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Active Vibration Control of Thin Plate Using Optimal Dynamic Inversion Technique

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Abstract: Geometrically non-linear von Karman plate vibrations are suppressed using optimal dynamic inversion technique. Two types of controller are considered, a continuous and finite discrete controllers in spatial domain to control the vibrations of the plate. Non-linear Finite Element (FE) method is used to transform the non-linear partial differential equations (PDE) into a set of non-linear algebraic equations and are solved. The non-linear PDE is directly used for controller design i.e. design-then-approximate (DTA) method is followed which ensures the stability and controllability of the system. The simuation study shows the effectiveness of controlling plate vibrations using continuous and discrete controllers.

Keywords: von Karman plate, vibration control, optimal dynamic inversion, non-linear control, non-linear plates

1. INTRODUCTION

The distributed parameter modelling is considered necessary in a wider classes of problems in science and engineering. One such problem is the vibration control of thin plates which inturn is applicable for analysis of ship hulls, airplane wings, etc. The bending of thin plates with large displacements and small strains is governed by ***Föppl-von Kármán equations***. Use of this plate theory is justified by Ciarlet (1980) for such problems.

There are two types of approaches in determining the control forces, *approximate-then-design* (ATD), where a system is discretized to approximate models and the controller is designed for them, and *design-then-approximate* (DTA), where the controller is designed using the governining differential equation and the system is solved using approximate methods. The merits and demerits of both the approaches are given by Burns et al. (1994). The ATD method produce erroneous results due to the lumping of the parameters which fail to capture the fundamental properties of the system like stability, controllability and observability whereas the DTA approach tends to become computationally difficult but robust control laws can be applied. In DTA approach, the fundamental properties are ensured when deriving the control forces and numerical techniques are applied to solve the system.

Control of slewing beam as a distributed parameter system (DPS) with sensors and actuators using Linear Quadratic Regulator (LQR) technique is studied by Yang and Jeng (1998). To accommodate discrete sensors and actuators, an output feedback algorithm is presented by them which transforms functional Riccati equation into set of algebraic equations. Lin and Huang (1999) investigated on vibration control of beam-plates with bonded piezoelectric sensors and actuators using LQR objective functional for Lyapunov energy function to design the controller. Li et al. (2003) designed μ-synthesis controller, which quantifies model uncertainties by uncertainty weights, for vibration control of the plate coupled with piezoelectric patches. The controller is designed in Laplacian domain including the uncertainties to achieve robustness and stability. De Abreu et al. (2004) presented static and dynamic behaviour of numerical modelled composite plate structure using Kirchhoff's plate theory and finite element method coupling piezoelectric sensors and actuators. Hamiltonian principle is applied to arrive at the governing equations for the mechanical-electrical coupled system.

Optimal dynamic inversion control design for nonlinear DPS with continuous and discrete actuators, using DTA approach, is studied by Padhi and Balakrishnan (2007). Dynamic inversion and variational optimization is combined to design the feedback controller which is applied to the continuous and discrete actuators in the spatial domain. It is ensured that the formulation does not lead to any singularity for continuous controller, but for discrete controller, the problem of singularity arises which can be overcome by using dynamic inversion. Using this approach, Ali and Padhi (2009) examined the active vibration control of non-linear Euler-Bernoulli beams as DPS. The governing non-linear PDE of motion is directly utilized for design of controllers as in DTA approach which ensures the system free from approximation errors and closed form solutions. Vibration behaviour of beam with continuous and finite discrete actuators is studied by simulating the system with controller using an implicit finite difference technique with unconditional stability.

Shirazi et al. (2011) investigated active vibration control of simply supported rectangular plate made of functionally

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graded material (FGM) using fuzzy logic controllers and compared the results with that of the system controlled by proportional-integral-derivative (PID) controller. Patches of piezoelectric sensors and actuators are modelled with the plate which is derived from classical plate theory, whose natural frequencies are derived from double Fourier series. Bratland et al. (2014) expanded modal analysis of active flexible multibody system with collocated sensors and actuators in FE environment, developed by Bratland et al. (2011) , for non-collocated sensors and actuators, damping and steady-state error elimination. They solved the multiple degrees of freedom (DOF) FE model using multiple-input multiple-output (MIMO) PID feedback controllers. Active vibration control of clamped circular plates, developed from classical plate theory, equipped with piezoelectric patches excited by plane sound wave is presented by Khorshidi et al. (2015). The transverse displacement of the circular plate is controlled using LQR and fuzzy logic controller (FLC) feedback control design techniques. Here also closed form solution is obtained for the controller which reduces approximation error and provides more stability.

2. NON-LINEAR PLATE MODEL

The geometric non-linearity of the thin plates under large deformations are described as in Park et al. (2009) by a set of non-linear partial differential equations named ***Föppl-von Kármán equations***. The equations are of the following form:

$$D\Delta^2 w - h[w, F(w)] = P \tag{1}$$
$$\Delta^2 F(w) = -\frac{E}{2}[w, w]$$

where w denotes the transverse displacement of the plate, Δ is the Laplace operator, the flexural rigidity $D = \frac{Eh^3}{12(1-\nu^2)}$, h is depth, E is modulus of elasticity, ν represents Poisson's ratio.

The von Kármán bracket $[w, \phi]$ is given by

$$[w, \phi] \equiv w_{xx}\phi_{yy} + w_{yy}\phi_{xx} - 2w_{xy}\phi_{xy} \tag{2}$$

The essential boundary conditions for simply supported plate vibration are given in the equation 3

$$\begin{aligned} &w(0, y, t) = 0; w(x, 0, t) = 0; \\ &w(a, y, t) = 0; w(x, b, t) = 0; \end{aligned} \tag{3}$$

where a and b are the size of plate in x and y directions.

3. SYSTEM DYNAMICS

The system dynamics of the geometrically non-linear thin plate can be represented by

$$\ddot{w} + \frac{c}{m}\dot{w} + \frac{D}{m}\Delta^2 w - \frac{h}{m}[w, F(w)] = u(x, y, t) \tag{4}$$

where u is the control variable to achieve the target displacement profile using continuous actuator.

For set of discrete actuators controlling plate, the control variable can be taken as

$$u(x, y, t) = \frac{1}{m}\sum_{n=1}^{N} \bar{u}_n S(x, y, x_n, y_n, s_{xn}, s_{yn}) \tag{5}$$

$S(x, y, x_n, y_n, s_{xn}, s_{yn})$ is a step function defined in the equation 6, which denotes the position of the discrete actuators with sizes (s_{xn}, s_{yn}) at (x_n, y_n).

$$S := \begin{cases} 1, & \forall (x_n - \frac{s_{xn}}{2}) \le x \le (x_n + \frac{s_{xn}}{2}) \\ & \&(y_n - \frac{s_{yn}}{2}) \le y \le (y_n + \frac{s_{yn}}{2}) \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

It is assumed that in the discrete controller design, the actuators are not placed near the boundaries and do not overlap with other actuators, u_n is having a constant magnitude within the interval and 0 outside, which is forced by the step function $S(x, y, x_n, y_n, s_{xn}, s_{yn})$.

4. DESIGN OF CONTROLLER

A control force can be applied through the actuators to the system to attenuate the vibrations of the plate i.e. $w(x, y, t) \to 0$ and $\dot{w}(x, y, t) \to 0$ as $t \to \infty$. An output function, described by Ali and Padhi (2009), is extended to two dimensions $Z(t)$ and defined as in equation 7 which ensures $w(x, y, t) \to 0$ and $\dot{w}(x, y, t) \to 0$ throughout the domain of interest as $Z(t) \to 0$.

$$Z(t) = \frac{1}{2}\int_0^b \int_0^a \left([w \ \dot{w}]\, \Phi \begin{bmatrix} w \\ \dot{w} \end{bmatrix} \right) dxdy \tag{7}$$

with a weighing matrix $\Phi = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{12} & \phi_{22} \end{bmatrix}$ which is a positive definite matrix of designer specific coefficients since this ensures the convergence of w and $\dot{w}$ when $Z(t) \to 0$. A stable error dynamics can be set for the Z function to achieve the target displacement. A simple decay phenomenon will satisfy the objective of the controller as in equation 8.

$$\dot{Z(t)} + \lambda Z = 0 \tag{8}$$

where $\lambda > 0$ is a gain generally taken as $1/\tau$, τ being the 'time constant'for the decay system.

4.1 Continuous controller

Substituting equation 7 and 4 in 8, the simplified form obtained is

$$\int_0^b \int_0^a \left(\frac{1}{m}(\phi_{12} w + \phi_{22}\dot{w}) u \right) dxdy - \gamma = 0 \tag{9}$$

where,

$$\begin{aligned} \gamma = \int_0^b \int_0^a \Bigg[&(\phi_{11} w\dot{w} + \phi_{12}\dot{w}^2) - (\phi_{12} w + \phi_{22}\dot{w}) \\ &\times \left(\frac{c}{m}\dot{w} + \frac{D}{m}\Delta^2 w - \frac{h}{m}[w, F(w)] \right) \\ &+ \frac{\lambda}{2}\left([w \ \dot{w}]\, \Phi \begin{bmatrix} w \\ \dot{w} \end{bmatrix} \right) \Bigg] dxdy \end{aligned} \tag{10}$$

It is now necessary to obtain the optimal solution for the control forces subjected to the constraint given by the equation 9. So, a cost function is introduced to optimize the control parameter u with the constraints using Lagrangian multiplier method as

$$\begin{aligned} J = &\frac{1}{2}\int_0^b \int_0^a r u^2 dxdy \\ &+ \Lambda \left[\int_0^b \int_0^a \left(\frac{1}{m}(\phi_{12} w + \phi_{22}\dot{w}) \right) dxdy - \gamma \right] \end{aligned} \tag{11}$$

where Λ is the Lagrangian multiplier used to convert the constrained optimization into free optimization problem. r

is the relative importance factor for controller at different locations. For optimal solution, the necessary condition is

$$\nabla_{u,\Lambda} J = 0 \quad (12)$$

Derivative of J with respect to u and Λ leads to the following equations

$$ru + \frac{\Lambda}{m}(\phi_{12}w + \phi_{22}\dot{w}) = 0 \quad (13)$$

$$\int_0^b \int_0^a \left(\frac{1}{m}(\phi_{12}w + \phi_{22}\dot{w})u \right) dxdy - \gamma = 0$$

Solving for u and Λ from the equations 13 and considering equal relative importance for all locations, gives,

$$u = \frac{m\gamma(\phi_{12}w + \phi_{22}\dot{w})}{\int_0^b \int_0^a (\phi_{12}w + \phi_{22}\dot{w})^2 \, dxdy} \quad (14)$$

4.2 Proof of convergence of continuous controller

At any point $x_0, y_0 \in \Omega$, the control solution for continuous controller can be written as

$$u = \frac{m\gamma(x_0,y_0)(\phi_{12}w(x_0,y_0) + \phi_{22}\dot{w}(x_0,y_0))}{\left[\int_0^b \int_0^a (\phi_{12}w + \phi_{22}\dot{w})^2 \, dxdy\right]_{(x_0,y_0)}} \quad (15)$$

For analysing the solution when $w = 0$ and $\dot{w} = 0$ $\forall x_0, y_0 \in \Omega$, without loss of generality, it can be analysed in the limit when $w \to 0$ and $\dot{w} \to 0$ for $x_0, y_0 \in [x_0 - \epsilon/2, x_0 + \epsilon/2, y_0 - \epsilon/2, y_0 + \epsilon/2] \in \Omega, \epsilon \to 0$, and $w = 0$ and $\dot{w} = 0$, everywhere else. The control solution for the small limited area can be defined as

$$\bar{u}(x_0,y_0,t) = \frac{-m(\phi_{12}w(x_0,y_0) + \phi_{22}\dot{w}(x_0,y_0))}{\left[\int_{y_0-\epsilon/2}^{y_0+\epsilon/2} \int_{x_0-\epsilon/2}^{x_0+\epsilon/2} (f)^2 \, dxdy\right]} \times \left[\int_{y_0-\epsilon/2}^{y_0+\epsilon/2} \int_{x_0-\epsilon/2}^{x_0+\epsilon/2} \left\{ (\phi_{11}w\dot{w} + \phi_{12}\dot{w}^2) - (\phi_{12}w + \phi_{22}\dot{w}) \times \left(\frac{c}{m}\dot{w} + G(w)\right) + \frac{\lambda}{2}\left([w\dot{w}]\, \Phi \begin{bmatrix} w \\ \dot{w} \end{bmatrix} \right) \right\} dxdy \right] \quad (16)$$

where f is the integrand of the denominator function $(\phi_{12}w + \phi_{22}\dot{w})$.

$$\bar{u}(x_0,y_0,t) = \frac{-m \times \epsilon^2 \times (\phi_{12}w(x_0,y_0) + \phi_{22}\dot{w}(x_0,y_0))}{f(x_0,y_0)^2 \times \epsilon^2} \times \left[\left\{ (\phi_{11}w(x_0,y_0)\dot{w}(x_0,y_0) + \phi_{12}\dot{w}(x_0,y_0)^2) - (\phi_{12}w(x_0,y_0) + \phi_{22}\dot{w}(x_0,y_0)) \times \left(\frac{c}{m}\dot{w}(x_0,y_0) + G(w(x_0,y_0))\right) + \frac{\lambda}{2}\left([w(x_0,y_0)\dot{w}(x_0,y_0)]\, \Phi \begin{bmatrix} w(x_0,y_0) \\ \dot{w}(x_0,y_0) \end{bmatrix} \right) \right\} \right] \quad (17)$$

Expanding the above equation, we get

$$\bar{u}(x_0,y_0,t) = m\left[\frac{\phi_{11}w(x_0,y_0)\dot{w}(x_0,y_0) + \phi_{12}\dot{w}^2(x_0,y_0)}{\phi_{12}w + \phi_{22}\dot{w}} + \frac{\lambda}{2}\frac{\phi_{11}w(x_0,y_0)^2 + 2\phi_{12}\dot{w}(x_0,y_0) + \phi_{22}\dot{w}^2(x_0,y_0)}{\phi_{12}w + \phi_{22}\dot{w}} - \left(\frac{c}{m}\dot{w}(x_0,y_0) + G(w(x_0,y_0))\right) \right] \quad (18)$$

The third term $\left(\frac{c}{m}\dot{w}(x_0,y_0) + G(w(x_0,y_0))\right) \to 0$ as $w \to 0$ and $\dot{w} \to 0$. Disgarding the third term, the remaining solution is a function of only two variables w and $\dot{w}$. The system states are zero everywhere other than the interval $x, y \in [x_0 - \epsilon/2, x_0 + \epsilon/2, y_0 - \epsilon/2, y_0 + \epsilon/2]$. The system states are small in this region. Hence, a linear behaviour is assumed in the region, i.e. $\dot{w}(x_0,y_0) = aw(x_0,y_0)$, where a is a constant. Substituting this in the equation 19, we obtain

$$\bar{u}(x_0,y_0,t) = \lim_{w(x_0,y_0)\to 0} m\frac{(\phi_{11}a + \phi_{12}a^2)w^2(x_0,y_0)}{(\phi_{12} + \phi_{22}a)w(x_0,y_0)} + \frac{m\lambda}{2}\frac{(\phi_{11} + 2\phi_{12}a + \phi_{22}a^2)w^2(x_0,y_0)}{(\phi_{12} + \phi_{22}a)w(x_0,y_0)} = 0 \quad (19)$$

Hence, $u(x,y,t) \to 0$ as $w \to 0$ and $\dot{w} \to 0, \forall x_0, y_0 \in [x_0 - \epsilon/2, x_0+\epsilon/2, y_0-\epsilon/2, y_0+\epsilon/2]$. This shows the convergence of the controller of interest.

4.3 Discrete controller

For discrete controllers, the formulation is changed to incorporate the discrete nature of the systems combined with the continuous plate system. The constraint equation can be derived by substituting the equations 5, 6, 7 in 8.

$$\langle I_n, \bar{u}_n \rangle = \gamma \quad (20)$$

where,

$$I_n = \int_{y_n - s_{yn}/2}^{y_n + s_{yn}/2} \int_{x_n - s_{xn}/2}^{x_n + s_{xn}/2} \left(\frac{1}{m}(\phi_{12}w + \phi_{22}\dot{w}) \right) dxdy \quad (21)$$

For a minimum control effort, a cost function of control variable along with the constraints is optimized. The cost function is in the form,

$$J = \frac{1}{2}\sum_{n=1}^{N}(r_n\Omega_n\bar{u}_n^2) + \Lambda[\langle I_n, \bar{u}_n\rangle - \gamma] \quad (22)$$

where $\Omega_n = s_{xn} * s_{yn}$, area of the n^{th} controller.
The necessary condition for the optimal solution for control variable $\bar{u}_n$ is,

$$\nabla_{\bar{u}_n,\Lambda} J = 0 \quad (23)$$

Solving for the necessary condition as in equation 23, the control variable is derived as

$$\bar{u}_n = \frac{I_n\gamma}{r_n\Omega_n \sum_{n=1}^{N}(I_n^2/r_n\Omega_n)} \quad (24)$$

4.4 Control singularity

In the optimized solutions for control variable in equation 24, the denominator approaches zero faster than the numerator. This can be avoided by introducing an error function $E := w_i(t) - w_i^{\#}(t)$, where $w_i(t)$ is the observed displacements of the dynamical system and $w_i^{\#}(t)$ is the target response of the observed state variables. As presented by Yeh et al. (1995), dynamic inversion is applied through enforcing a dynamics to the error function as in equation 25 such that it ensures $E(t) \to 0$ as $t \to 0$.

$$\ddot{E} + K_v\dot{E} + K_pE = 0 \quad (25)$$

The K_v and K_p matrices can be chosen to be diagonal matrices as $K_{pii} = \omega_n^2$ and $K_{vii} = 2\xi_n\omega_n$, where ω_n is the desired natural frequency and ξ_n is the damping ratio of the error dynamics. Keeping target state of $w_i^{\#}(t) = 0$ and

$\dot{w}_i^{\#}(t) = 0$, the final expression of the control parameter can be obtained as

$$\bar{u}_n = -\left[\left(K_p - M^{-1}K\right)\ \left(K_v - M^{-1}C\right)\right]\begin{bmatrix} w \\ \dot{w}\end{bmatrix} \quad (26)$$

Combining the equations, 24 and 26 for discrete controllers, it can be written as

$$\bar{u}_n = \begin{cases} \dfrac{I_n \gamma}{r_n \Omega_n \sum_{n=1}^{N}(I_n^2/r_n\Omega_n)} > \delta \\ -\left[\left(K_p - M^{-1}K\right)\ \left(K_v - M^{-1}C\right)\right]\begin{bmatrix} w \\ \dot{w}\end{bmatrix}, otherwise \end{cases} \quad (27)$$

where δ is a tolerance value provided by the control designer to bye-pass the singularity condition of the controllers.

5. NUMERICAL RESULTS

The approach handled here is DTA where the control forces are derived analytically and added to the PDE of plate and then simulated for numerical results using finite element analysis. Plates of parameters given in table 1 is considered for numerical simulation. In continuous controller case, the actuator is considered throughout on the plate surface. For discrete controller, a $0.2m \times 0.2m$ patch of nodes in the centre of the plate is chosen to apply the control forces. The dimensions of the plate and position of actuator (for discrete case) is given in the figure 1.

Table 1. Table of parameters

Description	Parameter	Valye
Modulus of Elasticity	E	$200GPa$
Mass per unit area	m	$15.7kg/m$
Dimension of plate	$a \times b \times h$	$0.5 \times 0.5 \times 0.002$
Damping coefficient	c	$2\% of critical$
Displacement of plate	w	State variable

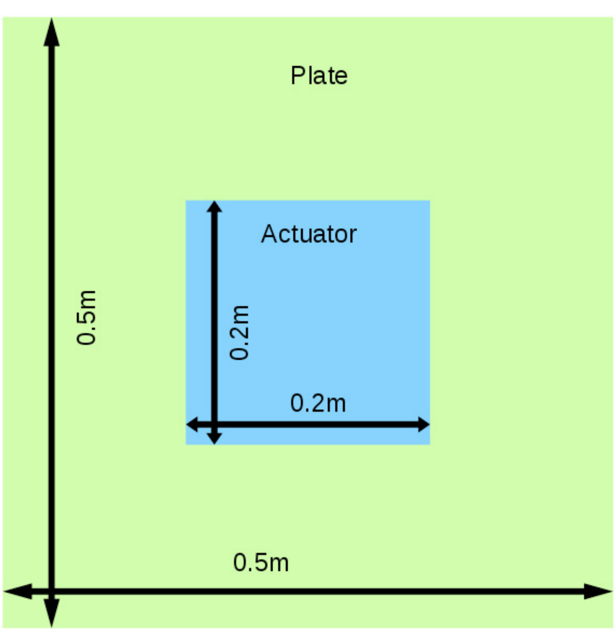

Fig. 1. Dimension of plate and discrete actuator

A linear analysis if done to obtain the frequencies and vibration modes of the plate. The vibration of the plate is set with an initial deflection profile similar to the first mode of vibration as shown in the figure 2. The vibrations of the plate with and without controllers are simulated using non-linear finite element method. The whole plate is discretized into 100 four-nooded C^1 continuous elements. The displacement field of the elements are approximated

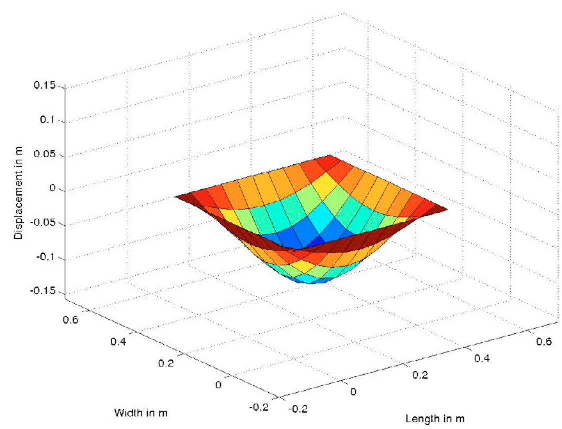

Fig. 2. First mode of vibration

with $2-d$ polynomial interpolation function represented in Einstein notation in 28.

$$w = \psi_i \Delta_i \quad (28)$$

where Δ_i is the degrees of freedom of the plate element and ψ_i is the shape function. Thus, the PDE is converted into a set of non-linear second order ODEs.

$$[M]\ddot{\Delta} + [C]\dot{\Delta} + [K(\Delta)]\Delta = \{U\} \quad (29)$$

where $[M], [C]\&[K]$ are the mass, damping and stiffness matrices of the system. The damping matrix is obtained using Rayleigh proportional damping $C = \alpha K + \beta M$, where the coefficients α and β can be obtained from

$$\begin{vmatrix} \xi_i \\ \xi_j \end{vmatrix} = \frac{1}{2}\begin{bmatrix} \dfrac{1}{\omega_i} & \omega_i \\ \dfrac{1}{\omega_j} & \omega_j \end{bmatrix}\begin{bmatrix} \beta \\ \alpha \end{bmatrix} \quad (30)$$

ξ_i is the damping ratio associated with ω_i frequency. The gain value $\lambda = 50$ is taken and the Φ matrix is taken as

$$\Phi = \begin{bmatrix} 100 & 36.8 \\ 36.8 & 15 \end{bmatrix}$$

The gain matrices K_p and K_v, are chosen to be diagonal matrices.

The displacement and velocity norms of the plate with and without controllers are compared. Figure 3 shows that the control goal is achieved using both continuous and discrete controllers where the displacement is quickly brought to zero. Figure 4 shows the achievement of target velocity by applying control forces. It is evident from the figures 3 and 4, that, in continuous controller, $w\&\dot{w} \to 0$ as the integral error $Z(t) \to 0$, but, in discrete controller, though $Z(t) \to 0$, $w\&\dot{w}$ still suffer non-zero values which can be compared with the figure 5. For avoiding such cases, more discrete actuators should be given so that, the target state values can be achieved with guarantee. This is because, the controller approaches to become a continuous one. From the figures, it can be concluded that the optimal dynamic

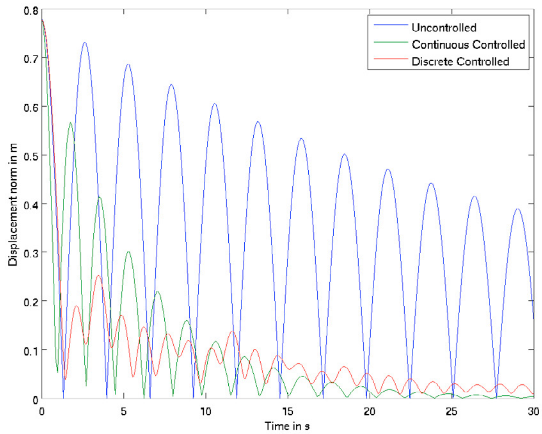

Fig. 3. Comparison of displacement norm.

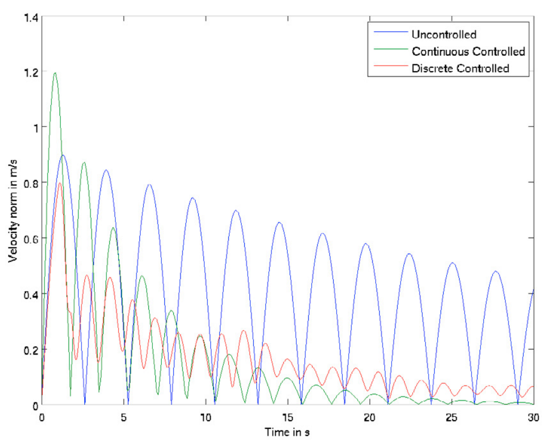

Fig. 4. Comparison of velocity norm.

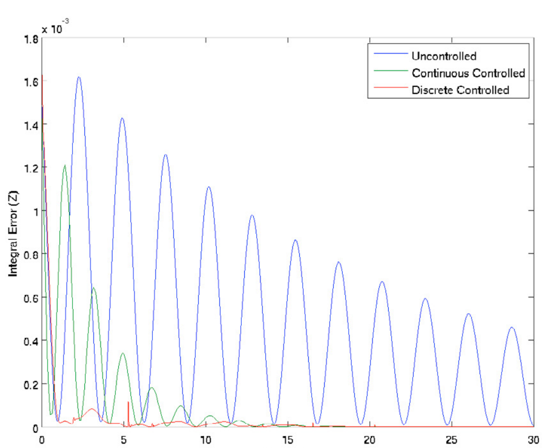

Fig. 5. Comparison of integral error values.

inversion technique can effectively control the vibrations of structural components with ensured stability and free from approximation errors.

6. CONCLUSION

A set of PDE called Föppl-von Kármán equations, has been used to derive the governing differential equation of the geometrically nonlinear simply supported plate. A constraint law has been induced such that the displacements and the velocities of the plate approaches zero. Using variational optimization, the control forces have been derived in closed form using the system PDE to reduce the approximation errors. Continuous and finite number of discrete actuators have been considered to apply the control forces to the structure. Numerical simulation has been done using finite element approximation for the PDE. The results have shown that as the integral error tends to zero, the displacement and velocity of the plate also tends to zero in continuous controller case whereas the plate suffers from vibration in the discrete controller case. Increasing the number of discrete controllers will reduce such errors since they work together as a continous one.
The method followed in this work shows significant use in the implementation of controllers for controlling distributed parameter systems. Since the control forces are of closed form, the method can be implimented in realtime problems.

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