambda_2 = 2\pi r/2\pi\varrho = r/\varrho = 1/Q$; and $\lambda_3 = dz/(dz/\lambda) = \lambda$
$Q^2/\lambda = 1 + K/r^2, \delta W_1/\delta I = Ae^{kI}, I = I_1 = \lambda^2 + Q^2/\lambda^2 + 1/Q^2$
we obtain

$$P = \frac{2Ae^{k\lambda^2}}{\lambda}\int_{r_1}^{r_2} e^{k/\lambda}\left[1 + K/r^2 + \frac{1}{1 + K/r^2}\right]\left[1 + K/r^2 - \frac{1}{1 + K/r^2}\right] dr/r \qquad (18)$$

3.2 *Velocity of Pulse Wave Propagation*

We now define the velocity of pulse propagation (C), in terms of the distensibility of the tube, as follows:

$$C^2 = 1/\varrho_F\, \Delta P/\Delta S \cdot S = r_2/2\varrho_F\,(\Delta P/\Delta r_2) \qquad (19)$$

where S = cross-sectional area of the vessel = πr_2^2
ϱ_F = density of the fluid (blood) in the tube
ΔS = change in the cross-sectional area corresponding to the change ΔP (due to pressure pulse) in the internal pressure P.

In order to obtain an expression for C (the pulse wave velocity in the arterial tube), we need to obtain an expression for $\Delta P/\Delta r_2$.

Evaluating ΔP from (18) using Leibnitz's rule

$$\Delta P = \frac{2Ae^{k\lambda^2}}{\lambda}\Big\{e^{k/\lambda}\left[1 + K/r_2^2 + \frac{1}{1 + K/r_2^2}\right]\left[1 + K/r_2^2 - \frac{1}{1 + K/r_2^2}\right]\Delta r_2/r_2$$
$$- e^{k/\lambda}\left[1 + K/r_1^2 + \frac{1}{1 + K/r_1^2}\right]\left[1 + K/r_1^2 - \frac{1}{1 + K/r_1^2}\right]\Delta r_1/r_1$$
$$+ \int_{r_1}^{r_2} e^{k/\lambda}\left[1 + K/r_2 + \frac{1}{1 + K/r_2}\right]\left[\left(\Delta K/r^2 + \frac{\Delta K/r^2}{(1 + K/r^2)^2}\right)\right.$$
$$+ \left(1 + K/r_2 - \frac{1}{1 + K/r_2}\right)\frac{k}{\lambda}\left(\Delta K/r^2 - \frac{\Delta K/r^2}{(1 + K/r^2)^2}\right) dr/r\Big\} \qquad (20)$$

where in

$$K = \frac{a_1^2}{\lambda} - r_1^2 = \frac{a_2^2}{\lambda} - r_2^2, \quad \eta = \frac{\lambda r_2^2}{a_2^2},$$

and $\Delta K = -2r_1\,\Delta r_1 = -2r_2\,\Delta r_2$
and substituting (20) into (19), we obtain after simplification, the expression for pulse velocity

$$C^2 = \frac{Ae^{k\lambda^2}}{\lambda\varrho_F}\Big[e^{k/\lambda\,(1/\eta + \eta)}(1/\eta - \eta)$$
$$- e^{k/\lambda}\,\{1/\eta + \eta + 2h/r_2(1 - 1/\eta - \eta^2 + \eta) + (h/r_2)^2$$
$$(3/\eta - 3 + \eta - 5\eta^2 + 4\eta^3)\}$$
$$[(1/\eta - \eta) + 2h/r_2(1 - 2/\eta + \eta^2) + (h/r_2)^2$$
$$(10/\eta - 7 + \eta^2 - 4\eta^3)]$$

$$1/\eta\,\{1 + 2(\eta - 1)h/r_2 - 3(\eta - 1)(h/r_2)^2\}$$
$$+ 1/1 - \eta \int_{1/\eta} e^{k/\lambda\, z(1 + 1/z^2)}[(1 + 1/z^2) + k/\lambda\, z(1 - 1/z^2)^2]\,dz \qquad (21)$$

where $h = (r_1 - r_2)$, $z = (1 + K/r_2)$, and $\eta = \dfrac{\lambda r_2^2}{a_2^2}$

Thus, the pulse velocity is expressed in terms of (i) the vessel medium's material parameters A and k (ii) the density of the fluid ϱ_F (iii) the prestrain λ and (iv) the geometrical variables of the vessel (r_1, r_2, a_2), as

$$C = F(r_1, r_2; a_2; A, k) \qquad (22)$$

Table 1: Pulse Velocity (C) in m/sec

K = 2.27 A = 22.05 $\lambda = 1.53$ r_2 = deformed internal radius of artery
a_2 = undeformed internal radius of artery
h = deformed thickness of tube

h/r_2 r_2/a_2	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20
1.00	0.8182	1.1280	1.3513	1.5310	1.6844	1.8206	1.9453	2.0624	2.1748	2.2848
1.01	0.8431	1.1161	1.3898	1.5738	1.7311	1.8712	2.0000	2.1216	2.2391	2.3547
1.02	0.8695	1.1961	1.4307	1.6194	1.7809	1.9252	2.0584	2.1847	2.3075	2.4293
1.03	0.8977	1.2333	1.4741	1.6678	1.8338	1.9825	2.1204	2.2518	2.3803	2.5087
1.04	0.9275	1.2728	1.5200	1.7190	1.8898	2.0432	2.1861	2.3229	2.4577	2.5933
1.05	0.9590	1.3745	1.5686	1.7731	1.9489	2.1074	2.2555	2.3982	2.5396	2.68304
1.06	0.9923	1.3585	1.6198	1.8302	2.0113	2.1751	2.3288	2.4778	2.6264	2.7782
1.07	1.0274	1.4048	1.6737	1.8902	2.0770	2.2464	2.4062	2.5619	2.7182	2.8792
1.08	1.0643	1.4535	1.7304	1.9534	2.1461	2.3214	2.4876	2.6505	2.8152	2.9861
1.09	1.1031	1.5047	1.7899	2.0196	2.2187	2.4003	2.5732	2.7438	2.9175	3.0993
1.10	1.1438	1.5583	1.8523	2.0892	2.2948	2.4831	2.6633	2.8421	3.0255	3.2192
1.11	1.1865	1.6145	1.9177	2.1620	2.3745	2.5699	2.7578	2.9455	3.1395	3.3461
1.12	1.2313	1.6735	1.9861	2.2383	2.4581	2.6609	2.8570	3.0543	3.2598	3.4806
1.13	1.2782	1.7351	2.0577	2.3181	2.5455	2.7563	2.9611	3.1686	3.3866	3.6229
1.14	1.3274	1.7996	2.1326	2.4015	2.6370	2.8561	3.0703	3.2888	3.5203	3.7738
1.15	1.3788	1.3670	2.2108	2.4887	2.7327	2.9606	3.1847	3.4151	3.6615	3.9338
1.16	1.4326	1.9375	2.2925	2.5797	2.8326	3.0699	3.3047	3.5479	3.8104	4.1035
1.17	1.4888	2.0111	2.3778	2.6748	2.9370	3.1842	3.4304	3.6875	3.9677	4.2838
1.18	1.5477	2.0879	2.4669	2.7741	3.0461	3.3037	3.5621	3.8343	4.1338	4.4754
1.19	1.6092	2.1681	2.5599	2.8777	3.1600	3.4287	3.7001	3.9887	4.3094	4.6794
1.20	1.6735	2.2519	2.6568	2.9858	3.2789	3.5594	3.8448	4.1511	4.4953	4.8967
1.21	1.7406	2.3394	2.7580	3.0985	3.4029	3.6959	3.9964	4.3220	4.6920	5.1286
1.22	1.8108	2.4306	2.8635	3.2161	3.5324	3.8387	4.1554	4.5021	4.9005	5.3764
1.23	1.8841	2.5257	2.9735	3.3387	3.6676	3.9880	4.3221	4.6918	5.1218	5.6418
1.24	1.9667	2.6250	3.0883	3.4666	3.8086	4.1440	4.4970	4.8919	5.3568	5.9263
1.25	2.0406	2.7286	3.2078	3.5998	3.9558	4.3072	4.6805	5.1031	5.6068	6.2319
1.26	2.1242	2.8366	3.3325	3.7389	4.1094	4.4778	4.8732	5.3261	5.8732	6.5609
1.27	2.2115	2.9494	3.4625	3.8838	4.2697	4.6563	5.0756	5.5619	6.1573	6.9158
1.28	2.3027	3.0669	3.5980	4.0349	4.4369	4.8431	5.2884	5.8115	6.4608	7.2996
1.29	2.3979	3.1896	3.7392	4.1924	4.6115	5.0386	5.5512	6.0760	6.7858	7.7155
1.30	2.4974	3.3174	3.8865	4.3565	4.7937	5.2432	5.7470	6.3566	7.1343	8.1674
1.31	2.6013	3.4508	4.0399	4.5277	4.9840	5.4575	5.9956	6.6546	7.5087	8.6597
1.32	2.7098	3.5900	4.1999	4.7062	5.1827	5.6821	6.2571	6.9717	7.9119	9.1974
1.33	2.8232	3.7352	4.3667	4.8922	5.3901	5.9174	6.5328	7.3094	8.3469	9.7863
1.34	2.9416	3.8866	4.5406	5.0863	5.6068	6.1642	6.8241	7.6698	8.8174	10.4331
1.35	3.0654	4.0446	4.7218	5.2886	5.8332	6.4232	7.1319	8.0550	9.3274	11.1458
1.36	3.1947	4.2094	4.9107	5.4997	6.0698	6.6951	7.4577	8.4674	9.8817	11.9334
1.37	3.3299	4.3813	5.1077	5.7198	6.3171	6.9807	7.8029	8.9099	10.4857	12.8066
1.38	3.4711	4.5607	5.3132	5.9494	6.5757	7.2808	8.1691	9.3854	11.1454	13.7776
1.39	3.6187	4.7479	5.5273	6.1889	6.8462	7.5966	8.5579	9.8976	11.8683	14.8612
1.40	3.7730	4.9433	5.7507	6.4389	7.1292	7.9290	8.9717	10.4505	12.6625	16.0743

4 Application of the Analysis

4.1 Determination of the undeformed internal radius »a_2« and the material parameters k and A:

The method outlined below is designed to enable determination of the undeformed internal radius »a_2« of the vessel, in terms of three independent sets of deformed radii (internal and external) and the associated pulse wave velocities. In that case, by using the notation of equation (22), we can obtain two equations of form:

$$C^I/C^{II} - G_1(r_1^I, r_2^I, r_1^{II}, r_2^{II}, a_2, k/\lambda) = 0$$
$$C^I/C^{III} - G_2(r_1^I, r_2^I, r_1^{III}, r_2^{III}, a_2, k/\lambda) = 0 \quad (23)$$

where C^I, C^{II}, C^{III} are the three monitored pulse wave velocities (at independent instants I, II, III), and G_1 and G_2 are the pulse wave velocity functions (= F^I/F^{II} and F^I/F^{III}, respectively), dependent on the internal undeformed radius a_2, material parameter k and the deformed radii. Hence if the deformed radii and the pulse wave velocity are monitored, equations (23) become functions of two unknowns a_2 and k. Hence, when equations (22) are plotted on the λ/a_2^2 vs k/λ coordinate plane (see Fig. 2) (for various values of k/λ and λ/a_2^2 their intersection yields the material parameter

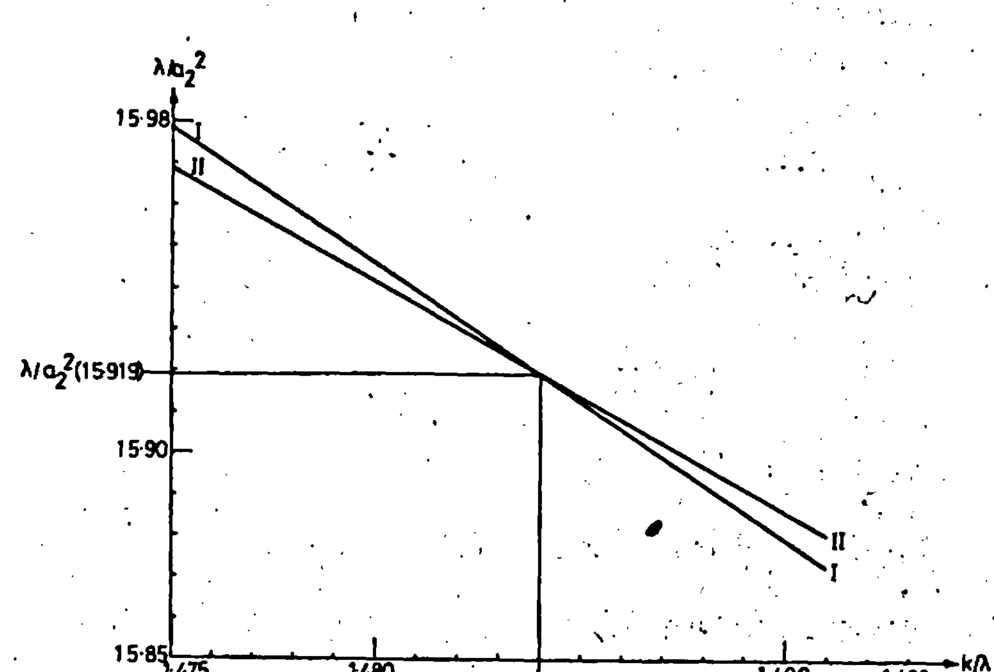

Fig. 2. Plot for determination of the arterial material parameter k and the undeformed radius.

k/λ and λ/a_2^2. Thus, one can noninvasively obtain the undeformed internal radius and material parameter »k« (for a given extension ratio λ corresponding to the strain energy density function chosen). Once the values of a_2 and k are determined, we can substitute their values in equation (21) for any one of the three instants I, II, III. This equation will now become an unknown in the parameter A only. Thus we can find the material constants of the artery namely the A and k for given extension ratio λ, characterising the stiffness of the artery, if pulse wave velocity C''^{s} and the deformed radii are monitored by any non-invasive procedure, possibly by photoplethysmographic transducers of the reflecting type, Weinmann et al. [9], and ultrasound monitoring of the diameter, Charleton et al. [2]; respectively.

4.2 *Numerical Example and Discussion*

The implementation of the analysis is demonstrated by means of a numerical example based on the in-vitro measurements of the arterial diameters (Table 2). Numerical analysis is carried out to calculate the values of the pulse wave velocities for tubes of various dimensions using for the elastic arterial material properties A and k; the values obtained by Simon et al. (for the canine abdominal aorta): $A = 22.05$ dyne/cm^2, $k = 2.27$ (for $\lambda = 1.53$).

Table 2
Pressure Radius Data from the Arterial Studies of Simon et al.
$\lambda = 1.53$, $K = 2.27$, $A = 22.05$ dyne/cm^2

Sl. No.	Pressure P in mm Hg	Internal deformed radius r_2 in cms.	External deformed radius r_1 in cms.	Mean deformed radius r_m in cms.
1.	25	0.348	0.401	0.375
2.	50	0.396	0.445	0.420
3.	75	0.425	0.473	0.449
4.	100	0.442	0.485	0.464
5.	150	0.467	0.510	0.489
6.	200	0.483	0.524	0.508

For finite deformation sustaining arterial tubes, equation (21) is numerically solved by Simpson's integration technique to obtain values of the pulse wave velocities

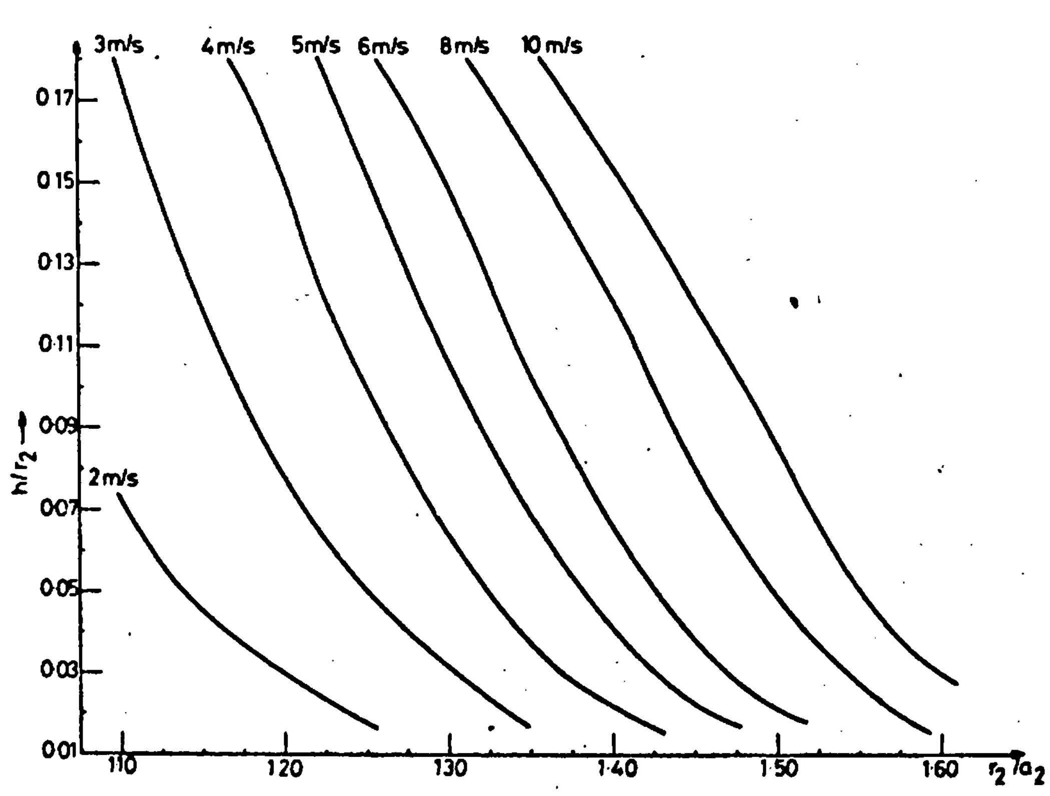

Fig. 3. Constant pulse velocity contours for various h/r_2 and r_2/a_2.

(Table 1) for (i) the wall thickness ratio parameter $h/r_2 = 0.02$ to 0.2, and (ii) the nondimensional internal radius $r_2/a_2 = 1.0$ to 1.4. Therefrom, pulse wave velocity contours are plotted as shown in Fig. (3), once the monitored measurable deformed internal and external radii are known. Next, graphs of pulse wave velocity (C) vs h/r_2 and C vs r_2/a_2 are plotted in Figs. 4 and 5; they

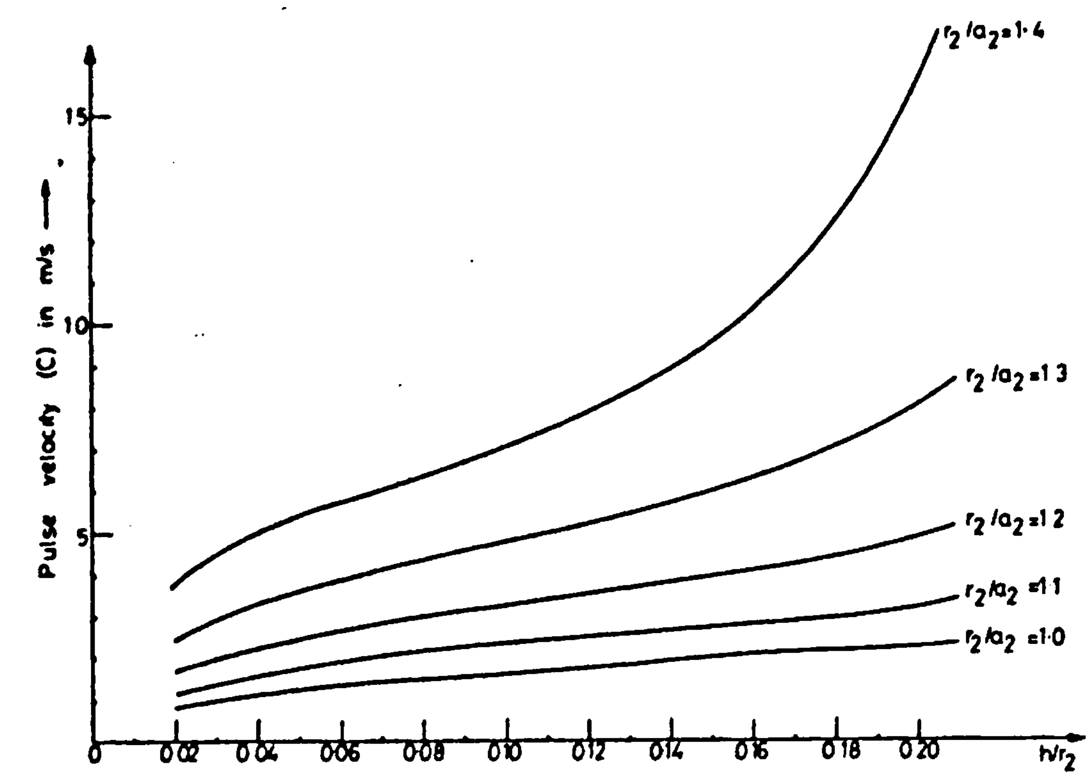

Fig. 5. Pulse velocity C vs r_2/a_2 for various h/r_2.

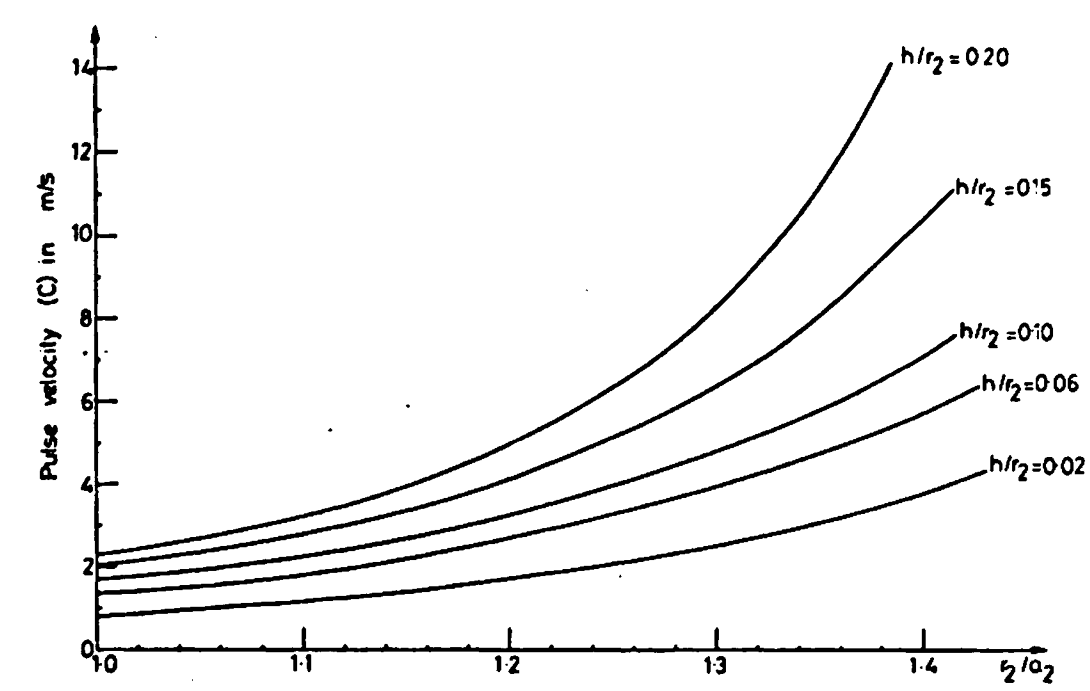

Fig. 4. Pulse velocity C vs h/r_2 for different r_2/a_2.

Table 3
Pulse Velocity (C) vs Arterial Pressure P

Sl. No.	Pressure P in mm Hg	Pulse velocity (C) in m/s from studies of Simon et al. CW_m	CW_2	CW_1	Pulse velocity for transversely isotropic medium (Present one)
1.	25	0.9	2.5	3.0	3.3
2.	50	1.0	4.3	5.0	5.35
3.	75	1.125	5.7	6.75	7.1
4.	100	1.25	7.0	8.0	8.5
5.	150	1.375	9.0	10.125	11.25
6.	200	1.50	10.375	11.625	13.95

indicate that the pulse wave velocity increases as both h/r_2 and r_2/a_2 increase. Finally, figure (6) shows a plot of pulse wave velocity C vs arterial pressure P (Table 3) obtained by means of our equations (18) and (21). The graph shows a value of C slightly higher than that obtained by Mirsky, wherein the pressure radius relationship which is of the exponential form is linear

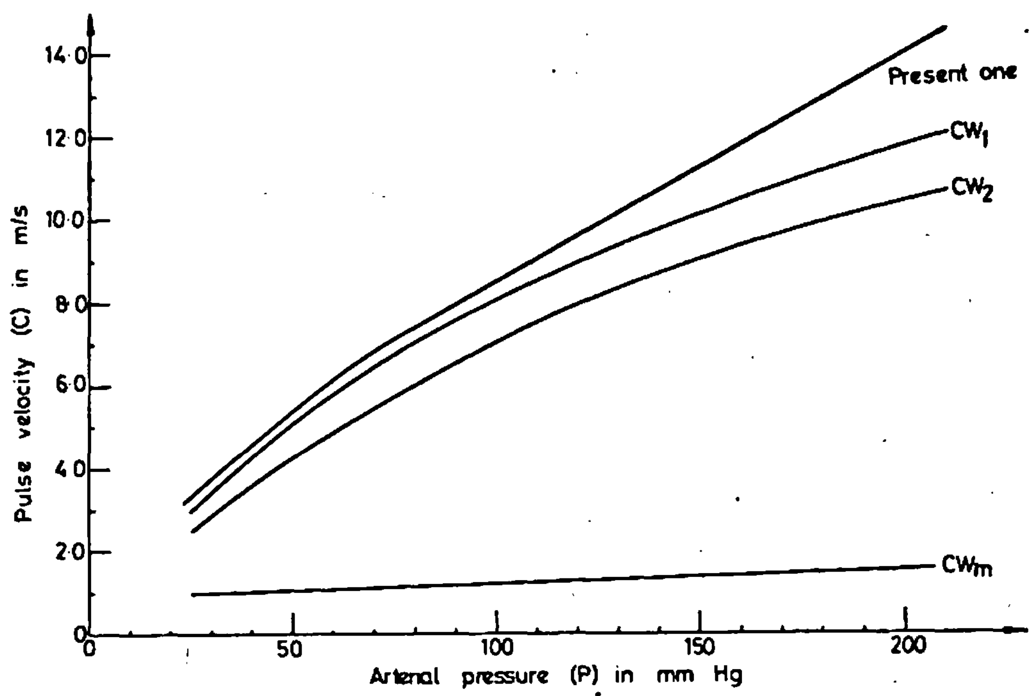

Fig. 6. Pulse velocity C-arterial pressure P · CW_m, CW_1 and CW_2 from Mirsky's paper (Ref. 7).

in the deformed radius while in the present formulation the pressure radius relationship is non-linear in the deformed radius.

5 Conclusions

1. A method to determine the in-vivo non-linear elastic parameters of an artery is presented which could characterise the state of the artery and hence a diagnostic indication of the atheromatous involvement of the blood vessel.
2. The pulse wave velocity C is shown to increase
   i) with increasing arterial pressure and
   ii) with increasing thickness to deformed radius ratio (h/r_2) and also to deformed to undeformed radius ratio (r_2/a_2).

References

[1] Attinger, F. M. L.: Some Problems Concerning the Analysis of the Mechanical Properties of the Blood Vessel Wall. – Ph. D. Thesis, Univ. of Pennsylvania (1964)
[2] Charleston, R. A., R. W. Sessions, and J. S. Graettinger: Diameter of heart measured by intracavity ultrasound, Medical Research Engineering (1969), pp. 28–32
[3] Fenn, W. O.: Changes in Length of Blood Vessels on Inflation. In Tissue Elasticity, Ed. J. Remington – Am. Physiol. Soc. (1957), pp. 154–167
[4] Mirsky, I.: Pulse Velocities in an Orthotropic Elastic Tube. Bull. of Math. Biophysics, 29 (1967b), pp. 311–318
[5] Mirsky, I.: Pulse Velocities in Initially Stressed Cylindrical Rubber Tubes. – Bull. of Math. Biophysics, 30 (1968), pp. 299–308
[6] Mirsky, I.: Pulse Velocities in Cylindrical Tapered and Curved Anisotropic Elastic Arteries. – Bull. of Math. Biology, Vol. 35 (1973), pp. 495–511
[7] Mirsky, I.: Ventricular and Arterial Wall Stresses Based on Large Deformation Analysis. – Biophysical Journal, Vol. 13 (1973), pp. 1141–1159
[8] Simon, B. R., A. S. Kobayashi, D. E. Strandness, and C. A. Wiederhielm: Large deformation Analysis of Arterial Cross-Section – Journal of Basic Engg., Trans. ASME, Vol. 93, Series D, No. 2 (1971), pp. 138–146
[9] Weinmann, J., and D. Sapozhikov: Equipment for continuous measurement of pulse wave velocities. Medical and Biological Engg., Vol. 9 (1971), pp. 125–138

472

Korrespondenzanschrift der Autoren:
S. Radhakrishnan
Biomedical Engineering Division
Department of Applied Mechanics
Indian Institute of Technology
Madras 600 036. India
D. N. Ghista
Department of Biomedical Engineering,
Chedoke-McMaster Hospital,
McMaster University,
Hamilton, Ont., Canada
G. Jayaraman
Department of Structural Engineering
College of Engineering
Perarignar Anna University of Technology
Guindy, Madras 600 025. India
K. M. Patil
Biomedical Engineering Division
Department of Applied Mechanics
Indian Institute of Technology
Madras 600 036. India