Mathl. Comput. Modelling Vol. 17, No. 6, pp. 83–90, 1993
Printed in Great Britain.
0895-7177/93 $6.00 + 0.00

AN INTERMITTENTLY USED TWO-UNIT SYSTEM WITH SEMI-MARKOV NEED PROCESS

M. SHARAF ALI
Department of Management Studies
University of Madras, Madras 600 005, India
R. NATARAJAN
Department of Mathematics, Presidency College
Madras 600 005, India
P. R. PARTHASARATHY
Department of Mathematics, Indian Institute of Technology
Madras 600 036, India

(Received July 1991; revised and accepted March 1992)

Abstract—An intermittently used two-unit system with a single repair facility is considered. The number of units required for the system performance at any time is governed by a particular semi-Markov process. Identifying suitable regeneration points, expressions are obtained for the distribution function of the time to the first disappointment and the mean rate of disappointments.

NOMENCLATURE

D	event denoting a disappointment.
E_i	event that the need process enters state "i."
E_{ik}	event that the repair for a unit commences and the number of failed units in the system is "i" (including the one on which the repair commences), $i = 1, 2$, and the need process is in state k, $k = 1, 2$.
E_{0k}	event that the need process enters into state k, $k = 1, 2$, and both the units are operable.
$N(\eta, t)$	number of η events in $(0, t)$, $\eta = D$, E_{ik} or E_i.
$X(t)$	number of units in the repair facility at t, including the one undergoing repair.
$Y(t)$	state of the need process at t.
$Z(t)$	state of the other unit when one unit is under repair at t; $= \begin{cases} 0, & \text{if operable,} \\ 1, & \text{if failed.} \end{cases}$
$u^*(s)$	the Laplace transform of any function $u(t)$.
$u^{(n)}(t)$	the n-fold convolution of $u(t)$ with itself.

1. INTRODUCTION

In the analysis of redundant systems, the emphasis had been all along on reliability and pointwise availability. However, it had also been realized [1] that while the concept of reliability may be quite important in certain systems, there are abundant situations wherein continuous failure-free performance may not be necessary. Gaver put forward the concept that the system should be available whenever it is needed. This concept of reliability was introduced to drive home the point that the system or facility may not be needed all the time. Thus, it is worthwhile to assume that the intervals during which the system is in the down state do not cause any disappointment if the system is not required during such periods. Such systems are called intermittently used systems.

The authors thank the referee for his encouraging comments and suggestions which have greatly improved the presentation of the paper.

Typeset by AMS-TEX

To characterize these systems, Gaver laid stress on the point event called a 'disappointment,' characterized by the entry of the system to either a down state during a usage period or the need arising for the system when it is in the down state. Later, Srinivasan [2] extended Gaver's analysis to two-unit cold standby systems. Nakagawa *et al.* [3] made a further study of these systems. Bhaskar [4] made a detailed study of two-unit intermittently used systems. Subramanian *et al.* [5] analyzed an intermittently used n-unit standby redundant system.

It is usually assumed that in these intermittently used systems the number of units required to perform the desired system operation is a constant, and that the need and the no-need periods alternate. However, real life situations do exist wherein this assumption is not true. For instance, to increase the thermal power plant availability, an additional induced draft (ID) fan may be installed in 200 MW sets, though two ID fans are normally used to handle flue gas and fly ash during full load operation of the plant [6].

Thus, it is possible that in multi-unit systems there may be situations that may warrant the use of none, or one or more units for the satisfactory performance of the system, depending upon the environmental stress and different weather conditions. In such situations, the number of units needed at any time may be described by a stochastic process called the "need process." Sharaf Ali *et al.* [7] introduced this concept in a two-unit system with exponential life and repair time distributions where the need process was governed by a Markov process. They further assumed that, when the need was for one unit, the system behaved like a one-unit cold standby system, and that, when the need was for two units, it behaved like a series system. Also, during the system down, the need process would not wait for system recovery.

In this paper, some of these assumptions have been generalized. Specifically the need process is governed by a particular semi-Markov process. A sort of dependence of the need process with the system has been introduced, in the sense that, during disappointment, the need process would wait indefinitely till the required number of operable units are available. The system behavior when the need is for one unit has also been generalized. Expressions for the distribution function of the time to the first disappointment and the mean number of disappointments are obtained. Section 2 describes the model and the assumptions, and Section 3 considers the operating characteristics of the system.

2. THE MODEL AND THE ASSUMPTIONS

The system consists of two units subject to failure and a single repair facility. The need process is governed by a special semi-Markov process. Precisely, we have the following assumptions.

(i) There are two statistically identical units. The failure rate of each unit is a constant, equal to "a" during usage and "b" during the time the unit is not in use.
(ii) There is only one repair facility and the repairs are taken in FIFO order.
(iii) Each unit is new after repair.
(iv) The repair time is arbitrarily distributed with probability density function (pdf) $g(\cdot)$ and cumulative distribution function (cdf) $G(\cdot)$.
(v) The need process $\{Y(t), t \geq 0\}$ has the state space $\{0, 1, 2\}$, where $Y(t)$ is the number of units required for the satisfactory performance of the system at t and is a particular semi-Markov process with the semi-Markov matrix

$$A(t) = \begin{bmatrix} 0 & a_{01}\, G_0(t) & a_{02}\, G_0(t) \\ a_{10}\,\left(1 - e^{-\lambda_1 t}\right) & 0 & a_{12}\,\left(1 - e^{-\lambda_1 t}\right) \\ a_{20}\,\left(1 - e^{-\lambda_2 t}\right), & a_{21}\,\left(1 - e^{-\lambda_2 t}\right) & 0 \end{bmatrix}, \tag{2.1}$$

where $G_0(t)$ (with pdf $g_0(t)$) is the distribution function of the duration of stay in the no-need state "0," and a_{ij} is the conditional probability of entering into state j, given that it was in state i at the last transition, with

$$\sum_{\substack{j=0 \\ j \neq i}}^{2} a_{ij} = 1. \tag{2.2}$$

When the need is for one unit, the system behaves like a two-unit warm standby system. When the need is for two units, the system behaves like a series system.

(vi) When a disappointment occurs, the need process waits indefinitely till the number of operable units are available, and then it lasts for a span of time governed by the same exponential distribution (with parameter λ_1 and λ_2 according to (2.1)).

(vii) The failure of a unit will be detected only when the need process is in a state of need, i.e., in the state 1 or 2. The failure remains undetected till the demand process leaves the state "0"; only then the failed unit would be taken up for repair.

OPERATING CHARACTERISTICS OF THE SYSTEM

3.1. General Considerations

To start with, let us assume that E_{11} occurs at $t = 0$. When one unit is under repair, the behavior of the other unit can be studied independently. To do this, let us introduce the functions $\pi_{11}(t)$, $g_{10}(t)$, $d_{11}(t)$, $d^0_{12}(t)$, and $d^1_{12}(t)$. The use of these functions will be restricted to a repair time interval or a part thereof.

It is possible that a disappointment may or may not occur during the span of a repair. Let us first consider the case when no disappointment intervenes during a repair. Define

$$\pi_{11}(t) = \text{Pr } \{Y(t) = 1,\ N(D,t) = 0 \mid E_{11} \text{ at } t = 0\}, \quad \text{and}$$

$$q_{10}(t) = \lim_{\Delta\to 0} \frac{1}{\Delta} \text{ Pr } \{Y(t+\Delta) = 0,\ Y(t) \neq 0,\ N(D,t) = 0 \mid E_{11} \text{ at } t = 0\}.$$

Since no disappointment is to occur in $(0,t)$, the need process must sojourn in state "1" or "0" and the operable unit does not fail during the repair in $(0,t)$, whether in the usage or nonusage state. Using renewal theoretic arguments, one obtains

$$\pi_{11}(t) = e^{-(a+\lambda_1)t} + \sum_{n=1}^{\infty} \int_0^t f^{(n)}_{11}(u)\, e^{-(a+\lambda_1)(t-u)}\, du \tag{3.1}$$

and

$$q_{10}(t) = \lambda_{10}\, \pi_{11}(t), \tag{3.2}$$

where

$$f_{11}(t) = \int_0^t a_{01}\lambda_{10}\, g_0(u)\, e^{-bu}\, e^{-(\lambda_1+a)(t-u)}\, du. \tag{3.3}$$

Next, let us consider the case in which a disappointment occurs during the repair in $(0,t)$. A disappointment occurs if:

(1) the other unit fails when the need process is in state "1,"

(2) the other unit does not fail but the need process enters state "2," or

(3) the other unit fails during a non-usage period and then the need process enters state "1".

Accordingly, define

$$d_{11}(t) = \lim_{\Delta\to 0} \frac{1}{\Delta} \text{ Pr } \{\text{a } D \text{ event in } (t, t+\Delta),\ Y(t) = 1 \mid E_{11} \text{ at } t = 0\},$$

$$d^0_{12}(t) = \lim_{\Delta\to 0} \frac{1}{\Delta} \text{ Pr } \{Y(t+\Delta) = 2 \neq Y(t),\ Z(t) = 0 \mid E_{11} \text{ at } t = 0\}, \quad \text{and}$$

$$d^1_{12}(t) = \lim_{\Delta\to 0} \frac{1}{\Delta} \text{ Pr } \{Y(t+\Delta) = 2 \neq Y(t),\ Z(t) = 1 \mid E_{11} \text{ at } t = 0\}.$$

These functions represent the probability of occurrence of a disappointment, when one unit is under repair. Probabilistic arguments lead to

$$d_{11}(t) = a\, \pi_{11}(t) + \int_0^t a_{01}\, q_{10}(u)\, g_0(t-u) \left(1 - e^{-b(t-u)}\right) du, \tag{3.4}$$

$$d^0_{12}(t) = \lambda_{12}\pi_{11}(t) + a_{02} \int_0^t q_{10}(u)\, g_0(t-u)\, e^{-b(t-u)}\, du, \quad \text{and} \tag{3.5}$$

$$d^1_{12}(t) = \int_0^t a_{02}\, q_{10}(u)\, g_0(t-u) \left(1 - e^{-b(t-u)}\right) du. \tag{3.6}$$

Now, assuming that initially both units are operable, one needs to consider the behavior of the system up to the commencement of the repair of a unit. This implies that one has to study the behavior of the need process with the condition that both the units are operable throughout. With this condition, let $\{\tau_n, n = 0,1,2,\dots;\ \tau_0 = 0\}$ be the sequence of epochs of change of state of the need process, and let Y_n be the state entered at τ_n. The process $\{Y_n, \tau_n\}$ terminates when one of the units fails during usage period, or when the process leaves the state "0" to find at least one failed unit. Let $L = 0,1,2,\dots$ be the total number of transitions the need process undergoes in its life. Exploiting the memoryless property of the failure free times of the units, it is easy to recognize that $\{Y_n, \tau_n; L\}$ is a Markov renewal process induced by the semi-Markov matrix

$$\mathbf{v}(t) = (v_{ij}(t)),$$

where

$$v_{ij}(t)\,dt = \text{ Pr } \{Y_{n+1} = j,\, t < \tau_{n+1} - \tau_n \le t + dt \mid Y_n = i\}$$

almost everywhere on $\{L > n\}$. We obtain

$$\mathbf{v}(t) = \begin{bmatrix} 0 & a_{01}\, g_0(t)\, e^{-2bt} & a_{02}\, g_0(t)\, e^{-2bt} \\ \lambda_{10}\, e^{-(\lambda_1+a+b)t} & 0 & \lambda_{12}\, e^{-(\lambda_1+a+b)t} \\ \lambda_{20}\, e^{-(\lambda_2+2a)t} & \lambda_{21}\, e^{-(\lambda_2+2a)t} & 0 \end{bmatrix}. \tag{3.7}$$

The higher order transition probabilities can now be obtained as

$$\begin{aligned} v_{ij}^{(n)}(t)\,dt &= \text{ Pr } \{Y_n = j,\, t < T_n \le t + dt,\, L > n \mid Y_0 = i\} \\ &= \sum_{k=0}^{2} dt \int_0^t v_{ik}^{(n-1)}(u)\, v_{kj}(t-u)\, du. \end{aligned} \tag{3.8}$$

If $\gamma_{ij}(t)\,dt$ is the probability of a transition into j in $(t, t+dt)$, given that the process initially started in i, then $\gamma_{ij}(t)$ is given by

$$\gamma_{ij}(t) = v_{ij}(t) + \sum_{k=0}^{2} \int_0^t v_{ik}(u)\, \gamma_{kj}(t-u)\, du.$$

Taking Laplace transforms, we get

$$\boldsymbol{\gamma}^*(s) = (\gamma_{ij}^*(s)) = \mathbf{v}^*(s)\,[\mathbf{I} - \mathbf{v}^*(s)]\,. \tag{3.9}$$

Define now

$$u_{ij}(t) = \lim_{\Delta\to 0} \frac{1}{\Delta} \text{ Pr } \{X(t+\Delta) = 1,\ X(u) = 0 \text{ for } u \in (0,t],\ Y(t) = j \mid E_i \text{ at } t = 0,\ X(0) = 0\}\,,$$
$$j = 0,1,2,;\ i = 1,2,$$

and

$$\pi_{ij}^k(t) = \lim_{\Delta\to 0} \frac{1}{\Delta} \text{ Pr } \{Y(t+\Delta) = j,\ Y(t) = 0,\ X(t) = k \mid E_i \text{ at } t = 0,\ X_0 = 0\}\,,$$
$$j = 0,1,2,;\ i,k = 1,2.$$

We note that $u_{ij}(t)dt$ is the probability that the process $\{Y_n, \tau_n\}$ terminates at t when in state j, by a failure of a unit, and $\pi_{ij}^k(t)\,dt$ is the probability that it terminates by a transition into j from "0" at t, with $k (k = 1,2)$ failed units. By considering the cases that either

$$(i)\quad \tau_1 \le t \qquad \text{or} \qquad (ii)\quad \tau_1 > t,$$

we obtain

$$u_{11}(t) = (a+b)\, e^{-(a+b+\lambda_1)t} + \int_0^t \gamma_{11}(u)\,(a+b)\, e^{-(a+b+\lambda_1)(t-u)}\, du, \tag{3.10}$$

$$u_{12}(t) = 2a \int_0^t \gamma_{12}(u)\, e^{-(\lambda_2+2a)(t-u)}\, du, \tag{3.11}$$

$$u_{21}(t) = (a+b) \int_0^t \gamma_{21}(u)\, e^{-(\lambda_1+a+b)(t-u)}\, du, \quad \text{and} \tag{3.12}$$

$$u_{22}(t) = 2a\, e^{-(\lambda_2+2a)t} + 2a \int_0^t \gamma_{22}(u)\, e^{-(\lambda_2+2a)(t-u)}\, du. \tag{3.13}$$

Similar arguments lead to, for $j, k = 1, 2$,

$$\begin{aligned} \pi_{0j}^k &= a_{0j} \binom{2}{k} e^{-b(2-k)t} \left(1 - e^{-bt}\right)^k g_0(t) \\ &\quad + a_{0j} \binom{2}{k} \int_0^t \gamma_{00}(u)\, e^{-b(2-k)(t-u)} \left(1 - e^{-b(t-u)}\right)^k g_0(t-u)\, du \end{aligned} \tag{3.14}$$

and

$$\pi_{ij}^k(t) = a_{0j} \binom{2}{k} \int_0^t \gamma_{i0}(u)\, e^{-b(2-k)(t-u)} \left(1 - e^{-b(t-u)}\right)^k g_0(t-u)\, du, \qquad i = 1, 2. \tag{3.15}$$

It is now possible to derive expressions for some of the operating characteristics of the system.

3.2. Time to the First Disappointment

The probability density function of the interval between two successive E_{11} events during which no D event occurs can be obtained from

$$\begin{aligned} {}_D\phi_1(t) &= \lim_{\Delta \to 0} \frac{1}{\Delta} \Pr \left\{ N(E_{11}, t+\Delta) = 1,\ N(E_{11}, t) = 0,\ N(D, t) = 0 \mid E_{11} \text{ at } t = 0 \right\}, \\ {}_D\bar{\Phi}(t) &= \int_t^\infty {}_D\phi_1(w)\, dw, \end{aligned} \tag{3.16}$$

and

$${}_DR(t) = \Pr \left\{ N(D, t) = 0 \mid E_{11} \text{ at } t = 0 \right\}. \tag{3.17}$$

Now considering all the probabilities, it follows

$$\begin{aligned} {}_D\phi_1(t) = &\int_0^t \pi_{11}(u)\, g(u)\, u_{11}(t-u)\, du + \int_0^t u_{11}(t-w)\, dw \int_0^w a_{01}\, q_{10}(x)\, g_0(w-x)\, dx \\ &\times \int_x^w g(y)\, e^{-b(2w-x-y)}\, dy \\ &+ \int_0^t a_{01}\, q_{10}(u)\, g_0(t-u) \left(1 - e^{-b(t-u)}\right) du \int_u^t g(v)\, e^{-b(t-v)} dv \\ &+ \int_0^t a_{01}\, q_{10}(u)\, g_0(t-u)\, e^{-b(t-u)} du \int_u^t g(v) \left(1 - e^{-b(t-v)}\right) dv \\ &+ \int_0^t u_{21}(t-w)\, dw \int_0^w a_{02}\, q_{10}(x) g_0(w-x)\, e^{-b(w-x)}\, dx \\ &\times \int_x^w g(y)\, e^{-b(w-y)}\, dy, \end{aligned} \tag{3.18}$$

and

$${}_D\bar{R}(t) = {}_D\bar{\Phi}_1(t) + \sum_{n=1}^\infty \int_0^t {}_D\phi_1^{(n)}(u)\, {}_D\bar{\Phi}_1(t-u)\, du. \tag{3.19}$$

Also, we obtain

$$E[\text{Time to the first disappointment}] = \lim_{s\to 0} {}_D\bar{R}^*(s) = -\left[\frac{\frac{d}{ds}\left[{}_D\phi_1^*(s)\right]}{1-{}_D\phi_1^*(s)}\right]_{s=0}. \tag{3.20}$$

3.3. Product Density of a Disappointment

One of the ways of characterizing a general stochastic process is through product densities [8]. These densities are analogues of the renewal density in the case of non-renewal processes. If $N(x,t)$ represent the number of events of a point process in the interval $(t, t+x)$, then the product density of order one is defined as

$$h_1(x) = \lim_{\Delta\to 0} \frac{E[N(\Delta, x)]}{\Delta}.$$

This represents the probability of an event in the interval $(x, x+\Delta)$. A similar definition can be given for the n^{th} order product densities. Even though these functions are called densities, it is important to note that their integration will not give probabilities but will yield the factorial moments.

Now, let $\{\xi(t)\}$ denote the vector process $\{X(t), Y(t)\}$. Consider the times of occurrence of the events $E_{ik} (i = 0,1,2,; k = 1,2,)$, and denote by $T_0 < T_1 < T_2 < \cdots$ these epochs. Also, let $U_n = \xi(T_n+)$ and $E = \{(i,j), i = 0,1,2; j = 1,2\}$. Then it is easily seen that $\{U_n, T_n, n = 0,1,2,\dots\}$ is a Markov renewal process on E. Let the semi-Markov kernel of this process be

$$\Theta(t) = \left[\int_0^t \theta(m,j,u \mid \ell, i)\, du\right],$$

where

$$\theta(m,j,t \mid \ell,i)\, dt = \Pr\ \{U_{n+1} = (m,j),\ t < T_{n+1} \le t + dt \mid U_n = (\ell, i)\}.$$

Renewal theoretic arguments yield to

$$\theta(m,j,t \mid 0,i) = \begin{cases} (1-\delta_{ij})\, v_{ij}(t) + \int_0^t v_{i0}(u)\, a_{0j}\, g_0(t-u)\, e^{-2b(t-u)}\, du, & m = 0;\ i,j = 1,2, \\ \delta_{i1}\,(a+b)\, e^{-(\lambda_1+a+b)t} + \delta_{i2}\, 2a\, e^{-(\lambda_2+2a)t} & \\ \quad + \int_0^t v_{i0}(u)\, a_{0j}\, g_0(t-u)\, 2e^{-b(t-u)}\left(1-e^{-b(t-u)}\right) du, & m = 1;\ j = i, \\ \int_0^t v_{i0}(u)\, a_{0j} g_0(t-u) \binom{2}{m} e^{b(2-m)(t-u)} \left(1-e^{-b(t-u)}\right) du, & \text{for other } (m,j), \end{cases} \tag{3.21}$$

$$\theta(m,j,t \mid 1,2) = \begin{cases} g(t)\, e^{-bt}, & m = 0;\ j = 2, \\ g(t)\left(1-e^{-bt}\right), & m = 1;\ j = 2, \\ 0, & \text{otherwise}, \end{cases} \tag{3.22}$$

$$\theta(m,j,t \mid 2,i) = \begin{cases} g(t), & i = 1,2;\ m = 0,1;\ j = 1,2, \\ 0, & \text{otherwise}, \end{cases} \tag{3.23}$$

and

$$\theta(m,j,t \mid 1,1) = \begin{cases} \delta_{j1}\pi_{11}(t)\, g(t) + \delta_{j2}\, g(t) \int_0^t d_{12}^0(w)\, e^{-b(t-w)}\, dw & \\ + \int_0^t a_{0j}\, q_{10}(w)\, g_0(t-w)\, dw & \\ \times \int_w^t g(v)\, e^{-b(2t-v-w)}\, dv, & m = 0;\ j = 1,2, \\ \delta_{j1}\, g(t) \int_0^t d_{11}(w)\, dw + \delta_{j2}\, g(t) \int_0^t d_{12}^1(w)\, dw & \\ \times \int_w^t \left[e^{-b(t-v)}\left(1-e^{-b(t-w)}\right) + e^{-b(t-w)}\left(1-e^{-b(t-v)}\right)\right] dv, & m = 1;\ j = 1,2, \\ \int_0^t a_{0j}\, q_{10}(w)\, g_0(t-w)\, dw & \\ \times \int_w^t g(v)\left(1-e^{-b(t-v)}\right)\left(1-e^{-b(t-v)}\right) dv & m = 2;\ j = 1,2. \end{cases} \tag{3.24}$$

Now, let $F(m,j,t \mid \ell,i)$ be the distribution function of first passage time from the state (ℓ,i) into (m,j). Then its Laplace-Steiltjes transform is given by

$$\hat{F}(m,j,s \mid \ell,i) = \begin{cases} \frac{\hat{R}(m,j,s|\ell,i)}{1-\hat{R}(\ell,i,s|\ell,i)}, & (m,j) = (\ell,i), \\ 1 - \frac{1}{\hat{R}(\ell,i,s|\ell,i)}, & (m,j) = (\ell,i), \end{cases} \tag{3.25}$$

where the matrix $\hat{\mathbf{R}}(s)$ is

$$\hat{\mathbf{R}}(s) = \left(\hat{R}(m,j,s \mid \ell,i)\right) = \left[\mathbf{I} - \hat{\Theta}(s)\right]^{-1}. \tag{3.26}$$

If $\hat{\Theta}(s)$ is partitioned as

$$\hat{\Theta}(s) = \begin{array}{c} \\ (0,1) \\ (0,2) \\ (1,1) \\ (1,2) \\ (2,1) \\ (2,2) \end{array} \begin{array}{c} \begin{array}{cccccc} (0,1) & (0,2) & (1,1) & (1,2) & (2,1) & (2,2) \end{array} \\ \left(\begin{array}{c:c} \hat{\Theta}_{11}(s) & \hat{\Theta}_{12}(s) \\ \hdashline \hat{\Theta}_{21}(s) & \hat{\Theta}_{22}(s) \end{array}\right) \end{array}, \tag{3.27}$$

then

$$\hat{\mathbf{R}}(s) = \left(\begin{array}{c:c} \mathbf{I} + \hat{\Theta}_{12}(s)\,\hat{\Theta}_{21}(s)\,\hat{\Theta}_{11}^{-1}(s)\,\hat{\mu}(s) & -\hat{\Theta}_{12}(s)\,\hat{\mu}(s) \\ \hdashline -\hat{\Theta}_{21}(s)\,\hat{\Theta}_{11}^{-1}(s)\hat{\mu}(s) & \hat{\mu} \end{array}\right), \tag{3.28}$$

with

$$\hat{\mu}(s) = \left[\hat{\theta}_{22}(s) - \hat{\theta}_{21}(s)\hat{\theta}_{11}(s)\hat{\theta}_{21}(s)\right]^{-1} \tag{3.29}$$

It is now possible to derive $h_D(t)$, the first order product density of a disappointment at t. Recalling that $h_D(t)\,dt$ is the probability that a disappointment occurs in $(t, t+dt)$ and considering the following mutually exclusive cases

(i) no E_{11} event occurs up to t, or
(ii) at least one E_{11} event occurs before t,

it follows

$$h_D(t) = \int_0^t {}_D r(t-u)\,R(1,1,du \mid 1,1,), \tag{3.30}$$

where the functions

$${}_D r(t) = -\frac{d}{dt}\left[{}_D\bar{R}(t)\right]$$

and $R(1,1,t \mid 1,1)$ are respectively given by Equations (3.19) and (3.28). The expected number of disappointments in $(0,t)$ is then given by

$$E\left[N(D,t)\right] = \int_0^t h_D(u)\,du. \tag{3.31}$$

REFERENCES

1. D.P. Gaver, Time to failure and availability of parallel systems with repair, *IEEE Trans. Rel.* **R-12**, 30–38 (1963).
2. V.S. Srinivasan, The effect of standby redundancy in system's failure with repair maintenance, *Oper. Res.* **14**, 1024–1036 (1966).
3. T. Nakagawa, Y. Sawa and Y. Suzuki, Reliability analysis of intermittently used systems when failures are detected only during usage period, *Microelec. Rel.* **15**, 35–38 (1976).
4. D. Bhaskar, Probabilistic analysis of intermittently used systems, Ph.D. Thesis, Dept. of Mathematics, Indian Institute of Technology, Madras, India, (1979).
5. R. Subramanian, Y.V.S. Sarma and R. Natarajan, An intermittently used n-unit redundant system, *Proceedings of the Second International Conference on Reliability and Exploitation of Computer Systems, Relcomex '81,* (Ksiaz Castle, Poland), Proc. I, pp. 63–67, (1981).
6. K.K. Das and D. Acharya, Optimal replacement policy for induced draft fans, *Microelec. Reliab.* **28** (4), 519–523 (1988).
7. M. Sharaf Ali, R. Natarajan and P.R. Parthasarathy, A complex two-unit system under varying demand, *Microelec. Rel* **28**, 359–361 (1988).
8. S.K. Srinivasan, *Stochastic Point Processes and their Applications*, Griffin, London, (1974).