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A mixed integer linear programming model for the vehicle routing problem with simultaneous delivery and pickup by heterogeneous vehicles, and constrained by time windows

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Abstract. In this work, we consider the Vehicle Routing Problem with Simultaneous Delivery and Pickup, and constrained by time windows, to improve the performance and responsiveness of the supply chain by transporting goods from one location to another location in an efficient manner. In this class of problem, each customer demands a quantity to be delivered as a part of the forward supply service and another quantity to be picked up as a part of the reverse recycling service, and the complete service has to be done simultaneously in a single visit of a vehicle, and the objective is to minimize the total cost, which includes the traveling cost and dispatching cost for operating vehicles. We propose a Mixed Integer Linear Programming (MILP) model for solving this class of problem. In order to evaluate the performance of the proposed MILP model, a comparison study is made between the proposed MILP model and an existing MILP model available in the literature, with the consideration of heterogeneous vehicles. Our study indicates that the proposed MILP model gives tighter lower bound and also performs better in terms of the execution time to solve each of the randomly generated problem instances, in comparison with the existing MILP model. In addition, we also compare the proposed MILP model (assuming homogeneous vehicles) with the existing MILP model that also considers homogeneous vehicles. The results of the computational evaluation indicate that the proposed MILP model gives much tighter lower bound, and it is competitive to the existing MILP model in terms of the execution time to solve each of the randomly generated problem instances.

Keywords. Supply chain; transportation; vehicle routing problem; simultaneous delivery and pickup; time windows; integer programming model.

1. Introduction

Logistics plays an important role in supply chain, and the three main drivers of logistics that contribute to supply chain profitability/surplus are facilities, inventory and transportation; among these drivers, transportation focuses on moving the inventory from one facility to another facility in the supply chain [1]. In this work, we study the Vehicle Routing Problem (VRP) with the consideration of simultaneous delivery and pickup for each customer, and the service to the customer is further constrained by time windows. We consider this class of problem to identify the cost-effective routes to improve the performance of the supply chain. In the basic VRP, the objective of the problem is to find the set of routes with minimum cost for the given vehicles with finite capacity, so that each customer is served with the respective demand. We refer Toth and Vigo [2] for review of the basic problem and its variants.

The study focuses on the Vehicle Routing Problem with Simultaneous Delivery and Pickup, and constrained by time windows (VRPSDPTW), and the VRPSDPTW is a special case of the VRP that comes under the class of Vehicle Routing Problems with Backhauls (VRPB). The VRPB can be further classified into four types (based on the customer requirements as regards delivery and pickup). In VRPB, shipment/goods to be delivered to linehaul customers are loaded at the depot, and goods to be picked up from backhaul customers are transported to the depot.

The first subclass deals with the problem where a vehicle has to serve the group of linehaul/delivery customers before any backhaul/pickup customer, and the problem is referred to as the Vehicle Routing Problem with Clustered Backhauls (VRPCB). The second subclass deals with the problem where a vehicle can serve customers in a mixed order, and the problem is referred to as the Vehicle Routing Problem with Mixed Backhauls (VRPMB). In both VRPCB and VRPMB, customers are of either pickup or delivery customers, and cannot be of both categories. In the next two

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subclasses, each customer requires both the services (delivery and pickup). The third subclass deals with the problem where each customer is associated with both pickup quantity and delivery quantity, and the vehicle can visit each customer more than once, and the problem is referred to as the Vehicle Routing Problem with Divisible Delivery and Pickup (VRPDDP). The fourth subclass deals with the problem where each customer is associated with both pickup quantity and delivery quantity, and every customer has to be visited only once, and the problem is referred to as the Vehicle Routing Problem with Simultaneous Delivery and Pickup (VRPSDP). For a detailed survey of VRPB and its variants, we refer to the survey article by Parragh *et al* [3].

We develop a Mixed Integer Linear Programming (MILP) model that is computationally more efficient (in relation to existing MILP models) for solving the VRPSDPTW; in order to save space in the paper, we specifically present an overview of the literature pertaining to MILP models for addressing the VRPSDPTW, even though many attempts have concentrated on the development of heuristic approaches in view of the problem being NP-hard [4].

Min [5] first studied the possibility of having simultaneous delivery and pickup at the same customer node, developed an MILP model and a solution procedure to handle the VRPSDP, and validated the model and its performance through a case study dealing with public library distribution system. Dethloff [6] studied the VRPSDP problem in which a fleet of identical vehicles in terms of the capacity were assumed to be available at the depot, and proposed an MILP model for solving the same. Ganesh and Narendran [7] studied the Travelling Salesman Problem (TSP) with the consideration of simultaneous delivery and pickup at each customer node, and discussed the various applications of this class of problem. They proposed an MILP model for solving this problem, discussed the complexity of the problem and classified this problem as NP-complete.

Ai and Kachitvichyanukul [8] studied the VRPSDP and proposed an MILP model as an extension to the basic VRP, and they claimed that this model is a generalized version of the models proposed previously in the literature for the VRPSDP. Subsequently, Wang and Chen [4] suggested that the model proposed by Ai and Kachitvichyanukul [8] could be simplified by reducing the redundant constraints and variables, and they proposed an MILP model and a genetic algorithm for solving the VRPSDPTW.

Liu *et al* [9] studied a specific problem in health care domain where drugs and medical devices were considered for delivery service, and biological samples, medical wastes and unused drugs were considered for pickup service, and they proposed MILP models for solving this specific problem. Wang *et al* [10] considered the MILP model for the VRPSDPTW in the case of identical vehicles, and the model was derived from Dethloff [6] and Kallehauge *et al* [11], and they also proposed an algorithm based on simulated annealing for solving the same.

Polat *et al* [12] proposed an MILP model and a variable neighbourhood search heuristic for solving the VRPSDP with Time Limit (VRPSDPTL) in which each customer can be visited at any point in time (but only once), and the vehicles need to be returned to the depot within the time limit. They extended this problem to additionally consider the service time with respect to each and every customer. In their work they assumed that a fleet of homogeneous vehicles is available at the depot.

In this work, we propose an MILP model for the VRPSDPTW and compare the efficiency of the model in terms of the CPU time to find the optimal solution, and also in terms of the lower bound that we obtain through Linear Programming (LP) relaxation. For this purpose, we choose the most appropriate models in terms of relevance and recency; we first carry out a computational experiment between our MILP model and the MILP model by Wang and Chen [4] by considering the heterogeneous vehicles in terms of capacity and cost of vehicles, and then we carry out a computational experiment between our MILP model and the MILP model by Polat *et al* [12] by considering the homogeneous vehicles. The results confirm that our model is able to perform better with respect to CPU time to find the optimal solution and also in terms of the lower bound that we obtain through LP relaxation, based on the problem instances considered in the comparison study.

2. Findings from the literature

The VRPSDPTW has been studied in the past by various researchers; the generalized MILP models were studied/modelled for solving the VRPSDPTW with the consideration of heterogeneous vehicles [4, 8], and the MILP models focusing on homogeneous vehicles were also studied [6, 10, 12]; a few attempts have also been made on the MILP models for solving the specific domain/problem [9]; as the problem is NP-hard, many attempts have been focusing on efficient heuristic and meta-heuristic algorithms for solving this class of problem.

The study on the generalized MILP model for the VRPSDPTW with the consideration of heterogeneous vehicles (in terms of capacity/cost), with the focus on more efficient MILP models in terms of execution time and lower bound (through LP relaxation), is very less in the literature. These additional aspects with respect to heterogeneous vehicles, simultaneous delivery and pickup, and time windows with respect to depot and each customer, make the problem difficult to mathematically model to be computationally more efficient in terms of CPU time and also to be better in terms of the lower bound through LP relaxation.

3. Contributions of the study

The contributions of this work are four-fold.

- The work proposes an efficient MILP model for solving the VRPSDPTW.

We propose an MILP model for the VRPSDPTW and compare the efficiency of the model in terms of the CPU time to find an optimal solution, and the model uses node-specific variables to model the capacity and time window constraints, without using vehicle index on the decision variable that tracks the use/existence of a particular arc (in the transportation network) in the final solution; this approach helps in reducing the number of binary variables, as against the model by Wang and Chen [4] for solving the VRPSDPTW, and also against the model by Polat *et al* [12] for solving the VRPSDPTL.

- The work reports better performance of the proposed MILP model against the models available in the literature, in terms of the CPU time, to find the optimal solution.

In order to evaluate the performance of the proposed MILP model, a comparison study is made between the proposed MILP model and the MILP model available in literature for the VRPSDPTW [4] with the consideration of heterogeneous vehicles. Our study indicates that the proposed MILP model performs better in terms of the execution time to solve each of the randomly generated problem instances, compared with the MILP model developed by Wang and Chen [4]; the respective results are presented in tables 7 and 8 (in section 7.1).

We also carry out a comparison study between the proposed MILP model and the MILP model available in literature for the VRPSDPTL [12] with the consideration of homogeneous vehicles. Our study indicates that the proposed MILP model is competitive in terms of the execution time to solve each of the randomly generated problem instances, compared with the MILP model developed by Polat *et al* [12]; the respective results are presented in table 9 (in section 7.2).

- The work reports the superior performance of the proposed MILP model against the models available in the literature, in terms of the lower bound that the models generate through LP relaxation.

Our study indicates that the proposed MILP model (with the consideration of heterogeneous vehicles) gives superior lower bound (through LP relaxation, when we relax the binary constraints on the respective models) compared with the MILP model developed by Wang and Chen [4]; the respective results are presented in tables 7 and 8 (in section 7.1). The study also indicates that the proposed MILP model (with the consideration of homogeneous vehicles) reports tighter lower bound (through LP relaxation, when we relax the binary constraints on the respective models), compared with the MILP model developed by Polat *et al* [12]; the respective results are presented in table 9 (in section 7.2).

- The work also discusses a specific scenario with respect to the feasibility of existing MILP model by Wang and Chen [4], and proposes a modification to resolve the feasibility issue.

4. Problem description and assumptions

The VRPSDPTW is a routing problem with additional constraints in which a customer requires a simultaneous service of delivery and pickup, the complete service has to be done on a single visit by a vehicle and also the service has to be offered in the time window preferred by the respective customer. This additional aspect of considering the specific time window for each customer enables the customer to accept/receive the service at a preferred/convenient time window.

The objective of the problem is to find optimal routes in terms of routing cost such that the cumulative load of the vehicle is maintained below its capacity for every customer and also at the depot; the objective is also to route/schedule the vehicle such that the service is offered at the preferred time window specified by each customer.

The VRPSDPTW is generally defined as follows: customers are given, and each customer requires a given shipment to be delivered as a part of the forward supply service and another load to be picked up as a part of the reverse recycling service, on the same visit to the customer within a defined time period. A set of vehicles (with capacity restrictions) is ready to provide the service to the customers at the distribution centre (DC); each vehicle can be assigned at most to a single route as assumed in VRP class of problems [2]; each vehicle begins the trip from DC, provides service to the customers in the route and completes the trip at collection centre (CC). The objective of the problem is to minimize the number of vehicles and total travelling costs in order to satisfy both pickup service and delivery service such that the total load of the vehicle is below the vehicle capacity at each and every point in the route.

The VRPSDPTW is illustrated in figure 1. In this example, two vehicles are available at the depot with different capacities; the pickup/delivery service has to be offered to 7 customers, and each customer has a preferred time period/window in which the service has to be offered; in this example, vehicle 1 starts from the depot, provides the service to customer 5, then proceeds to customer 2, subsequently visits customer 6, finally provides the service to customer 1 and returns to the depot; vehicle 2 starts from the depot, provides the service to customer 7, subsequently

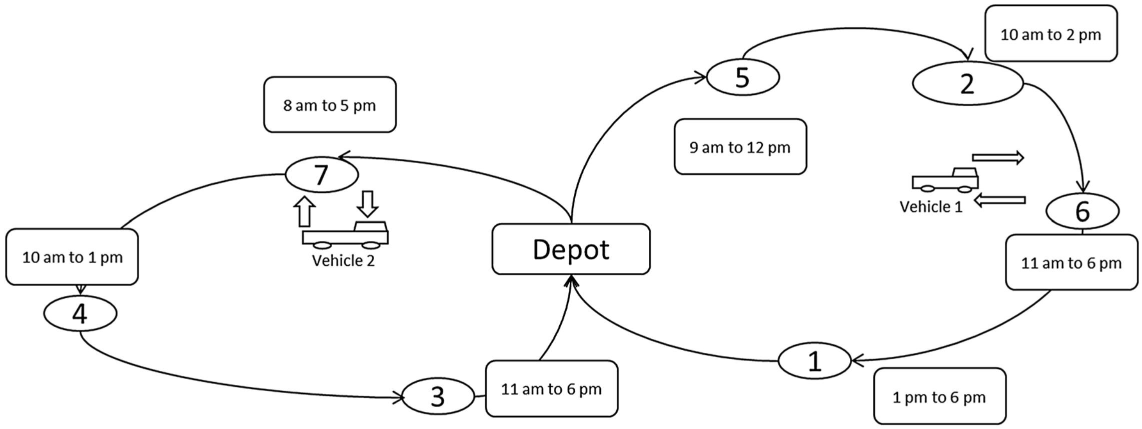

Figure 1. Illustrative example for the VRPSDPTW.

provides the service to customer 4, finally offers service to customer 3 and returns to the depot.

In section 5, we propose an MILP model for the VRPSDPTW; in section 6, we discuss the salient features of the proposed MILP model and the advantages of the proposed MILP model against the models available in the literature; subsequently, in section 7, we present a comparison study of our MILP model with the models available in the literature for the VRPSDPTW and VRPSDPTL.

5. Proposed MILP model for the VRPSDPTW

5.1 *Notations*

N	total number of customers; DC/CC is represented by node 0
V	total number of available vehicles
$c_{i,j}$	travel distance (i.e., routing cost) between node i and j
$t_{i,j}$	travelling time between node i and j
a^0	earliest start time of any vehicle from DC
a^i	earliest start time of service at customer i
b^0	latest arrival time of any vehicle to CC
b^i	latest start time of service at customer i
C^k	capacity of vehicle k (i.e., heterogeneous vehicles are assumed)
s_i	service time of customer i
p_i	pickup quantity of customer i
d_i	delivery quantity of customer i
$fixed_k$	dispatching cost of vehicle k, a fixed/overhead cost
α	a constant representing the trade-off between dispatching cost and routing cost
node $N+k$	a fictitious start node with respect to vehicle k, acting corresponding to DC as the start of the route; we have $c_{N+k,j} = c_{0,j}, \forall j, k$ and $t_{N+k,j} = t_{0,j}, \forall j, k$
node $N+V+k$	a fictitious end node with respect to vehicle k, acting corresponding to CC as the end of the route; we have $c_{i,N+V+k} = c_{i,0}, \forall i, k$ and $t_{i,N+V+k} = t_{i,0}, \forall i, k$
$M1$	a large value, and its value is defined as follows: $(\max_{0 \le i \le N, 0 \le j \le n} c_{i,j}) \times (N+2)$
$M2$	a large value, and its value is defined as follows: $\left(\max_{0 \le i \le N, 0 \le j \le n} t_{i,j}\right) \times (N+2) + \sum_{i=1}^{N} s_i$
$M3$	a large value, and its value is defined as follows: $2 \times (\max_{1 \le k \le V} c^k)$

5.2 *Decision variables*

$x_{i,j}$	assigned with the value 1 when the arc between customer i and customer j is selected as a part of the routing plan, 0 otherwise
$x_{N+k,j}$	assigned with the value 1 when the arc between node $N+k$ and customer j is selected as a part of the routing plan, 0 otherwise
$x_{i,N+V+k}$	assigned with the value 1 when the arc between customer i and node $N+V+k$ is selected as a part of the routing plan, 0 otherwise
$del_{i,k}$	assigned with the value 1 when customer i is assigned to vehicle k, 0 otherwise
$delv_k$	assigned with the value 1 when vehicle k is used in the solution, 0 otherwise
$load_k^0$	load of vehicle k when it starts from DC
$start_k^0$	starting time of vehicle k when it starts from DC
ld_i	load on vehicle after completing the service at customer i
$dist_i$	total distance travelled up to customer i
st_i	starting time of the service at customer i

The proposed MILP model uses node-specific variables to model the load of the vehicle and time window constraints, and uses three sets of two-dimensional binary variables to track the routes and their associated vehicles.

The proposed MILP model also uses fictitious node for each vehicle with respect to DC and CC in order to model the start of the trip from DC and end of the trip to DC. The objective of the problem is to find the optimal routing plan with respect to the total cost, which includes the routing cost and the fixed cost for the vehicles, as given in Eq. (1) (the same as that given in Wang and Chen [4]).

Minimize

$$Z = \alpha \times \sum_{i=1}^{N}\sum_{j=1}^{N}(x_{i,j} \times c_{i,j}) + \alpha \times \sum_{j=1}^{N}\sum_{k=1}^{V}\left(x_{N+k,j} \times c_{0,j}\right) + \alpha \times \sum_{i=1}^{N}\sum_{k=1}^{V}\left(x_{i,N+V+k} \times c_{i,0}\right) + (1-\alpha) \times \sum_{k=1}^{V}(delv_k \times fixed_k) \quad (1)$$

subject to the following constraints.

Constraint (2) restricts that each customer should be allocated to only a single vehicle:

$$\sum_{k=1}^{V} del_{i,k} = 1,. \quad i = 1,2,\ldots\ldots,N \quad (2)$$

Constraints (3) and (4) are introduced with respect to the fictitious nodes (specific for each vehicle), in order to allocate them to the corresponding vehicle:

$$del_{N+k,k} = 1, \quad k = 1,2,\ldots\ldots,V \quad (3)$$

$$del_{N+V+k,k} = 1, \quad k = 1,2,\ldots\ldots,V \quad (4)$$

Constraint (5) represents that a customer can be allocated to a vehicle only when the corresponding vehicle is used in the final solution:

$$del_{i,k} \le delv_k, \quad i = 1,2,\ldots\ldots,N \text{ and } k = 1,2,\ldots\ldots,V \quad (5)$$

Constraints (6)–(9) ensure that the customers belonging to the same route are allocated to the same vehicle; specifically, Constraints (6)–(8) ensure that when the route is directly between two nodes in the transportation network, then they should be allocated to the same vehicle (for example, $del_{i,k} = del_{j,k}$). If nodes i and j are not allocated to the same vehicle k, then $x_{i,j} = 0$; however, if they are allocated to the same vehicle k, then $x_{i,j}$ can be 0 or 1, based on the routing plan.

$$del_{i,k} - del_{j,k} \le \left(1 - x_{i,j}\right), \quad i,j = 1,2,\ldots,N, \ i \ne j \text{ and } k = 1,2,\ldots,V \quad (6)$$

$$del_{N+k,k} - del_{i,k} \le \left(1 - x_{N+k,i}\right), \quad i = 1,2,\ldots,N \text{ and } k = 1,2,\ldots,V \quad (7)$$

$$del_{N+V+k,k} - del_{i,k} \le \left(1 - x_{i,N+V+k}\right), \ i = 1,2,\ldots,N \text{ and } k = 1,2,\ldots,V \quad (8)$$

$$del_{i,k} \le 1 - x_{N+k,N+V+k}, i = 1,2,\ldots,N \text{ and } k = 1,2,\ldots,V \quad (9)$$

Constraints (10) and (11) ensure that either each vehicle starts from a depot to serve a particular customer, or it is not being utilized, and thereby the vehicle directly makes a visit between fictitious nodes:

$$\sum_{j=1}^{N} x_{N+k,j} + x_{N+k,N+V+k} = 1, \quad k = 1,2,\ldots,V \quad (10)$$

$$\sum_{i=1}^{N} x_{i,N+V+k} + x_{N+k,N+V+k} = 1, \ k = 1,2,\ldots,V \quad (11)$$

Constraints (12) and (13) indicate that the total number of arcs selected in the final solution, before and after visiting a customer/fictitious node, should be equal to 1:

$$\sum_{j=1}^{N} x_{i,j} + \sum_{k=1}^{V} x_{i,N+V+k} = 1, \quad i = 1,2,\ldots,N \quad (12)$$

$$\sum_{i=1}^{N} x_{i,j} + \sum_{k=1}^{V} x_{N+k,j} = 1, \quad j = 1,2,\ldots,N \quad (13)$$

Constraint (14) ensures that either the vehicle is being utilized for providing service to the customer or the vehicle directly makes a visit between fictitious nodes:

$$\sum_{i=1}^{N} del_{i,k} + x_{N+k,N+V+k} \ge 1, \quad k = 1,2,\ldots,V \quad (14)$$

The approach of having a separate set of fictitious nodes, specific to each vehicle for DC and CC, helps modelling Constraints (15) and (16). These constraints make sure that when a customer is visited as first/last customer in the route, then the respective customer is also allocated to the same vehicle.

$$x_{N+k,i} \le del_{i,k}, \ i = 1,2,\ldots,N \text{ and } k = 1,2,\ldots,V \quad (15)$$

$$x_{i,N+V+k} \le del_{i,k}, \quad i = 1,2,\ldots,N \text{ and } k = 1,2,\ldots,V \quad (16)$$

Constraints (17)–(21) are introduced to restrict the sub-tours in the route sequence in addition to the distance-related constraints (Constraints (22)–(24)), which further governs the sub-tour elimination:

$$x_{i,i} = 0, \quad i = 1,2,\ldots,N \quad (17)$$

$$x_{i,j} + x_{j,i} \le 1, \quad i = 1,2,\ldots,N-1 \text{ and } j = i+1, i+2,\ldots,N \quad (18)$$

$$x_{N+V+k,j} = 0,\ j = 1,2,\ldots,N \text{ and } k = 1,2,\ldots,V \quad (19)$$

$$x_{i,N+k} = 0,\ i = 1,2,\ldots,N \text{ and } k = 1,2,\ldots,V \quad (20)$$

$$x_{N+V+k,N+k} = 0,\ \ k = 1,2,\ldots,V \quad (21)$$

Constraints (22)–(24) represent that if the arc between two nodes is selected/available as a part of the travel plan, then the distance travelled to reach the second node from the first node is equal to the travel distance between them:

$$dist_i \geq c_{0,i} - M1 \times \left(1 - x_{N+k,i}\right),\ \ i = 1,2,\ldots,N \text{ and } k = 1,2,\ldots,V \quad (22)$$

$$dist_j \geq dist_i + c_{i,j} - M1 \times \left(1 - x_{i,j}\right),\ \ i,j = 1,2,\ldots,N \text{ and } i \neq j \quad (23)$$

$$dist_{N+V+k} \geq dist_i + c_{i,0} - M1 \times \left(1 - x_{i,N+V+k}\right),\ \ i = 1,2,\ldots,N \text{ and } k = 1,2,\ldots,V \quad (24)$$

Constraints (25)–(27) denote that if the arc between customers i and j is selected as a part of the routing plan, then the time to provide service at customer j should be later than the time to provide service at customer i, and should be more than the time to travel from customer i to customer j:

$$st_i \geq start_k^0 + t_{0,i} - M2 \times \left(1 - x_{N+k,i}\right),\ \ i = 1,2,\ldots,N \text{ and } k = 1,2,\ldots,V \quad (25)$$

$$st_j \geq st_i + s_i + t_{i,j} - M2 \times \left(1 - x_{i,j}\right),\ \ i,j = 1,2,\ldots,N \text{ and } i \neq j \quad (26)$$

$$st_i + s_i + t_{i,0} \times x_{i,N+V+k} \leq b^0,\ \ i = 1,2,\ldots,N \text{ and } k = 1,2,\ldots,V \quad (27)$$

Constraint (28) ensures that the load of a vehicle at any point in route should be less than the capacity of the vehicle:

$$ld_i \leq \sum_{k=1}^{V} del_{i,k} \times C^k,\quad i = 1,2,\ldots,N \quad (28)$$

Constraint (29) limits the initial load of the vehicle when it starts from the DC:

$$load_k^0 = \sum_{i=1}^{N} del_{i,k} \times d_i,\ \ k = 1,2,\ldots,V \quad (29)$$

Constraints (30) and (31) represent the cumulative load of a vehicle after visiting a particular customer:

$$ld_i \geq load_k^0 - d_i + p_i - M3 \times \left(1 - x_{N+k,i}\right),\ \ i = 1,2,\ldots,N \text{ and } k = 1,2,\ldots,V \quad (30)$$

$$ld_j \geq ld_i - d_j + p_j - M3 \times \left(1 - x_{i,j}\right),\ i,j = 1,2,\ldots,N \text{ and } i \neq j \quad (31)$$

Constraints (32)–(36) represent the bounds for the respective decision variables:

$$a^0 \leq start_k^0 \leq b^0,\ \ k = 1,2,\ldots,V \quad (32)$$

$$a^i \leq st_i \leq b^i,\ \ i = 1,2,\ldots,N \quad (33)$$

$$0 \leq load_k^0 \leq C^k,\ k = 1,2,\ldots,V \quad (34)$$

$$0 \leq ld_i \leq \max_{1 \leq k \leq V} C^k,\ \ i = 1,2,\ldots,N \quad (35)$$

$$0 \leq dist_i \leq M1,\ \ i = 1,2,\ldots,N \quad (36)$$

5.3 *Additional observations*

In the proposed MILP model, Constraints (25) and (26) are used to define the starting time of the service for each customer, and these constraints define only the lower limit for starting the service at the respective customer (as assumed/modelled by Wang and Chen [4]), and the same is observed and discussed in section 6.4; if the objective of the problem is also to get the exact schedule for providing the service for each customer, without inserting any waiting/idle time during the travel, then we have to define Constraints (37) and (38) to obtain the exact schedule for providing the service at the respective customer:

$$st_i \leq start_k^0 + t_{0,i} + M2 \times \left(1 - x_{N+k,i}\right),\ \ i = 1,2,\ldots,N \text{ and } k = 1,2,\ldots,V \quad (37)$$

$$st_j \leq st_i + s_i + t_{i,j} + M2 \times \left(1 - x_{i,j}\right),\ \ i,j = 1,2,\ldots,N \text{ and } i \neq j \quad (38)$$

6. Discussion on the proposed MILP model

In section 6.1, we discuss the salient features of the proposed MILP model, and in section 6.2, we discuss the advantages of the proposed MILP model over the MILP models by Wang and Chen [4] and Polat *et al* [12]. In section 6.3, we present a numerical example for the VRSDPTW; in section 6.4, we discuss the possible infeasibility with respect to the implementation of an existing MILP model by Wang and Chen [4] for a particular scenario, and propose a constraint to their model to address the feasibility issue.

6.1 *Salient features of the proposed MILP model*

We have modelled the VRPSDPTW using three sets of two-dimensional binary variables, and the purpose/usage of these variables is described in this section.

- In the proposed MILP model, the first set of binary variables (namely $x_{i,j};, x_{N+k,j}$ and $x_{i,N+V+k}$) is used to track the immediate precedence between customers in the final route.
- The second set of binary variables ($del_{i,k}$) is used to model the allocation of customers to specific vehicle, and its related constraints.
- The third set of binary variables ($delv_k$) is used to make a decision about the usage of a particular vehicle in the final solution.

These variables are linked through binding constraints (5) and (6)–(9), and these constraints also act as a set of tight constraints to produce a tighter lower bound (when we relax the binary variables through LP relaxation). Constraint (5) links the binary variables $del_{i,k}$ and $delv_k$ to make sure that a customer can be allocated to vehicle only when the corresponding vehicle is selected in the final solution. Constraints (6)–(9) link the binary variables $x_{i,j}$ and $del_{i,k}$ to make sure that whenever the arc between two customer is selected in the route, then the respective customers are also allocated to the same vehicle.

6.2 *Advantages of the proposed MILP model*

Following are the advantages of the proposed MILP model (for having three sets of two-dimensional binary variables) in comparison with the MILP model by Wang and Chen [4] and Polat *et al* [12]:

- In the proposed MILP model there are $V + (N + V \times 2)^2 + (N + V \times 2) \times V$ in total, out of which V binary variables ($delv_k$) are used to track usage of vehicle in the final solution, $(N + V \times 2)^2$ binary variables ($x_{i,j}$) are used to track the direct precedence between two customers in the route and $(N + V \times 2) \times V$ binary variables ($del_{i,k}$) are used to model the allocation-related constraints.
- In the MILP models by Wang and Chen [4] and Polat *et al* [12], there are $(N + 1)^2 \times V$ binary variables, which is mainly due to the fact that the same set of binary variables ($x_{i,j,k}$) has been used to track the direct precedence as well as to track the allocation of customers to vehicle.
- In the proposed MILP model, we use three different set of binary variables ($delv_k, del_{i,k}$ and $x_{i,j}$) and link them through the binding constrains (Constraints (5)–(9)), and this helps to reduce the number of binary variables significantly to $V + (N + V \times 2)^2 + (N + V \times 2) \times V$ from $(N + 1)^2 \times V$.
- In the proposed MILP model, the active number of binary variables with respect to fictitious nodes is further reduced using Constraints (19)–(21).
- The separate set of binary variables specific to vehicle usage ($delv_k$) in the proposed MILP model helps to obtain a tighter lower bound (through LP relaxation by relaxing the binary variables), in comparison with the lower bound that we obtain from the MILP models by Wang and Chen [4] and Polat *et al* [12]; this is because in the proposed MILP model, the entire set of variables ($x_{i,j}; x_{N+k,j}$ and $x_{i,N+V+k}$) in the final route is indirectly used to derive/determine the vehicle usage ($delv_k$), whereas only a single variable per route $\left(x_{0,j,k}\right)$ that tracks the direct precedence between DC and customer node is used to track the vehicle usage in the model by Wang and Chen [4]. This improvement in the lower bound also helps the solver to execute the proposed MILP model relatively faster, in comparison with the MILP model by Wang and Chen [4].

6.3 *A numerical example for the VRPSDPTW: proposed MILP model*

For the purpose of numerical illustration, we have taken a specific problem instance; details related to the co-ordinates of the nodes (in (X,Y) plane) are given in table 1, and the travel time/distance for any pair of nodes is calculated as the straight line distance between these nodes. The details specific to pickup and delivery quantities and time window and service time with respect to each customer are also presented in table 1. The details related to vehicle-specific parameters are presented in table 2. The cost of travel ($c_{i,j}$) between customer i and customer j is expressed in terms of time ($t_{i,j}$) in this problem instance.

When we execute the proposed MILP model for this example, two routes are formed; the corresponding values of the decision variables are presented in table 3; the Z value for the solution is 198.10 and we have assigned 0.5 as the value for the trade-off parameter α with respect to cost functions as assumed by Wang and Chen [4].

- Route for Vehicle 1 with respect to customer sequence is (9,10); the vehicle starts from depot, serves (simultaneous service of delivery and pickup) the set of customers {9,10}, in this sequence, and returns to depot.
- Route for Vehicle 2 with respect to customer sequence is (2,6,7,8,5,3,1,4); the vehicle starts from depot, serves (simultaneous service of delivery and pickup) the set of customers {2,6,7,8,5,3,1,4}, in this sequence, and returns to depot.

6.4 *Discussion of the existing MILP model by Wang and Chen [4], and our proposed modification*

We observe that the MILP model proposed by Wang and Chen [4] results in an infeasible solution in the scenario

Table 1. Problem instance details – customers.

Node	X	Y	Delivery quantity	Pickup quantity	Earliest start time	Latest arrival time	Service time
0	40	31	0	0	0	240	0
1	25	85	20	10	67	191	10
2	22	75	30	32	32	97	10
3	22	85	10	12	101	146	10
4	20	80	40	35	71	193	10
5	20	85	20	29	40	113	10
6	18	75	20	11	55	164	10
7	15	75	20	28	69	118	10
8	15	80	10	14	56	155	10
9	10	35	20	15	51	160	10
10	10	40	30	40	90	177	10

Table 2. Problem instance details – vehicles.

Vehicle	Dispatching cost	Capacity
1	94	200
2	104	200

Table 3. Values of the decision variables: proposed MILP model.

Decision variable	Value	Decision variable	Value
$x_{2,6}$	1	$del_{1,2}$	1
$x_{6,7}$	1	$del_{2,2}$	1
$x_{7,8}$	1	$del_{3,2}$	1
$x_{8,5}$	1	$del_{4,2}$	1
$x_{5,3}$	1	$del_{5,2}$	1
$x_{3,1}$	1	$del_{6,2}$	1
$x_{1,4}$	1	$del_{7,2}$	1
$x_{12,2}$	1	$del_{8,2}$	1
$x_{4,14}$	1	$del_{9,1}$	1
$x_{11,9}$	1	$del_{10,1}$	1
$x_{9,10}$	1	Z	198.10
$x_{10,13}$			

where the dispatching cost of a particular vehicle is less compared with other vehicles and the capacity of that particular vehicle is large enough to accommodate more customers. In this scenario, the model tries to optimize the objective function by assigning the same vehicle for multiple routes because the objective function concentrates on minimizing the number of vehicles used in the final solution in addition to minimizing the total travel distance. All these are explained with the help of the following example. In addition, the model yields an infeasible solution that is evident from the simultaneous visit of two customers during the same time window, in spite of these two customers being in different routes.

In the example presented in tables 4–6, dispatching centre (DC) is the same as collection centre (CC), which is node 0, and there are two customers who require service from DC/CC. Two vehicles are available at the DC to provide service to the customers. We have assigned 0.5 as the value for the trade-off parameter α with respect to cost functions. The problem instance details with respect to the vehicles are given in table 4, and the problem instance details for customers and the travel distance/time with respect to transportation network are given in tables 5 and 6, respectively. The cost of travel $(c_{i,j})$ between customer i and customer j is expressed in terms of time $(t_{i,j})$ in this problem instance.

Table 4. Problem instance details – vehicles.

Vehicle number k)	Dispatching cost per vehicle	Earliest departure time	Latest arrival time	Capacity
1	90	0	400	400
2	100	0	400	400

Table 5. Problem instance details – customers.

Customer number i	Pickup quantity	Delivery quantity	Earliest departure time	Latest arrival time	Service time
1	180	120	0	70	10
2	80	150	0	70	10

When we execute the model by Wang and Chen [4] for this problem instance, two routes are formed for vehicle 1, and the routes are as follows:

- vehicle 1 starts from DC, serves customer 1 and then returns to CC, and the starting time of service at customer 1 is 70;
- vehicle 1 starts from DC, serves customer 2 and then returns to CC, and the starting time of service at customer 2 is 70.

The Z value for this solution is 180. This solution is infeasible in the problem instance because the same vehicle (vehicle 1) is utilized for two routes (i.e., DC – customer 1 – CC and DC – customer 2 – CC), and also the decision variables with respect to the starting time of service at customers 1 and 2 are 70 and 70, respectively. The former aspect contradicts the fact that the same vehicle is utilized for two different routes. The second aspect of the overlap of start and finish times of service at customers 1 and 2 by the same vehicle indicates infeasibility with respect to time.

To address this infeasibility, we include the following constraint to the MILP model by Wang and Chen [4]. In their model binary variable, $x_{0,j,k}$ indicates whether the vehicle k travels directly from DC (node 0) to customer node j. We now add the following:

$$\sum_{j=1}^{n} x_{0,j,k} \leq 1, \quad k = 1, 2, \ldots, V \tag{39}$$

This constraint states that for a given vehicle, the total number of arcs selected directly from the DC to the customer nodes should be less than or equal to 1. This constraint helps the model to assign a maximum of only one route to the vehicle. We include Constraint (39) to the model by Wang and Chen [4]; then when we execute the model, only one route is formed for each vehicle and the solution is presented here:

- vehicle 2 starts from DC, serves customer 1 and then returns to CC, and the starting time of service at customer 1 is 70;
- vehicle 1 starts from DC, serves customer 2 and then returns to CC, and the starting time of service at customer 2 is 70.

The Z value for this solution is 185 and the solution is feasible with respect to start and finish times of service at customers 1 and 2, without any overlap of start/finish times of service by the same vehicle. The same solution is obtained (Z value is 185) when we solve this example using our proposed MILP model; as discussed in section 5.3, the starting time of service at both customers 1 and 2 is 70; this is mainly because we define the lower limit only with respect to the starting time of service, which is 50 for customer 1, and 40 for customer 2.

7. Computational evaluation of our MILP model and existing MILP models

In this section, we describe the procedure to generate a test problem instance for the VRPSDPTW and VRPSDPTL. In section 7.1, we present a comparative computational study of the proposed MILP model for the VRPSDPTW and the MILP model available in the literature for the VRPSDPTW (by Wang and Chen [4]). In section 7.2, we present a comparative computational study of the proposed MILP model and the MILP model available in the literature for the VRPSDPTL (by Polat *et al* [12]).

In order to evaluate/compare our model with the MILP model by Wang and Chen [4], we generate 24 random problem instances, varying from 10 customer nodes to 30 customer nodes, by modifying the basic VRPTW instances given by Solomon [13]; later, to evaluate the proposed MILP models with the MILP model by Polat *et al* [12], we generate 13 random problem instances, varying from 10 customer nodes to 20 customer nodes, by modifying the basic VRPTW instances given by Solomon [13].

Each VRPTW instance by Solomon [13] belongs to one of the following six types:

- type C1 refers to clusters of customers (based on their locations), with shorter time windows and small vehicle capacity;
- type C2 refers to clusters of customers (based on their locations), with longer time windows and large vehicle capacity;
- type R1 refers to the set of customers whose locations are randomly generated, with shorter time windows and small vehicle capacity;
- type R2 refers to the set of customers whose locations are randomly generated, with longer time windows and large vehicle capacity;
- type RC1 refers to the set of customers whose locations are combination of both clustered and randomly placed locations, with shorter time windows and small vehicle capacity;
- type RC2 refers to the set of customers whose locations are combination of both clustered and randomly placed locations, with longer time windows and large vehicle capacity.

For generating a random problem instance specific to VRPSDPTW, first a specific VRPTW problem instance is selected and then the transportation network structure, time windows, capacity of the vehicles and the delivery quantity demanded by each customer are taken from the selected VRPTW instance, and the pickup quantities are randomly generated in the range of 50%–150% of respective delivery quantities. We split the VRPSDPTW problem instances into two categories and for the first category (12 problem instances), the dispatching cost of vehicle ($fixed_k$) is assumed to be the same for all vehicles; it is directly proportional to the average distance

Table 6. Problem instance details – transportation network.

Nodes	0	1	2
0	–	50	40
1	50	–	80
2	40	80	–

(*AVG_DISTANCE*) between any two nodes in the transportation network. The dispatching cost of a vehicle is set as follows:

$$fixed_k = N \times 0.25 \times AVG\_DISTANCE \qquad (40)$$

For the second category of problem instances, the dispatching cost of vehicle (i.e., $fixed_k$) is assumed to be different for each vehicle, and it is randomly generated in the range of 50%–150% of the ($N \times 0.25 \times AVG\_DISTANCE$), and the remaining parameters are the same as those of the respective instance from the first category.

For generating a random problem instance specific to VRPSDPTL, first a specific VRPTW problem instance is selected; later the transportation network structure, time window specific to depot and the delivery quantity demanded by each customer are taken from the selected VRPTW instance, and the pickup quantities are randomly generated in the range of 50%–150% of respective delivery quantities. The vehicles are assumed to be homogeneous in this case, and hence the capacity of the vehicles is set as the maximum capacity of the available vehicles in the selected VRPTW instance.

The computational experiments have been carried out using an Intel Core i7 processor, 2.80 GHz with 8 GB of RAM, and we have used ILOG CPLEX v12.7 (Academic Version) for solving the MILP models. Note that we have assigned 0.5 as the value for the trade-off parameter (α) with respect to cost functions for the VRPSDPTW instances, and we have assigned 1.0 as the value for the trade-off parameter (α) with respect to cost functions for the VRPSDPTL instances because the objective function of the VRPSDPTL focuses only on the routing cost. The cost of travel ($c_{i,j}$) between customer i and customer j is expressed in terms of time (($t_{i,j}$) in these problem instances.

7.1 *A comparative computational study of our MILP model and the existing MILP model for the VRPSDPTW*

To evaluate the performance of the proposed MILP models with the MILP model in the literature, we use the MILP model proposed by Wang and Chen [4] for the first set/category of instances. For the second set of instances, we include the proposed Constraint (39) to the model by Wang and Chen [4], because the problem instances in the second category require the proposed constraint (discussed earlier in section 6.4) to bring in feasibility with respect to solution generated. The problem instances specific to first category of the VRPSDPTW are provided in the link https://www.dropbox.com/sh/05c67kv6qqwdc4q/AABSrxW8r-YgkqkoSsTJCrwSa?dl=0, the problem instances specific to second category of the VRPSDPTW are provided in the link https://www.dropbox.com/sh/hggzv1ceas4d0di/AACt6KBChxF2Y1g-jTQApoURa?dl=0 and a sample set of problem instances are also provided in Appendix A.

The results of the comparison evaluation with respect to first set of instances and second set of instances are presented in tables 7 and 8, respectively. The results clearly indicate that the proposed MILP model performs better than the model proposed by Wang and Chen [4] in terms of the execution time to find an optimal solution, on the whole. In addition, our MILP model also performs well with the lower bound obtained through LP relaxation (i.e., relaxing the original problem by relaxing the restrictions on the binary variables as $0 \le x_{i,j} \le 1$, $0 \le delv_k \le 1$, and $0 \le del_{i,k} \le 1$ in our model and relaxing the restrictions on

Table 7. Computational experience with the proposed MILP model and an existing MILP model for the VRPSDPTW: first category of instances.

Instance Id	Number of customers	Number of vehicles	Optimal solution Z	Our Model: CPU time (s)	LB-Our Model	Existing MILP model: CPU time (s)	LB-Existing MILP model
1	10	2	286.70	5.42	161.63	2.81	73.60
2	10	2	240.10	2.66	119.99	6.69	22.45
3	10	2	190.83	9.45	119.99	2.78	24.92
4	20	4	191.91	1794.53	85.91	1693.23	44.18
5	20	4	231.95	7.13	85.91	13.06	44.12
6	20	4	252.59	6.55	208.02	130.13	115.58
7	20	4	166.79	5.64	137.78	12.28	63.35
8	20	4	564.28	5.88	208.02	11.08	114.87
9	20	4	420.94	459.08	154.51	#	42.74
10	30	6	257.00	76.44	103.92	#	43.19
11	30	6	263.78	110.88	103.92	126.84	43.19
12	30	6	810.18	16.86	243.30	218.47	150.20

Note:

LB-Our Model indicates the lower bound that we get from our MILP model through LP relaxation ($0 \le x_{i,j} \le 1;, 0 \le delv_k \le 1$ and $0 \le del_{i,k} \le 1$).

LB-Existing MILP model indicates the lower bound that we get from the MILP model by Wang and Chen [4] through LP relaxation ($0 \le x_{i,j,k} \le 1$).

For problem instances 9 and 10, when we have executed/implemented the model of Wang and Chen for 2 h, the solver is not able to find the optimum within 2 h.

Table 8. Computational experience with the proposed MILP model and the existing MILP model for the VRPSDPTW: second category of instances.

Instance Id	Number of customers	Number of vehicles	Optimal solution Z	Our Model: CPU time (s)	LB-Our Model	Existing MILP model: CPU time (s)	LB-Existing MILP model
1	10	2	236.70	4.59	133.63	2.09	73.60
2	10	2	198.10	1.44	96.94	5.26	22.45
3	10	2	167.33	10.95	96.49	3.53	24.92
4	20	4	157.91	478.08	68.69	#	44.18
5	20	4	186.45	7.55	68.69	12.08	44.12
6	20	4	216.09	6.63	171.52	126.48	115.58
7	20	4	139.79	5.11	110.78	11.27	63.35
8	20	4	462.78	7.91	173.49	8.81	114.87
9	20	4	325.94	435.22	119.81	#	42.74
10	30	6	198.50	61.97	83.96	#	43.19
11	30	6	205.28	104.36	83.96	94.01	43.19
12	30	6	672.68	13.34	207.52	336.05	150.20

Note:

LB-Our Model indicates the lower bound that we get from our MILP model through LP relaxation ($0 \le x_{i,j} \le 1;, 0 \le delv_k \le 1$ and $0 \le del_{i,k} \le 1$).

LB-Existing MILP model indicates the lower bound that we get from the MILP model by Wang and Chen [4] through LP relaxation ($0 \le x_{i,j,k} \le 1$).

For problem instances 4, 9 and 10, when we have executed/implemented Wang and Chen's model for 2 h, the solver is not able to find the optimum within 2 h.

the binary variable $x_{i,j,k}$ as $0 \le x_{i,j,k} \le 1$ in the model by Wang and Chen [4]).

We also observe that the lower bound obtained from the MILP model by Wang and Chen [4] is the same for both sets/categories 1 and 2, and this indicates that the solver assigns the value zero for the variables $\left(x_{0,j,k}\right)$, which tracks the direct precedence between depot node and customer node while obtaining the lower bound, as they directly contribute to the objective function/lower bound. As discussed in section 6.2, the proposed MILP model uses a separate set of binary variables ($delv_k$) to track the vehicle usage, which helps in obtaining a tighter lower bound, and this is mainly due to the fact that routing variables ($x_{i,j}$) in the travel plan (in our proposed model) are indirectly used to derive/determine the vehicle usage ($delv_k$), through binding constraints (Constraints (5)–(9)). We have also used the same and appropriate M value in the model by Wang and Chen [4], assuming the same value $M2$ with respect to the time window constraints as used in our model and the same value $M3$ with respect to the capacity constraints as used in our model, to have/make the evaluation of lower bounds generated from the models comparable.

7.2 *A comparative computational study of our MILP model and the existing MILP model for the VRPSDPTL*

In the VRPSDPTL, each customer can be visited at any point in time (but only once), and the vehicles need to return to the depot within the specified time limit with respect to the depot (see Polat *et al* [12]). In order to evaluate the performance of the proposed MILP models with the MILP model available in the literature for the VRPSDPTL, we consider Constraints (41)–(43) in the proposed MILP model instead of Constraints (28), and (33)–(35) for defining the bounds of the decision variables with respect to starting time of vehicle k, starting time of the service at customer i and load of the vehicle at customer i:

$$0 \le start_k^0 \le b^0, \quad k = 1, 2, \ldots, V \tag{41}$$

$$0 \le st_i \le b^0, i = 1, 2, \ldots, N \tag{42}$$

$$0 \le ld_i \le C^0, \ i = 1, 2, \ldots, N \tag{43}$$

The problem instances specific to the VRSDPTL are provided in the link https://www.dropbox.com/sh/ki31s9uqz1xzk41/AABapMk52aUgROH3X-t02usLa?dl=0 and a sample set of problem instances are also provided in Appendix A.

The results of the comparison evaluation are presented in table 9. The results indicate that the proposed MILP model is competitive to the model proposed by Polat *et al* [12] in terms of the execution time to find an optimal solution, on the whole. However, the proposed MILP model performs much better than the model proposed by Polat *et al* [12] in terms of the lower bound that the models generate for each problem instance using the LP relaxation (i.e., relaxing the binary variables such as $0 \le x_{i,j} \le 1$, $0 \le delv_k \le 1$ and $0 \le del_{i,k} \le 1$ in the proposed model, and relaxing the binary variable $x_{i,j}^k$ as $0 \le x_{i,j}^k \le 1$ in the model by Polat *et al* [12]).

Table 9. Computational experience with the proposed MILP model and the existing MILP model for the VRPSDPTL with homogeneous vehicles: third category of instances.

Instance Id	Number of customers	Number of vehicles	Optimal solution Z	Our Model: CPU time (s)	LB-Our Model	Existing MILP model: CPU time (s)	LB-Existing MILP model
1	10	2	45.61	0.67	44.48	1.58	21.04
2	10	2	189.18	2.91	157.26	2.59	41.83
3	10	2	195.85	5.36	98.97	0.98	107.63
4	10	2	131.66	1.53	118.00	6.05	8.51
5	10	2	160.48	0.75	157.26	0.86	8.37
6	10	2	151.23	8.94	98.97	2.72	21.53
7	15	3	113.99	4.45	71.25	9.44	53.37
8	15	3	264.11	13.67	212.92	81.91	58.55
9	15	3	217.51	13.13	120.63	8.98	141.64
10	15	3	165.81	5.55	139.66	30.34	17.80
11	15	3	215.86	1.36	212.92	1.13	11.71
12	20	4	178.04	15.34	150.22	134.67	28.25
13	20	4	253.63	3.05	248.05	8.08	16.69

Note:

LB-Our Model indicates the lower bound that we get from our MILP model through LP relaxation ($0 \leq x_{i,j} \leq 1; , 0 \leq delv_k \leq 1$ and $0 \leq del_{i,k} \leq 1$).

LB-Existing MILP model indicates the lower bound that we get from the MILP model by Polat *et al* [12] through LP relaxation ($0 \leq x^k_{i,j} \leq 1$).

8. Practical implications

In supply chain, logistics plays an essential role and its role and contributions continue to grow and evolve even in the firms that follow electronic-commerce type of business model [14]. This study mainly focuses on transportation to improve the performance and the responsiveness of the supply chain. and the study specifically concentrates on the pickup and delivery problems that commonly arise in the VRP. The study proposes an efficient MILP model (in terms of the CPU time) for the routing problem with the consideration of simultaneous delivery and pickup, and practitioners can make use of this model and its relaxed version to evaluate the solution quality of the exiting methods applied in the industry for the real-life problem instances (i.e., large-sized problem instances).

9. Conclusions

In supply chain, transportation refers to the movement of goods from one facility to another facility; in this work, we studied the Vehicle Routing Problem with Simultaneous Delivery and Pickup, and constrained by time windows (VRPSDPTW) in order to find the cost-effective routes to improve the performance and responsiveness of the supply chain. As a part of this study, we have proposed an MILP model for the VRPSDPTW. The proposed MILP model performs quite well, compared with existing models available in the literature for the VRPSDPTW (with the consideration of heterogeneous vehicles) and the VRPSDPTL (with the consideration of homogeneous vehicles), in terms of the execution time to solve each of the randomly generated problem instances. In addition, the corresponding LP relaxation of the proposed MILP model gives a tighter lower bound compared with the MILP models available in literature for VRPSDPTW and VRPSDPTL, and this lower bound may be useful for evaluating heuristics in the case of large-sized problem instances. The superior performance of our proposed MILP model is primarily attributed to the use of two-dimensional binary variable (with respect to precedence relationship between two given customers), instead of the use of three-dimensional binary variable (with respect to precedence relationship between two given customers) in existing MILP models.

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Appendix A

A sample set of problem instances for VRPSDPTW

Problem instance 1 for the VRPSDPTW

Table A1. Problem instance details – customers.

Node	*X*	*Y*	Delivery quantity	Pickup quantity	Earliest start time	Latest arrival time	Service time
0	40	55	0	0	0	1236	0
1	45	68	10	5	85	1127	90
2	45	70	30	32	0	1125	90
3	42	66	10	12	0	1129	90
4	42	68	10	8	727	782	90
5	42	65	10	14	0	1130	90
6	40	69	20	11	621	702	90
7	40	66	20	28	0	1130	90
8	38	68	20	29	255	324	90
9	38	70	10	7	534	605	90
10	35	66	10	13	357	410	90
11	35	69	10	11	448	505	90
12	25	85	20	10	0	1107	90
13	22	75	30	19	30	92	90
14	22	85	10	6	567	620	90
15	20	80	40	57	384	429	90
16	20	85	40	42	475	528	90
17	18	75	20	28	99	148	90
18	15	75	20	29	179	254	90
19	15	80	10	14	278	345	90
20	30	50	10	12	10	73	90

Table A2. Problem instance details – vehicles.

Vehicle	Dispatching cost	Capacity
1	59	200
2	66	200
3	51	200
4	74	200

Problem instance 2 for the VRPSDPTW.

Table A3. Problem instance details – customers.

Node	*X*	*Y*	Delivery quantity	Pickup quantity	Earliest start time	Latest arrival time	Service time
0	35	18	0	0	0	1000	0
1	41	49	10	5	18	898	10
2	35	17	7	7	93	333	10
3	55	45	13	15	436	676	10
4	55	20	19	17	620	860	10
5	15	30	26	38	20	260	10
6	25	30	3	1	345	585	10
7	20	50	5	7	251	491	10
8	10	43	9	13	323	563	10
9	55	60	16	12	329	569	10
10	30	60	16	21	485	725	10
11	20	65	12	13	146	386	10
12	50	35	19	9	167	407	10
13	30	25	23	14	639	879	10
14	15	10	20	12	32	272	10
15	30	5	8	11	118	358	10
16	10	20	19	20	203	443	10
17	5	30	2	2	682	922	10
18	20	40	12	17	286	526	10
19	15	60	17	24	204	444	10
20	45	65	9	11	504	744	10

Table A4. Problem instance details – vehicles.

Vehicle	Dispatching cost	Capacity
1	111	1000
2	124	1000
3	95	1000
4	139	1000

A sample set of problem instances for VRPSDPTL
Problem instance 1 for the VRPSDPTL

Table A5. Problem instance details – customers.

Node	*X*	*Y*	Delivery quantity	Pickup quantity	Service time
0	40	31	0	0	0
1	25	85	20	10	10
2	22	75	30	32	10
3	22	85	10	12	10
4	20	80	40	35	10
5	20	85	20	29	10
6	18	75	20	11	10

Table A5. continued

Node	X	Y	Delivery quantity	Pickup quantity	Service time
7	15	75	20	28	10
8	15	80	10	14	10
9	10	35	20	15	10
10	10	40	30	40	10

Table A6. Problem instance details – depot and vehicles.

Number of vehicles	2
Capacity of vehicles	1000
Time limit (for depot)	960

Problem instance 2 for the VRPSDPTL

Table A7. Problem instance details – customers.

Node	X	Y	Delivery quantity	Pickup quantity	Service time
0	40	55	0	0	0
1	45	68	10	5	90
2	45	70	30	32	90
3	42	66	10	12	90
4	42	68	10	8	90
5	42	65	10	14	90
6	40	69	20	11	90
7	40	66	20	28	90
8	38	68	20	29	90
9	38	70	10	7	90
10	35	66	10	13	90
11	35	69	10	11	90
12	25	85	20	10	90
13	22	75	30	19	90
14	22	85	10	6	90
15	20	80	40	57	90

Table A8. Problem instance details – depot and vehicles.

Number of vehicles	3
Capacity of vehicles	200
Time limit (for depot)	1236

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