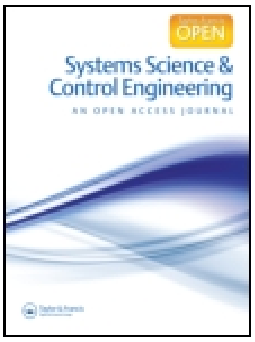

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A non-continuum lumped-parameter dynamic model applied to Indian traffic

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Dynamic traffic flow models are essential for obtaining information about the time evolution of variables describing the traffic flow phenomena and have a critical role in the development and implementation of real-time applications such as Intelligent Transportation Systems. Macroscopic traffic flow models that treat the traffic as a continuum are preferred for such applications. But, existing macroscopic models characterize homogeneous traffic, and may not be directly applicable to capture the vehicle heterogeneity seen on Indian roads. To address this issue, a non-continuum macroscopic dynamic traffic flow model based on the lumped-parameter approach was developed in this study. The model was developed based on the conservation of vehicles equation and a dynamic speed equation, incorporating an empirically developed traffic stream model, which is an important contribution of this study. Using this model, an estimation scheme has been developed based on the Kalman filtering technique to estimate traffic states in real time. The proposed scheme was implemented and corroborated for the heterogeneous traffic conditions existing in India. The performance of this scheme has been evaluated and the results obtained have been found to be promising.

Keywords: dynamic traffic flow modelling; lumped-parameter approach; extended Kalman filter; real-time traffic state estimation

1. Introduction

The exponential increase in quantity and complexity of road traffic, coupled with insufficient developments in infrastructure to match, has led to serious traffic problems on the Indian road. A cost-effective way to manage these problems is by utilizing the existing facility more efficiently through operational means. Such an approach hinges on a better understanding of the dynamics of traffic flow through traffic flow modelling. Traffic flow models mathematically study the interactions between vehicles, drivers and infrastructure to understand and predict the dynamic behaviour of traffic and develop optimal transportation patterns and networks. Such dynamic traffic models are particularly critical in the development and implementation of real-time applications such as Intelligent Transportation Systems (ITS).

A popular ITS application is the provision of real-time information about traffic states to travellers. Such a facility requires quantification of traffic congestion in terms of speed, delay, travel time and traffic density on the road. Among these, traffic density and travel time are the most important traffic parameters that can directly be used to determine the level of congestion. However, density and travel time (or speed) are spatial parameters that cannot be easily measured from the field using automated traffic sensors. They are usually estimated from other traffic parameters that can be measured using available sensors.

Accurate mathematical models are required to process the location-based information into spatial information. Model-based estimation schemes using techniques such as Kalman filters (KFs) are being increasingly used for the purpose because deterministic models have been found to be insufficient by themselves for accurate estimation of traffic variables in real time (Jabari and Liu, 2013; Wang, Li, Chen, & Ni, 2011). Such techniques have an additional advantage of being able to account for the uncertainty associated with traffic flow phenomena, which is of more relevance under traffic conditions such as in India where the randomness associated with traffic is high. Another advantage of this approach is that in order to estimate state variables at a given instant of time, one needs only the estimate from the previous instant of time and the measured data collected during that instant of time. Thus, unlike data or pattern-driven methods, the data measured during all the previous instants of time need not be stored, which is advantageous in places where the system is being implemented and hence a database is not available, such as under Indian conditions.

Traffic flow models have been classified into microscopic and macroscopic based on the level of detail used

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to describe traffic. Macroscopic traffic flow models are better suited for real-time applications and hence such an approach is attempted in this study. A majority of the macroscopic models reported in literature treat traffic as a continuum (Hoogerdoorn & Bovy, 2001). However, this continuum approach to model traffic flow has been criticized by several researchers (Papageorgiou, 1998; Tyagi, Darbha, & Rajagopal, 2009). The number of vehicles in a typical section of roadway is limited and cannot be described as a continuum, even in congested condition. Furthermore, continuum models allow the two-way propagation of disturbances, which is unrealistic for the traffic scenario. Because of these issues, alternative non-continuum approaches have been attempted for modelling traffic flow (Daganzo, 1994a, 1994b; Tyagi, Darbha, & Rajagopal, 2008). The present study proposed a non-continuum macroscopic dynamic traffic flow model for the estimation of traffic density using Kalman filtering technique. A few studies in this area of model-based density estimation are listed in the following table.

The modelling studies mentioned above have predominantly dealt with homogeneous traffic conditions. A few models developed for heterogeneous traffic conditions are discussed in this section. Logghe and Immers (2003) developed an extended Lighthill, Whitham and Richard (LWR) model to include different classes of vehicles. A microscopic theory of spatial-temporal congested traffic patterns in heterogeneous traffic flow with a variety of driver behavioural characteristics and vehicle parameters was presented by Kerner and Klenov (2004) based on three-phase traffic theory. A recent addition to heterogeneous traffic modelling is by Tang, Huangb, Zhao, and Shang (2009), where a new dynamic car-following model has been developed by applying the relationship between the microscopic and macroscopic variables. Studies reported from India include a few on the use of macroscopic models (Anand, Vanajakshi, & Subramanian, 2011; Padiath, Vanajakshi, & Subramanian, 2010; Padiath, Vanajakshi, Subramanian, & Manda, 2009; Tiwari, Fazio, Gaurav, & Chatteerjee, 2008). Other reported studies have focused on microscopic models that are not suited for real-time applications (Chakroborty & Kikuchi, 1999; Chakroborty & Maurya, 2008; Gupta, Chakroborty, & Mukherjee, 1998; Mallikarjuna & Rao, 2009; Mathew, Gundaliya, & Dhingra, 2006; Maurya & Chakroborty, 2008; Venkatesan, Gowri, & Sivanandan, 2008).

Thus, although there are a number of reports on macroscopic traffic flow, most of them focused on homogeneous traffic conditions. Reported heterogeneous traffic flow modelling has been mainly of the microscopic type, which is not applicable for real-time applications. Macroscopic models that are computationally tractable are particularly suitable for representing the real-time stream features such as traffic congestion on Indian urban

No.	Authors (year)	Facility type, data source and type of data	Parameter estimated and technique used
1.	Gazis and Knapp (1971)	Freeway, location-based, field data	Density, model-based extended Kalman filter (EKF) approach
2.	Nahi and Trivedi (1973)	Freeway, location-based, field data as well as simulated data	Density, model-based KF approach
3.	Gazis and Szeto (1974)	Multilane roadway, location-based, field data	Density and speed, model-based KF approach
4.	Houpt, Kurkjian, Gershwin, and Willsky (1979)	Freeway, location-based, Field data as well as simulated data	Density, model-based scalar KF approach
5.	Willsky et al. (1980)	Freeway, location-based, field data as well as simulated data	Density and speed, model-based KF approach
6.	Kurkjian et al. (1980)	Freeway, location-based, field data as well as simulated data	Density, model-based scalar KF approach
7.	Hoogendoorn and Bovy (2000)	Simulated data	Density and speed, method of moments
8.	Zhongke (2003)	Freeway, location-based, field data as well as simulated data	Density, model-based EKF approach
9.	Sun, Muñoz, and Horowitz (2004)	Highway, location-based and spatial, field data as well as simulated data	Density, mixture KF approach
10.	Wang and Papageorgiou (2005)	Freeway, location-based and field data as well as simulated data	Density and speed, model-based EKF approach
11.	Chu, Oh, and Recker (2005)	Freeway, location-based, field data as well as simulated data	Density and travel time, model-based adaptive Kalman filter approach
12.	Hegyi, Girimonte, Babuska, and DeSchutter (2006)	Freeway, location-based, field data as well as simulated data	Density and speed, METANET Model + EKF and unscented Kalman filter
13.	Wang, Papageorgiou, and Messmer (2007)	Freeway, location-based and field data as well as simulated data	Density and speed, model-based EKF approach
14.	Wang, Papageorgiou, and Messmer (2008)	Freeway, location-based and field data as well as simulated data	Density and speed, model-based EKF approach

roads. Due to the unsuitability of continuum models, non-continuum models are gaining importance in traffic modelling and this study presents one such model. A lumped-parameter macroscopic dynamic traffic flow model based on a non-continuum approach for the estimation of traffic density using the Kalman filtering technique is proposed here. The model formulation employed the law of conservation of vehicles inside a road section and a dynamic speed equation obtained using a traffic stream model. The chosen stream model was a two-regime steady-state speed–density relation that was developed for the specific traffic condition being analysed. Density and aggregate space mean speed were considered as the state variables of interest in this model. Using this model, an estimation scheme was developed based on the Kalman filtering technique. All traffic variables were quantified without considering traffic lanes in order to account for the lack of lane discipline. The scheme was first designed and implemented without considering the heterogeneity of traffic. In the next stage, the heterogeneity was incorporated into the scheme in two ways. In the first approach, heterogeneity was incorporated by expressing all variables in standard Passenger Car Unit (PCU) equivalent values (IRC, 1990). In the second approach, heterogeneity was incorporated by explicitly considering different categories of vehicles into the modelling and estimation processes. The above estimation schemes were corroborated using field data. The results showed the efficacy of the developed model and the estimation scheme in real-time estimation of traffic state under the heterogeneous traffic conditions existing in India.

2. The proposed model

The model proposed in this study is developed in a state-space form appropriate for the design of model-based scheme for estimation of traffic density using the Kalman filer. A dynamic non-continuum lumped-parameter macroscopic model was formulated for describing the flow of traffic. In the lumped-parameter approach, within a small section of roadway, the spatial variation of traffic variables (such as density, speed, etc.) is neglected and it is assumed that the variables depend only on time. The typical section length with automated data collection is less than 1 km and it is reasonable to make this assumption for this distance. The length of the section (L) used in this study is also in this range. To apply this procedure to a longer roadway, it must be divided into sections within which it is reasonable to neglect spatial variations in traffic characteristics. A schematic diagram of a typical road section is shown in Figure 1. The lumped-parameter approach results in the governing equations of the model being ordinary differential equations (ODEs) (in the continuous time domain) and ordinary difference equations (in the discrete time domain).

The number of vehicles inside the section per unit length (density) and the average space mean speed of traffic are spatial parameters that are difficult to measure in the field. However, being spatial in nature, they are good indicators of the state of traffic. Hence, they were considered as the macroscopic state variables in this study. The first governing equation of the model was formulated based on the conservation of vehicles inside the section as follows.

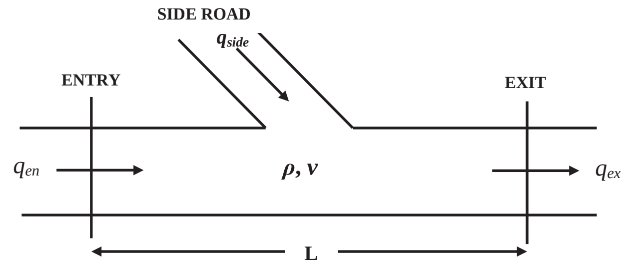

Figure 1. Schematic diagram of a typical road section.

Let $N(k)$ denote the number of vehicles inside the section at the kth instant of time. Then, the conservation of vehicles inside the section for a time step of h can be represented as

$$N(k+1) = N(k) + h(q_{\mathrm{en}}(k) - q_{\mathrm{ex}}(k) + q_{\mathrm{side}}(k)), \quad (1)$$

where $q_{\mathrm{en}}(k)$ is the flow rate at which vehicles are entering into the section, $q_{\mathrm{ex}}(k)$ is the flow rate at which vehicles are exiting from the section and $q_{\mathrm{side}}(k)$ is the net flow rate at which the vehicles are entering into the section from the side road in the time interval $(k, k+1)$.

Dividing Equation (1) by the length of the section (L) resulted in

$$\rho(k+1) = \rho(k) + \frac{h}{L}(q_{\mathrm{en}}(k) - q_{\mathrm{ex}}(k) + q_{\mathrm{side}}(k)), \quad (2)$$

where $\rho(k+1)$ denotes the density inside the section at the $(k+1)$th instant of time.

The second governing equation of the model is a dynamic speed equation formulated by incorporating the appropriate stream model for the specific traffic under study. A two-regime steady-state speed–density relationship was found to be the best fit for the traffic under study (Ajitha and Vanajakshi, 2012). The second governing equation was then obtained with the motive of minimizing the error (e) between the speed values estimated using this steady-state speed–density relation $v(\rho)$ and the observed speed values v, that is, $e = v(\rho) - v$. The time evolution of this error was hypothesized to behave as governed by

$$\frac{\mathrm{d}e}{\mathrm{d}t} = -a \cdot e(t), \quad (3)$$

where the parameter a was selected to be positive. This equation is a linear homogeneous ODE and it is well known that its unique solution is $e(t) = e(0)\exp(-at)$ (Coddington, 1989), where $e(0)$ is the initial error (can be either positive or negative). Thus, the error will converge to zero with time. Although there may be other choices for describing the time evolution of the error function, an exponentially decaying error function (an exponential function is a very commonly used function in many phenomenological studies) has been chosen in this study since its performance will be comparably good to any alternate choice. This approach has been applied in other studies involving the dynamical systems approach (Ioannou & Chien, 1993; Swaroop, Hedrick, Chien, & Ioannou, 1994).

Substituting $e = v(\rho) - v$ in Equation (3) and rearranging resulted in

$$\frac{\mathrm{d}(v(\rho))}{\mathrm{d}\rho} \cdot \frac{\mathrm{d}\rho}{\mathrm{d}t} - \frac{\mathrm{d}v}{\mathrm{d}t} = -a \cdot (v(\rho) - v). \quad (4)$$

Discretizing Equation (4) using a time step h resulted in

$$\frac{\mathrm{d}(v(\rho))}{\mathrm{d}\rho} \frac{(\rho(k+1) - \rho(k))}{h} - \frac{(v(k+1) - v(k))}{h} = -a(v(\rho) - v(k)). \quad (5)$$

By substituting $(\rho(k+1) - \rho(k))/h = (1/L)(q_{\mathrm{en}}(k) - q_{\mathrm{ex}}(k) + q_{\mathrm{side}}(k))$ from Equation (2), the dynamic equation for average space mean speed inside the section (i.e. the equation governing the evolution of v) was obtained as

$$v(k+1) = v(k) + ah(v(\rho) - v(k)) + \frac{h}{L}\frac{\mathrm{d}(v(\rho))}{\mathrm{d}\rho}(q_{\mathrm{en}}(k) - q_{\mathrm{ex}}(k) + q_{\mathrm{side}}(k)), \quad (6)$$

where $v(k + 1)$ denotes the average space mean speed of traffic inside the section at the $(k + 1)$th instant of time.

Thus, the general formulation of the non-continuum lumped-parameter model is represented by Equations (2) and (6). Here, the heterogeneity of traffic was not explicitly considered. The site-specific speed–density relationship $v(\rho)$ was also developed without considering heterogeneity and was incorporated in Equation (6). Brief descriptions of the developed speed–density relation and the details on incorporating it in Equation (6) are provided below.

Based on the field data collected from the study site, the best-fitting speed–density relation was identified empirically (Ajitha & Vanajakshi, 2012). To take into account the lack of lane discipline, the roadway was analysed without considering the traffic lanes and hence flow was expressed in veh/hr and density in veh/km. Using this data, the best-fitting traffic stream model was found to be a two-regime model with a constant speed in the free-flow regime and the speed decreasing nonlinearly in the congested regime up to the point where the jam density is reached. The functional form of this two-regime speed–density relation was obtained as

$$v = \begin{cases} 47 & \text{when } 0 \leq \rho \leq 156, \\ 20.226\left(\dfrac{519}{\rho} - 1\right) & \text{when } 156 \leq \rho \leq 519, \end{cases} \quad (7)$$

where the speed v is expressed in kmph and the density ρ in veh/km.

Incorporating this in Equation (6), the dynamic speed equation is reduced to

$$v(k+1) = \begin{cases} v(k) + ah(47 - v(k)) \quad \text{when } 0 \leq \rho(k) \leq 156, \\ v(k) + ah\left(20.226\left(\dfrac{519}{\rho(k)} - 1\right) - v(k)\right) \\ \quad - \dfrac{10497.29h}{L \cdot (\rho(k))^2}(q_{\mathrm{en}}(k) - q_{\mathrm{ex}}(k) + q_{\mathrm{side}}(k)) \\ \quad \text{when } 156 \leq \rho(k) \leq 519. \end{cases} \quad (8)$$

Equations (2) and (8) represent the complete model for traffic without considering heterogeneity. This model was represented in the state-space form appropriate for the design of a model-based estimation scheme using the KF (more explanation on the KF is given in Section 3). However, a necessary condition for the KF to work correctly is that the system for which the states are to be determined is observable. A system is said to be observable if the internal states of the system can be determined using only the knowledge about the system out puts. Observability for a discrete time linear system can be tested by checking the rank of the obsevability matrix.

Consider a linear system whose model takes the form:

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{B}\mathbf{u}_k + \mathbf{w}_k, \quad (9)$$

$$\mathbf{z}_k = \mathbf{H}\mathbf{x}_k + \mathbf{v}_k, \quad (10)$$

where, $\mathbf{x}_k$ is the system state, $\mathbf{z}_k$ is the system output, $\mathbf{u}_k$ is the system input, $\mathbf{w}_k$ is the process disturbance and $\mathbf{v}_k$ is the measurement noise at the kth instant of time. The matrix $\mathbf{A}$ relates the state at the kth instant of time to the state at $(k + 1)$th instant of time and the matrix $\mathbf{B}$ relates the input to the state.

Then the system governed by Equations (2) and (8) is observable if and only if the observability matrix ($\mathbf{O}$) given by

$$\mathbf{O} = \begin{bmatrix} \mathbf{H} \\ \mathbf{HA} \\ \mathbf{HA}^2 \\ \vdots \\ \mathbf{HA}^{n-1} \end{bmatrix},$$

has rank equal to n, the order of the system (Gopal, 1984).

For the system under study represented by Equations (3) and (9), the values of the parameters, $\mathbf{A}$ and $\mathbf{H}$ for the free-flow regime were derived as

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 - ah \end{bmatrix}, \quad \mathbf{H} = [0 \quad 1].$$

The corresponding observability matrix was obtained as

$$\mathbf{O} = \begin{bmatrix} 0 & 1 \\ 0 & 1 - ah \end{bmatrix}.$$

The rank of this observability matrix was found to be 1 and thus the system is found to be unobservable in the free-flow regime.

To make the system observable, the governing equation for density as represented by Equation (2) was modified by expressing the flow passing exit section q_{ex} in terms of the average space mean speed of traffic passing the exit location using the fundamental equation of traffic flow given by

$$q_{ex}(k) = \rho(k)v_{ex}(k), \tag{11}$$

where ρ is the density and v_{ex} is the average space mean speed of vehicles calculated using individual speed measurements at the exit. Substituting Equation (11) in Equation (2) provided:

$$\rho(k+1) = \rho(k) + \frac{h}{L}(q_{en}(k) - \rho(k)v_{ex}(k) + q_{side}(k)). \tag{12}$$

Incorporating this change in the dynamic equation for speed (Equation (7)), the governing equation for speed is reduced to

$$v(k+1) = \begin{cases} v(k) + ah(47 - v(k)) & \text{when } 0 \le \rho(k) \le 156, \\ v(k) + ah\left(20.226\left(\frac{519}{\rho(k)} - 1\right) - v(k)\right) & \\ + \frac{-10497.29h}{L \cdot (\rho(k))^2}(q_{en}(k) - \rho(k) \cdot v_{ex}(k) & \\ + q_{side}(k)) & \text{when } 156 \le \rho(k) \le 519. \end{cases} \tag{13}$$

The check for observability of the system governed by the modified state equations (Equations (12) and (13)) was again carried out for free-flow and congested regimes as detailed above and the rank for the new observability matrices was found to be equal to 2 and thus the system was observable. Equations (12) and (13) govern the traffic system without considering heterogeneity. This model was then modified to incorporate heterogeneity in two different ways as explained below.

One common approach to consider the mixture of different categories of vehicles in a traffic stream is to convert them into a homogeneous equivalent using standard PCU values (IRC, 1990). This approach has been first adopted in this study to account for the effect of presence of several vehicle types. Thus, in Equations (12) and (13), traffic flow was considered in PCU/hr and traffic density in PCU/km. The site-specific speed–density relationship $v(\rho)$ was developed using PCU converted data (Ajitha & Vanajakshi, 2012) and was incorporated in Equation (13). While developing $v(\rho)$ in this case, the data were measured separately for different categories of vehicles and then converted in to PCU units. The functional form of this speed–density relationship was obtained as

$$v = \begin{cases} 47 & \text{when } 0 \le \rho \le 143, \\ 23.5\left(\frac{429}{\rho} - 1\right) & \text{when } 143 \le \rho \le 429, \end{cases} \tag{14}$$

where the density ρ is in PCU/km.

Incorporating this in Equation (13), the dynamic speed equation is reduced to

$$v(k+1) = \begin{cases} v(k) + ah(47 - v(k)) & \text{when } 0 \le \rho(k) \le 143, \\ v(k) + ah\left(23.5\left(\frac{429}{\rho(k)} - 1\right) - v(k)\right) & \\ -\frac{10081.5h}{L \cdot (\rho(k))^2}(q_{en}(k) - \rho(k) \cdot v_{ex}(k) & \\ + q_{side}(k)) & \text{when } 143 \le \rho(k) \le 429. \end{cases} \tag{15}$$

Thus, the complete model formulation after PCU incorporation was represented by Equations (12) and (15) with flow rates and density expressed in PCU/hr and PCU/km, respectively.

Another way of introducing heterogeneity in traffic flow is to consider different categories separately in the modelling process. The classification considered in this study was the three vehicle groups, namely two wheelers (TWs), three wheelers (ThWs) and four wheelers (FWs). TWs include motorcycles, scooters and mopeds and ThWs include auto-rickshaws and small three-wheeled tempos. The FW category can be further subdivided into classes such as light passenger cars, heavy commercial vehicles, and others. However, due to the difficulty in manual data extraction, this classification was followed in the present study. The procedure can be easily extended for other classes, if data are available. The above developed model was modified by incorporating separate state equations for the three different classes of vehicles considered, namely TWs, ThWs and FWs. The first three equations of the modified model were based on the conservation of the three classes of vehicles inside the section and was obtained as

$$\rho_{TW}(k+1) = \rho_{TW}(k) + \frac{h}{L}(q_{en}^{TW}(k) - \rho_{TW}(k) \cdot v_{ex}^{TW}(k) + q_{side}^{TW}(k)), \tag{16}$$

$$\rho_{ThW}(k+1) = \rho_{ThW}(k) + \frac{h}{L}(q_{en}^{ThW}(k) - \rho_{ThW}(k) \cdot v_{ex}^{ThW}(k) + q_{side}^{ThW}(k)), \tag{17}$$

$$\rho_{FW}(k+1) = \rho_{FW}(k) + \frac{h}{L}(q_{en}^{FW}(k) - \rho_{FW}(k) \cdot v_{ex}^{FW}(k) + q_{side}^{FW}(k)). \tag{18}$$

The remaining three equations were dynamic speed equations formulated using the stream models developed for these three classes. The stream models developed for the three categories of vehicles (Ajitha & Vanajakshi, 2012) were

$$v_{\mathrm{TW}} = \begin{cases} 48 & \text{when } 0 \le \rho_{\mathrm{TW}} \le 87, \\ 18.316\left(\dfrac{315}{\rho_{\mathrm{TW}}} - 1\right) & \text{when } 87 \le \rho_{\mathrm{TW}} \le 315, \end{cases} \tag{19}$$

$$v_{\mathrm{ThW}} = \begin{cases} 40 & \text{when } 0 \le \rho_{\mathrm{ThW}} \le 14, \\ 18.065\left(\dfrac{45}{\rho_{\mathrm{ThW}}} - 1\right) & \text{when } 14 \le \rho_{\mathrm{ThW}} \le 45, \end{cases} \tag{20}$$

$$v_{\mathrm{FW}} = \begin{cases} 50 & \text{when } 0 \le \rho_{\mathrm{FW}} \le 57, \\ 23.208\left(\dfrac{180}{\rho_{\mathrm{FW}}} - 1\right) & \text{when } 57 \le \rho_{\mathrm{FW}} \le 180. \end{cases} \tag{21}$$

Using these equations, the formulated dynamic speed equations for TWs, ThWs and FWs were obtained as

$$v_{\mathrm{TW}}(k+1) = \begin{cases} v_{\mathrm{TW}}(k) + ah(48 - v_{\mathrm{TW}}(k)) \\ \quad \text{when } 0 \le \rho_{\mathrm{TW}}(k) \le 87, \\ v_{\mathrm{TW}}(k) + ah\left(18.316\left(\dfrac{315}{\rho_{\mathrm{TW}}(k)} - 1\right) \right. \\ \quad \left. - v_{\mathrm{TW}}(k)\right) - \dfrac{5769.54h}{L.(\rho_{\mathrm{TW}}(k))^2}(q_{\mathrm{en}}^{\mathrm{TW}}(k) \\ \quad -\rho_{\mathrm{TW}}(k) \cdot v_{\mathrm{ex}}^{\mathrm{TW}}(k) + q_{\mathrm{side}}^{\mathrm{TW}}(k)) \\ \quad \text{when } 87 \le \rho_{\mathrm{TW}}(k) \le 315, \end{cases} \tag{22}$$

$$v_{\mathrm{ThW}}(k+1) = \begin{cases} v_{\mathrm{ThW}}(k) + ah(40 - v_{\mathrm{ThW}}(k)) \\ \quad \text{when } 0 \le \rho_{\mathrm{ThW}}(k) \le 14, \\ v_{\mathrm{ThW}}(k) + ah\left(18.065\left(\dfrac{45}{\rho_{\mathrm{ThW}}(k)} - 1\right) \right. \\ \quad \left. - v_{\mathrm{ThW}}(k)\right) - \dfrac{812.925h}{L \cdot (\rho_{\mathrm{ThW}}(k))^2}(q_{\mathrm{en}}^{\mathrm{ThW}}(k) \\ \quad -\rho_{\mathrm{ThW}}(k) \cdot v_{\mathrm{ex}}^{\mathrm{ThW}}(k) + q_{\mathrm{side}}^{\mathrm{ThW}}(k)) \\ \quad \text{when } 14 \le \rho_{\mathrm{ThW}}(k) \le 45, \end{cases} \tag{23}$$

$$v_{\mathrm{FW}}(k+1) = \begin{cases} v_{\mathrm{FW}}(k) + ah(50 - v_{\mathrm{FW}}(k)) \\ \quad \text{when } 0 \le \rho_{\mathrm{FW}}(k) \le 57, \\ v_{\mathrm{FW}}(k) + ah\left(23.208\left(\dfrac{180}{\rho_{\mathrm{FW}}(k)} - 1\right) \right. \\ \quad \left. - v_{\mathrm{FW}}(k)\right) - \dfrac{4177.44h}{L \cdot (\rho_{\mathrm{FW}}(k))^2}(q_{\mathrm{en}}^{\mathrm{FW}}(k) \\ \quad -\rho_{\mathrm{FW}}(k) \cdot v_{\mathrm{ex}}^{\mathrm{FW}}(k) + q_{\mathrm{side}}^{\mathrm{FW}}(k)) \\ \quad \text{when } 57 \le \rho_{\mathrm{FW}}(k) \le 180. \end{cases} \tag{24}$$

Thus, it can be seen that all the above models have two components – one component that was derived using the conservation of vehicles and the hypothesis regarding the evolution of the error e, which will hold for any road segment. The second component was obtained from a traffic stream model. Such a stream model is not available for Indian road traffic conditions and hence is one of the contributions of this study. It is well known that traffic stream models are section-/location-dependent (Gartner, Messer, & Rathi, 2001). Hence, the model is transferable only to sections with similar characteristics. In other cases, the section-specific stream models need to be known or developed.

Next, the estimation scheme was developed using the Kalman filtering technique. The KF is an optimal state estimator applicable for dynamic systems. Although it was originally derived for linear systems, it can be extended to nonlinear systems and the filter so obtained is referred to as the EKF. In the present study, as the models developed are linear in the uncongested or free-flow regime and nonlinear in the congested regime, both the linear and the EKFs were used to estimate the traffic state as detailed below.

3. Estimation scheme

The KF (Kalman, 1960) is a popular tool for recursive estimation of variables that characterize a system (these variables are usually referred to as 'state variables'). The KF is a model-based estimation scheme that takes into account the stochastic properties of the process disturbance and the measurement noise. The process disturbance and the measurement noise are assumed to be independent of one another, white and normally distributed with zero mean. The KF works like a predictor corrector algorithm, that is, it first predicts an 'a priori' estimate of the state variables using the system model and the state estimate from the previous time interval, and then corrects the same using measurements to obtain an 'a posteriori' state estimate. The KF has been widely used in many disciplines including the field of transportation (Nanthawichit, Nakatsuji, & Suzuki, 2003; Okutani & Stephanedes, 1984; Wang & Papageorgiou, 2005; Wang, Papageorgiou, & Messmer, 2008). The KF is used for estimation and prediction when the governing equations of the system are linear. When the governing equations are nonlinear, an EKF (Jazwinsky, 1970) is commonly used. The EKF linearizes the governing equations at each time step about the estimate obtained from the previous time step.

Using the traffic flow models presented in the previous section, the estimation scheme was obtained as explained below. In the first case, where heterogeneity was not considered, the density of vehicles (ρ) expressed in veh/km and the average space mean speed (v) of vehicles inside the section in km/hr were taken as the state variables of interest. The output variable was taken as the measured values of the average space mean speed of vehicles. The rates at which vehicles enter into the section from upstream and from the side road in veh/hr were provided as inputs to the estimation scheme. Similarly, for the second case, where heterogeneity was incorporated by expressing all variables in PCU units, density (ρ) expressed in PCU/km and average space mean speed (v) in km/hr were considered as state variables, measured values of average space mean speed of vehicles as the output variable and flow passing the upstream and side road in PCU/hr as inputs to the scheme. For the third case, where different classes of vehicles were explicitly considered in modelling, density of TWs (ρ_{TW}), ThWs (ρ_{ThW}) and FWs (ρ_{FW}) and average space mean speed of TWs (v_{TW}), ThWs (v_{ThW}) and FWs (v_{FW}) were taken as the state variables of interest. The output variables were taken as the measured values of space mean speeds of these three classes of vehicles and inputs were taken as flow of these three classes of vehicles from upstream and through side roads.

Since the governing equations of the models in the uncongested regime were linear, the KF was used as below. Consider a linear system whose model takes the form:

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{B}\mathbf{u}_k + \mathbf{w}_k, \tag{25}$$

$$\mathbf{z}_k = \mathbf{H}\mathbf{x}_k + \mathbf{v}_k, \tag{26}$$

where $\mathbf{x}_k$ is the system state, $\mathbf{z}_k$ is the system output, $\mathbf{u}_k$ is the system input, $\mathbf{w}_k$ is the process disturbance and $\mathbf{v}_k$ is the measurement noise at the kth instant of time. The matrix $\mathbf{A}$ relates the state at the kth instant of time to the state at $(k+1)$th instant of time and the matrix $\mathbf{B}$ relates the input to the state.

Now the following steps are used recursively for estimation:

(1) The a priori estimate in the $(k+1)^{\text{th}}$ interval of time was obtained through

$$\hat{\mathbf{x}}_{k+1}^- = \mathbf{A}\hat{\mathbf{x}}_k^+ + \mathbf{B}\mathbf{u}_k.$$

(2) The a priori error covariance in the $(k+1)$th interval of time was obtained through

$$\mathbf{P}_{k+1}^- = \mathbf{A}\mathbf{P}_k^+\mathbf{A}^{\mathrm{T}} + \mathbf{Q}.$$

(3) The Kalman gain $\mathbf{K}_{k+1}$ was calculated through

$$\mathbf{K}_{k+1} = \mathbf{P}_{k+1}^-\mathbf{H}^{\mathrm{T}}(\mathbf{H}\mathbf{P}_{k+1}^-\mathbf{H}^{\mathrm{T}} + \mathbf{R})^{-1}.$$

(4) Then, the a posteriori state estimate was calculated through

$$\mathbf{x}_{k+1}^+ = \mathbf{x}_{k+1}^- + \mathbf{K}_{k+1}(\mathbf{z}_{k+1} - \mathbf{H}\mathbf{x}_{k+1}^-).$$

(5) Finally, the a posteriori error covariance was obtained through

$$\mathbf{P}_{k+1}^+ = (1 - \mathbf{K}_{k+1}\mathbf{H})\mathbf{P}_{k+1}^-.$$

Here, $\mathbf{Q}$ is the process disturbance covariance, $\mathbf{R}$ is the measurement noise covariance and $\mathbf{H}$ is the matrix which relates the state to the measurement. Also, $\hat{\mathbf{x}}_k^-$ and $\hat{\mathbf{x}}_k^+$ denote, respectively, the a priori estimate and the a posteriori estimate of the state variables at the kth instant of time. Similarly, $\mathbf{P}_k^-$ and $\mathbf{P}_k^+$ denote, respectively, the a priori and the a posteriori error covariance at the kth instant of time.

In the present study, since the models in the congested regime were nonlinear, an extended KF was used. The EKF linearizes the governing equations at each time step about the estimate obtained from the previous time step. Consider a nonlinear system whose model is given by

$$\mathbf{x}_{k+1} = \mathbf{f}(\mathbf{x}_k, \mathbf{u}_k, \mathbf{w}_{\mathbf{1}k}), \tag{27}$$

$$\mathbf{z}_k = \mathbf{g}(\mathbf{x}_k, \mathbf{v}_{\mathbf{1}k}), \tag{28}$$

where $\mathbf{f}$ represents the nonlinear function that relates the state at time step k to the state at time step $k+1$. Similarly, $\mathbf{g}$ is the nonlinear function that relates the state to the measurement. The above equations can be linearized using Taylor's Series expansion to result in

$$\mathbf{x}_{k+1} = \tilde{\mathbf{x}}_{k+1} + \mathbf{A}_\mathbf{1}(\mathbf{x}_k - \hat{\mathbf{x}}_k^+) + \mathbf{W}\mathbf{w}_{1k}, \tag{29}$$

$$\mathbf{z}_k = \tilde{\mathbf{z}}_k + \mathbf{H}_1(\mathbf{x}_k - \tilde{\mathbf{x}}_k) + \mathbf{V}\mathbf{v}_{\mathbf{1}k}, \tag{30}$$

where $\tilde{\mathbf{x}}$ and $\tilde{\mathbf{z}}$ are the approximate state and measurement variables without considering the process disturbance and measurement noise as indicated by Equations (31) and (32), $\mathbf{A}_\mathbf{1}$ is the matrix of the partial derivative of $\mathbf{f}$ with respect to $\mathbf{x}$, $\mathbf{W}$ is the matrix of the partial derivative of $\mathbf{f}$ with respect to $\mathbf{w}_\mathbf{1}$, $\mathbf{H}_\mathbf{1}$ is the matrix of the partial derivative of $\mathbf{g}$ with respect to $\mathbf{x}$ and $\mathbf{V}$ is the matrix of the partial derivative of $\mathbf{g}$ with respect to $\mathbf{v}_\mathbf{1}$. Thus,

$$\tilde{\mathbf{x}}_{k+1} = \mathbf{f}(\hat{\mathbf{x}}_k^+, \mathbf{u}_k, 0), \tag{31}$$

$$\tilde{\mathbf{z}}_k = \mathbf{g}(\tilde{\mathbf{x}}_k, 0). \tag{32}$$

Now the following steps were followed recursively for estimation using EKF:

(1) The a priori estimate in the $(k+1)$th interval of time was obtained through

$$\hat{\mathbf{x}}_{k+1}^- = \mathbf{f}(\hat{\mathbf{x}}_k^+, \mathbf{u}_k).$$

(2) The a priori error covariance in the $(k+1)$th interval of time was obtained through

$$\mathbf{P}_{k+1}^- = \mathbf{A}_\mathbf{1}\mathbf{P}_k^+\mathbf{A}_\mathbf{1}^{\mathrm{T}} + \mathbf{W}\mathbf{Q}\mathbf{W}^{\mathrm{T}}.$$

(3) The Kalman gain $\mathbf{K}_{k+1}$ was calculated through

$$\mathbf{K}_{K+1} = \mathbf{P}_{k+1}^-\mathbf{H}_1^{\mathrm{T}}[\mathbf{H}_1\mathbf{P}_{k+1}^-\mathbf{H}_1^{\mathrm{T}} + \mathbf{V}\mathbf{R}\mathbf{V}^{\mathrm{T}}]^{-1}.$$

(4) Then, the a posteriori state estimate was calculated through

$$\hat{\mathbf{x}}_{k+1}^+ = \hat{\mathbf{x}}_{k+1}^- + \mathbf{K}_{k+1}(\mathbf{z}_{k+1} - \mathbf{g}(\hat{\mathbf{x}}_{k+1}^-)).$$

(5) Finally, the a posteriori error covariance was obtained through

$$\mathbf{P}_{k+1}^{+} = [\mathbf{I} - \mathbf{K}_{k+1}\mathbf{H}_1]\mathbf{P}_{k+1}^{-}.$$

In the present study, the following parameters were identified in free-flow regime for the first case where heterogeneity was not considered:

$$\mathbf{x}(k) = \begin{bmatrix} \rho(k) \\ v(k) \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} q_{\text{en}} \\ q_{\text{side}} \end{bmatrix}, \quad \mathbf{z}(k) = v(k),$$

$$\mathbf{A} = \begin{bmatrix} 1 - \dfrac{h \cdot v_{\text{ex}}(k)}{L} & 0 \\ 0 & 1 - ah \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} \dfrac{h}{L} & \dfrac{h}{L} \\ 0 & 0 \end{bmatrix}, \quad \mathbf{H} = [0 \quad 1].$$

In the congested regime for this case, the parameters were obtained as

$$\mathbf{f} = \begin{bmatrix} \rho(k) + \dfrac{h}{L}(q_{\text{en}}(k) - \rho(k) \cdot v_{\text{ex}}(k) + q_{\text{side}}(k)) \\ v(k) + ah\left(20.226\left(\dfrac{519}{\rho(k)} - 1\right) - v(k)\right) \\ -\dfrac{10497.29h}{L \cdot (\rho(k))^2}(q_{\text{en}}(k) - \rho(k) \cdot v_{\text{ex}}(k) \\ +q_{\text{side}}(k)) \end{bmatrix},$$

$$\mathbf{A_1} = \begin{bmatrix} 1 - \dfrac{h \cdot v_{\text{ex}}(k)}{L} & 0 \\ \dfrac{-10497.29h}{(\rho(k))^2}\left[a - \dfrac{2}{L \cdot \rho(k)}(q_{\text{en}}(k) + q_{\text{side}}(k)) + \dfrac{v_{\text{ex}}(k)}{L}\right] & 1 - ah \end{bmatrix},$$

$$\mathbf{W} = \begin{bmatrix} h & 0 \\ 0 & h \end{bmatrix}, \quad \mathbf{H}_1 = [0 \quad 1], \quad \mathbf{V} = 1.$$

where density ρ is in veh/km and flow rates q_{en} and q_{side} are in veh/hr.

Similarly, for the second case, where all variables were considered in PCU units, the parameters for the free-flow regime were the same as that of the previous case except that the variables such as density and flow rates were considered in PCU units. In the congested regime, the parameters such as **W**, $\mathbf{H_1}$ and **V** remain same as that of the first case and the nonlinear function **f** and $\mathbf{A_1}$ was obtained as

$$\mathbf{f} = \begin{bmatrix} \rho(k) + \dfrac{h}{L}(q_{\text{en}}(k) - \rho(k) \cdot v_{\text{ex}}(k) + q_{\text{side}}(k)) \\ v(k) + ah\left(23.5\left(\dfrac{429}{\rho(k)} - 1\right) - v(k)\right) \\ -\dfrac{10081.5h}{L \cdot (\rho(k))^2}(q_{\text{en}}(k) - \rho(k) \cdot v_{\text{ex}}(k) \\ +q_{\text{side}}(k)) \end{bmatrix},$$

$$\mathbf{A_1} = \begin{bmatrix} 1 - \dfrac{h \cdot v_{\text{ex}}(k)}{L} & 0 \\ \dfrac{-10081.5h}{(\rho(k))^2}\left[a - \dfrac{2}{L \cdot \rho(k)}(q_{\text{en}}(k) + q_{\text{side}}(k)) + \dfrac{v_{\text{ex}}(k)}{L}\right] & 1 - ah \end{bmatrix},$$

where density ρ is in PCU/km and flow rates q_{en} and q_{side} are in PCU/hr.

For the third case, where different classes of vehicles were considered separately, the parameters in free-flow regime were obtained as

$$\mathbf{x}(k) = \begin{bmatrix} \rho_{\text{TW}}(k) \\ \rho_{\text{ThW}}(k) \\ \rho_{\text{FW}}(k) \\ v_{\text{TW}}(k) \\ v_{\text{ThW}}(k) \\ v_{\text{FW}}(k) \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} q_{\text{en}}^{\text{TW}} \\ q_{\text{side}}^{\text{TW}} \\ q_{\text{en}}^{\text{ThW}} \\ q_{\text{side}}^{\text{ThW}} \\ q_{\text{en}}^{\text{FW}} \\ q_{\text{side}}^{\text{FW}} \end{bmatrix}, \quad \mathbf{z}(k) = \begin{bmatrix} v_{\text{TW}}(k) \\ v_{\text{ThW}}(k) \\ v_{\text{FW}}(k) \end{bmatrix},$$

$$\mathbf{A} = \begin{bmatrix} 1 - \dfrac{h \cdot v_{\text{ex}}^{\text{TW}}}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 - \dfrac{h \cdot v_{\text{ex}}^{\text{ThW}}}{L} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 - \dfrac{h \cdot v_{\text{ex}}^{\text{FW}}}{L} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 - ah & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 - ah & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 - ah \end{bmatrix},$$

$$\mathbf{H} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

and in the congested regime for the same case, the nonlinear function **f** was obtained as

$$f = \begin{bmatrix} \rho_{\mathrm{TW}}(k) + \frac{h}{L}(q_{\mathrm{en}}^{\mathrm{TW}}(k) - \rho_{\mathrm{TW}}(k) \\ \cdot v_{\mathrm{ex}}^{\mathrm{TW}}(k) + q_{\mathrm{side}}^{\mathrm{TW}}(k)) \\ \rho_{\mathrm{ThW}}(k) + \frac{h}{L}(q_{\mathrm{en}}^{\mathrm{ThW}}(k) - \rho_{\mathrm{ThW}}(k) \\ \cdot v_{\mathrm{ex}}^{\mathrm{ThW}}(k) + q_{\mathrm{side}}^{\mathrm{ThW}}(k)) \\ \rho_{\mathrm{FW}}(k) + \frac{h}{L}(q_{\mathrm{en}}^{\mathrm{FW}}(k) - \rho_{\mathrm{FW}}(k) \\ \cdot v_{\mathrm{ex}}^{\mathrm{FW}}(k) + q_{\mathrm{side}}^{\mathrm{FW}}(k)) \\ v_{\mathrm{TW}}(k) + ah\left(18.316\left(\frac{315}{\rho_{\mathrm{TW}}(k)} - 1\right) - v_{\mathrm{TW}}(k)\right) \\ -\frac{5769.54h}{L \cdot (\rho_{\mathrm{TW}}(k))^2}(q_{\mathrm{en}}^{\mathrm{TW}}(k) - \rho_{\mathrm{TW}}(k) \\ \cdot v_{\mathrm{ex}}^{\mathrm{TW}}(k) + q_{\mathrm{side}}^{\mathrm{TW}}(k)) \\ v_{\mathrm{ThW}}(k) + ah\left(18.065\left(\frac{45}{\rho_{\mathrm{ThW}}(k)} - 1\right) - v_{\mathrm{ThW}}(k)\right) \\ -\frac{812.925h}{L \cdot (\rho_{\mathrm{ThW}}(k))^2}(q_{\mathrm{en}}^{\mathrm{ThW}}(k) - \rho_{\mathrm{ThW}}(k) \\ \cdot v_{\mathrm{ex}}^{\mathrm{ThW}}(k) + q_{\mathrm{side}}^{\mathrm{ThW}}(k)) \\ v_{\mathrm{FW}}(k) + ah\left(23.208\left(\frac{180}{\rho_{\mathrm{FW}}(k)} - 1\right) - v_{\mathrm{FW}}(k)\right) \\ -\frac{4177.44h}{L \cdot (\rho_{\mathrm{FW}}(k))^2}(q_{\mathrm{en}}^{\mathrm{FW}}(k) - \rho_{\mathrm{FW}}(k) \\ \cdot v_{\mathrm{ex}}^{\mathrm{FW}}(k) + q_{\mathrm{side}}^{\mathrm{FW}}(k)) \end{bmatrix}.$$

Other parameters such as $\mathbf{A_1}$, $\mathbf{W_1}$, $\mathbf{H_1}$ and $\mathbf{V}$ for this case were derived as before. The initial values of the state variables were assumed in all the three cases and the above estimation schemes were implemented. The results from these estimation schemes were compared with the actual values collected from the field.

4. Data collection and extraction

Data collection for the present study was carried out using the video recording technique on a stretch on the Rajiv Gandhi road, Chennai, India. The selected stretch was a six-lane roadway, with three lanes in each direction. For the present study, only one direction of traffic was considered. The section had one side road and the vehicles entering through it were counted manually. Video data were collected at the entry and exit points of the selected section of roadway during one hour each on five week days and two hours and three hours each on two other week days. The collected videos were later analysed in the laboratory to extract the required data. Data extraction was carried out manually due to lack of any reliable automated data extraction methods and was carried out separately for all the three classes of vehicles considered. The flow data passing the entry and exit sections were extracted for every one-minute interval by counting the number of TWs, ThWs and FWs travelling in all the three lanes. The spot speeds of TWs, ThWs and FWs passing the entry and exit sections were determined for every one minute by measuring the time taken to cross a known distance. The space mean speeds were then computed for these three classes by taking the harmonic mean (HM) of spot speeds (May, 1990). The average space mean speed values of vehicles were computed by averaging the HM values at the entry and exit sections. The actual densities of the three classes of vehicles at every one-minute interval required for corroboration of the estimation scheme were determined using input–output analysis (May, 1990). The initial numbers of TWs, ThWs and FWs present inside the section (required for the input output analysis) were measured by taking a still picture of the section at the start of the data collection. For implementing and corroborating the first scheme, data were considered without classifying them into different classes. In the case of the second scheme, PCU converted values were used and for the third scheme, the classified data were used.

5. Results

The estimated values of state variables were compared with the actual values obtained from field. The performance of this approach was quantified by calculating the mean absolute percentage error (MAPE) given by

$$\mathrm{MAPE} = \left[\frac{1}{N}\sum_{i=1}^{N}\frac{|x_{\mathrm{est}} - x_{\mathrm{obs}}|}{x_{\mathrm{obs}}}\right]100, \tag{33}$$

where x_{est} and x_{obs} are the estimated and observed values of the state variable, respectively, and N is the total number of observations. The estimated values of average space mean speed values were found to converge with the measured values in all the three cases. The plots of the estimated values of densities using PCU converted data against the actual values for two representative days are shown in Figures 2 and 3, respectively. The MAPE values for traffic density estimates for all days using the three different schemes are tabulated in Table 1.

According to Lewis' interpretation of MAPE results (Kenneth & Ronald, 1982), any forecast with a MAPE value of up to 10% is considered as highly accurate, 11–20% as good, 21–50% as reasonable and greater than 50% as weak and inaccurate. Based on this, it can be seen that the results are good for five out of the seven days and reasonable for the remaining two days for all the schemes.

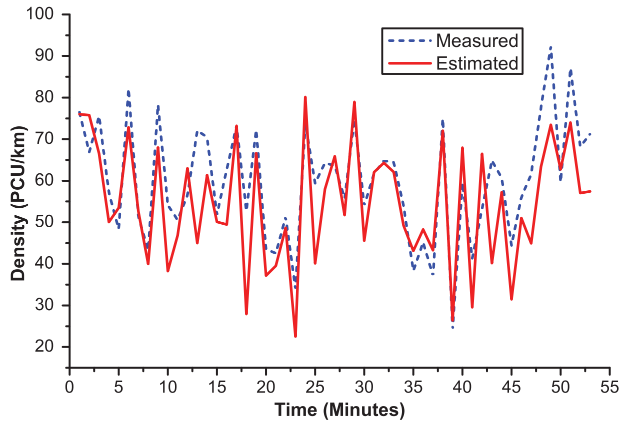

Figure 2. Comparison of the actual and estimated values of density of representative Day 1.

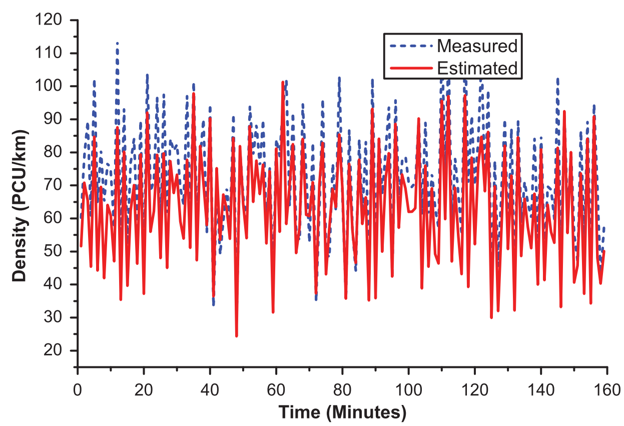

Figure 3. Comparison of the actual and estimated values of density of representative Day 2.

Table 1. MAPE for density estimation.

	MAPE (%)		
Day	Without considering heterogeneity	Heterogeneity introduced in terms of PCU	Heterogeneity by including different classes
Day 1	20.2	17.7	17.6
Day 2	13.1	11.6	14.0
Day 3	13.0	13.9	12.6
Day 4	24.1	23.8	23.7
Day 5	23.3	25.6	23.8
Day 6	15.1	14.2	14.6
Day 7	14.0	16.7	20.8

It can also be seen that introduction of heterogeneity in terms of PCU or considering different classes of vehicles separately improved results for a few days. However, the improvement may not be significant enough compared to the efforts involved in data collection and extraction. Considering the stochastic nature of Indian traffic, the performance of all the estimation schemes is promising.

6. Conclusions

A model-based approach was developed for the estimation of traffic states in Indian heterogeneous traffic conditions using the Kalman filtering approach. A non-continuum lumped-parameter macroscopic model was proposed using the law of conservation of vehicles and a dynamic speed equation incorporating an empirically developed traffic stream model. Traffic density and aggregate space mean speed were identified as the state variables of interest. The scheme was implemented initially without considering heterogeneity and then heterogeneity was incorporated in terms of standard PCU units as well as by explicitly considering different categories of vehicles into the modelling and estimation processes. It was observed that converting heterogeneous traffic into a homogeneous equivalent using constant values of PCU was a good representation in the proposed non-continuum macroscopic modelling approach, when compared with the alternative approach of including different categories of vehicles in the model. The model-based scheme with speed as measurement was performing better under all traffic conditions. The estimation scheme was corroborated using data collected from a road stretch in Chennai, India, using the videographic technique. The results obtained were compared with the actual values and were found to be promising. The space mean speed estimates can be used to estimate and predict the travel time of vehicles in a given road stretch and traffic density is useful in the prediction of congestion. An advantage of this scheme is that it can be easily integrated with real-time data, if available, and can be used for real-time implementations. Successful implementation of such systems will help in better management of traffic through real-time ITS applications. The dynamic model developed in this study can also be used to develop control schemes for regulating the flow of traffic on Indian roads.

Overall, the main contributions of this study are:

- This study is one of the first steps towards dynamic modelling of Indian Traffic by treating it as a non-continuum and applying it for real-time traffic state estimation.
- The developed traffic stream model is one of the first stream models for Indian urban road traffic conditions that helped in the development of a dynamic macroscopic model applied to Indian traffic conditions.
- The comprehensive non-continuum macroscopic model developed for describing the flow of traffic will overcome the limitations in modelling traffic as a continuum and will be useful in characterizing Indian traffic.
- The developed model-based scheme for real-time estimation of traffic density using the Kalman filtering technique can take into account the variability

arising from heterogeneity to a large extent, leading to better prediction accuracy.

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Disclosure statement

No potential conflict of interest was reported by the authors.

References

Ajitha, T., & Vanajakshi, L. (2012, January). *Development of optimized traffic stream models under heterogeneous traffic conditions*. Proc. 91st transportation research board annual meeting, Washington, DC, USA.

Anand, R. A., Vanajakshi, L., & Subramanian, S. C. (2011, June). *Traffic density estimation under heterogeneous traffic conditions using data fusion*. Intelligent vehicles symposium (IV), 2011 IEEE, pp. 31–36.

Chakroborty, P., & Kikuchi, S. (1999). Evaluation of the general motors based car-following models and a proposed fuzzy inference model. *Transportation Research C: Emerging Technologies*, *7*, 209–235.

Chakroborty, P., & Maurya, A. K. (2008). Microscopic analysis of cellular automata based traffic flow models and an improved model. *Transport Reviews*, *28*, 717–734.

Chu, L., Oh, S., & Recker, W. (2005, January). *Adaptive Kalman filter based freeway travel time estimation*. 84th TRB annual meeting, Washington, DC, USA.

Coddington, E. A. (1989). *An introduction to ordinary differential equations*. New York, NY: Dover Publications.

Daganzo, C. F. (1994a). The cell transmission model: A dynamic representation of highway traffic consistent with hydrodynamic theory. *Transportation Research B: Methodological*, *28*(4), 269–287.

Daganzo, C. F. (1994b). The cell transmission model, part II: Network traffic. *Transportation Research B*, *29*(2), 279–293.

Gartner, N., Messer, C. J., & Rathi, A. K. (2001). *Traffic flow theory a state-of-the-art report* (Transportation Research Board Special Report 165). Wasington, DC: Springer-Verlag.

Gazis, D. C., & Knapp, C. H. (1971). On-line estimation of traffic densities from time-series of flow and speed data. *Transportation Science*, *5*(3), 283–301.

Gazis, D. C., & Szeto, M. W. (1974). *Design of density-measuring systems for roadways*. Transportation Research Record 495, Transportation Research Board, pp. 44–52 (No. HS-015 791).

Gopal, M. (1984). *Modern control system theory*. New York, NY: John Wiley and Sons.

Gupta, S., Chakroborty, P., & Mukherjee, A. (1998). *Microscopic simulation of vehicular traffic on congested roads*. Proceedings of the international symposium on Industrial Robotic Systems-98, Bangalore, India.

Hegyi, A., Girimonte, D., Babuska, R., & De Schutter, B. (2006, September). *A comparison of filter configurations for freeway traffic state estimation*. Intelligent transportation systems conference. ITSC'06, Toronto, Canada, IEEE, pp. 1029–1034.

Hoogendoorn, S. P., & Bovy, P. H. L. (2000). Continuum modeling of multiclass traffic flow. *Transportation Research Part B: Methodological*, *34*, 123–146.

Hoogerdoorn, S. P., & Bovy, P. H. L. (2001). State-of-the-art of vehicular traffic flow modelling. Proceedings of Institution of Mechanical Engineers, Part I. *Journal of Systems and Control Engineering*, *215*, 283–304.

Houpt, P., Kurkjian, A., Gershwin, S. B., & Willsky, A. S. (1979, December). *Estimation of roadway traffic density on freeways using presence detector data*. Decision and control including the symposium on adaptive processes, 1979 18th IEEE, 18, pp. 524–525.

Ioannou, P. A., & Chien, C. C. (1993). Autonomous intelligent cruise control. *IEEE Transactions on Vehicular Technology*, *42*(4), 657–672.

IRC: 106. (1990). *Guidelines for capacity of urban roads in plain areas*. New Delhi: IRC.

Jabari, S. E., & Liu, H. X. (2013). A stochastic model of traffic flow: Gaussian approximation and estimation. *Transportation Research B: Methodological*, *47*, 15–41.

Jazwinsky, A. H. (1970). *Stochastic processes and filtering theory*. New York, NY: Academic Press.

Kalman, R. E. (1960). A new approach to linear filtering and prediction problems. *Journal of Basic Engineering*, *82*, 35–45.

Kenneth, D. L., & Ronald, K. K. (1982). *Advances in business and management forecasting*. London: Emerald Books.

Kerner, B. S., & Klenov, S. L. (2004). Spatial–temporal patterns in heterogeneous traffic flow with a variety of driver behavioural characteristics and vehicle parameters. *Journal of Physics A: Mathematical and General*, *37*, 8753–8788.

Kurkjian, A., Gershwin, S. B., Houpt, P. K., Willsky, A. S., Chow, E. Y., & Greene, C. S. (1980). Estimation of roadway traffic density on freeways using presence detector data. *Transportation Science*, *14*(3), 232–261.

Logghe, S., & Immers, L. H. (2003). *Heterogeneous traffic flow modeling with the LWR model using passenger–car equivalents*. Proc.10th world congress on ITS, Madrid.

Mallikarjuna, C., & Rao, K. R. (2009). Cellular automata model for heterogeneous traffic. *Journal of Advanced Transportation*, *43*, 321–345.

Mathew, T. V., Gundaliya, P., & Dhingra, S. L. (2006, August). *Heterogeneous traffic flow modeling and simulation using cellular automata*. Proceedings of 9th international conference on Applications of Advanced Technology in Transportation, Chicago.

Maurya, A. K., & Chakroborty, P. (2008, September). *Microscopic model for simulation of uninterrupted mixed traffic streams without lane discipline*. Proceedings of the international conference on Best Practices to Relieve Congestion on Mixed-Traffic Urban Streets in Developing Countries, Chennai, India.

May, A. D. (1990). *Traffic flow fundamentals*. Englewood Cliffs, NJ: Prentice Hall.

Nahi, N. E., & Trivedi, A. N. (1973). Recursive estimation of traffic variables: Section density and average speed. *Transportation Science*, *7*(3), 269–286.

Nanthawichit, C., Nakatsuji, T., & Suzuki, H. (2003, January). *Application of probe vehicle data for real time traffic state estimation and short term travel time prediction on a freeway*. 82nd transportation research board annual meeting, Washington, DC, USA.

Okutani, I., & Stephanedes, Y. J. (1984). Dynamic prediction of traffic volume through Kalman filtering theory. *Transportation Research B: Methodological*, *18*, 1–11.

Padiath, A., Vanajakshi, L., Subramanian, S. C., & Manda, H. (2009, October). *Prediction of traffic density for congestion analysis under Indian traffic conditions*. Intelligent transportation systems, 2009. ITSC'09. 12th International IEEE, pp. 1–6.

Padiath, A. S., Vanajakshi, L. D., & Subramanian, S. C. (2010, January). *Estimation of traffic density under Indian traffic conditions*. Transportation research board 89th annual meeting (No. 10–2877), Washington, DC, USA.

Papageorgiou, M. (1998). Some remarks on macroscopic traffic flow modeling. *Transportation Research A*, *32*, 323–329.

Swaroop, D., Hedrick, J. K., Chien, C. C., & Ioannou, P. (1994). A comparison of spacing and headway control laws for automatically controlled vehicles. *Vehicle System Dynamics*, *23*, 597–625.

Sun, X., Muñoz, L., & Horowitz, R. (2004, June). *Mixture Kalman filter based highway congestion mode and vehicle density estimator and its application*. American control conference, Boston, USA. IEEE, 3, pp. 2098–2103.

Tang, Q., Huangb, H. J., Zhao, S. G., & Shang, H. Y. (2009). A new dynamic model for heterogeneous traffic flow. *Physics Letters A*, *373*, 2461–2466.

Tiwari, G., Fazio, J., Gaurav, S., & Chatteerjee, N. (2008). Continuity equation validation for nonhomogeneous traffic. *Journal of Transportation Engineering ASCE*, *134*, 118–127.

Tyagi, V., Darbha, S., & Rajagopal, K. R. (2008). A dynamical systems approach based on averaging to model the macroscopic flow of freeway traffic. *Nonlinear Analysis: Hybrid Systems*, *2*, 590–612.

Tyagi, V., Darbha, S., & Rajagopal, K. R. (2009). A review of the mathematical models for traffic flow. *International Journal of Advances in Engineering Sciences and Applied Mathematics*, *1*, 53–68.

Venkatesan, K., Gowri, A., & Sivanandan, R. (2008). Development of microscopic simulation model for heterogeneous traffic using object oriented approach. *Transportmetrica*, *4*(3), 227–247.

Wang, H., Li, J., Chen, Q. Y., & Ni, D. (2011). Logistic modeling of the equilibrium speed-density relationship. *Transportation Research Part B*, *45*, 554–566.

Wang, Y., & Papageorgiou, M. (2005). Real-time freeway traffic state estimation based on extended Kalman filter: A general approach. *Transportation Research Part B: Methodological*, *39*(2), 141–167.

Wang, Y., Papageorgiou, M., & Messmer, A. (2007). Real-time freeway traffic state estimation based on extended Kalman filter: A case study. *Transportation Science*, *41*(2), 167–181.

Wang, Y., Papageorgiou, M., & Messmer, A. (2008). Real-time freeway traffic state estimation based on extended Kalman filter: Adaptive capabilities and real data testing. *Transportation Research Part A: Policy and Practice*, *42*(10), 1340–1358.

Willsky, A. S., Chow, E., Gershwin, S., Greene, C., Houpt, P., & Kurkjian, A. (1980). Dynamic model-based techniques for the detection of incidents on freeways. *IEEE Transactions on Automatic Control*, *25*(3), 347–360.

Zhongke, S. (2003, October). *A new estimation method for multi-section traffic states of freeway*. Intelligent transportation systems, 2003. IEEE, 1, pp. 618–623.