A mathematical model of tractor-occupant system with a new seat suspension for minimization of vibration response

Mothiram K. Patil

Biomedical Engineering Division, Department of Applied Mechanics, Indian Institute of Technology, Madras 600 036, India

M. S. Palanichamy

Department of Structural Engineering, College of Engineering, P.A. University of Technology, Guindy, Madras 600 025, India (Received July 1986; revised July 1987)

In this paper the tractor-occupant system is modelled as a lumped parameter system; the composite model is analysed by computer simulation for vertical vibrational responses for a new type of seat suspension. It is shown that the new tractor seat suspension system (by proper selection of parameters) drastically improves the tolerance to high-intensity vibrations, in the 0.5–11-Hz range, experienced by tractor occupants, by reducing the maximum (i) amplitude ratios and relative displacements of the body parts to 0.029 and 0.19 mm, respectively, and (ii) body parts "acceleration levels" to much below the ISO specified 7-h "exposure limit" curve.

Keywords: human body responses, vibration responses, tractor vibrations, new type seat suspension, mathematical model

Introduction

A tractor occupant is exposed to high-intensity vibration levels in the 0.5–11-Hz (discomfort) range for extended periods of time, which he is not physically equipped to tolerate.¹ It is therefore not surprising that a survey by orthopaedic surgeons¹ in the United States establishes that truck and tractor drivers suffer from a number of disorders of the spine and supporting structures. In fact, high incidences of osteoarthritis, traumatic fibrositis, herniated disks, coccygodynia, lumbosacral pain, abdominal pain, and intestinal disorders occur in drivers of trucks, tractors, motorcycles, and other vehicles or machinery in which appreciable vibrations and jolts occur.²

Vibration intensity is characterized by the amplitude

ratio, acceleration level, relative amplitude between the adjacent body parts and pitch of the tractor. Any isolation of vibration by providing a suspension should reduce all these characteristics. It is seen¹ that the acceleration levels in conventional tractors are about 0.5 to 1.5 g in the frequency range of 2 to 7 Hz, and standard seats (of different suspension parameters) give rise to amplitude ratios of 2.5 to 4.5. These vibration acceleration levels are of much higher intensity than the one minute 'exposure limit' proposed by the International Standards Organization (ISO). Therefore, it is proposed, in this study, to reduce the acceleration levels to much below 7-h 'exposure limit' proposed by ISO³ by provision of a new type of seat suspension and suitably selecting its parameters.

Earlier works^{4,5} reported design of tractor suspen-

sion systems for isolating pitch and vertical vibrations. These suspensions were not very effective, since the suspension system (for vibration isolation) were designed based only on the measurement of seat vibration transmissibilities. It is found⁶ that measurement of vibration on the suspended seat alone does not truly reflect the vibration level which the human body parts are exposed. Hence, designing the seat suspension alone without taking into account the combined effect of the vehicle and the occupant does not yield satisfactory results.6 Work at the National Institute of Agricultural Engineering, United Kingdom, has shown that a mechanical simulation of the human body characteristics together with the seat is necessary.⁷ Therefore, in this paper presented here, the occupant and the tractor are modelled together in the form of a lumped mass system interconnected by springs and dashpots. The composite model, consisting of human body, tractor, and its new seat suspension is subjected to sinusoidal (idealized field or road profile) vibration at the tire contact points. The resulting responses (transient and steady state) of each body part, found by computer simulation, are studied to select the parameters of the new seat suspension such that the occupant vibration intensity (characterized by human body acceleration levels, amplitude ratios, and relative displacements) is reduced to a minimum in the 0.5-11-Hz frequency range.

Analysis of tractor-occupant vibrational response

The human occupant model

The tractor-occupant acts as a lumped parameter model at low frequencies from 0.5 to 100 Hz^{1,8} and it is idealized as a seven-degrees-of-freedom nonlinear lumped parameter model.⁹ As shown in *Figure 1*, the lumped masses (of head, back, torso, thorax, diaphragm, abdomen, and pelvis) are connected by springs and dashpots, representing the elastic and damping properties of the connective tissue between the segments. The model proposed by Muksian and Nash,9 is modified in our study to include the damping and elasticity of the buttocks. The values of the tissue springs and dashpot parameters are obtained from studies on the characteristics of specific subsystem^{9,10} and are listed in Table 1. The validity of this model is established later (in the next section) by good agreement between our model response and that recorded experimentally by other investigators.

Tractor model

The tractor is idealized (as shown in Figure 1) by a new seat, chassis body and tire masses (lumped together) interconnected by springs and dashpots of the new seat suspension. The schematic diagram of the new type of seat suspension, as shown in Figure 1, consists of a heavy coiled compression spring (K_{sb}) between the centre block (seat) and the U block. Diametrically opposite (in the other side of U block and central block) to the spring is a dashpot (C_{st}) in parallel with a semicircular leaf spring (K_{st}) . The action of sem-



Figure 1 Occupant-tractor model with new type of seat suspension

icircular leaf spring (K_{si}) , on the seat, is in the opposite direction to that of the compression spring (K_{sb}) , facilitated by the action of lever mechanisms *DE* and *FG* (schematically shown in *Figure 1*). The parametric constants of each spring and the dashpot are adjusted to ensure that the central block (seat) stays at the centre without any appreciable vibration. The tires are represented by linear vertical springs in parallel with velocity-dependent dampers. The parameters for the tractor are obtained from Mathews,⁴ and they are listed

Table 1 Parameter values of occupant model

Mass <i>M</i>	Damping constant C	Spring constant K	
(kg)	(kN/m/s)	(kN/m)	
$ \begin{array}{rcl} M_h &=& 5.45 \\ M_b &=& 6.82 \\ M_t &=& 32.762 \\ M_{th} &=& 1.362 \\ M_d &=& 0.455 \\ M_s &=& 5.921 \\ M_n &=& 27.23 \\ \end{array} $	$C_{h} = 3.58$ $C_{b} = 3.58$ $C_{t}^{*} = 3.58$ $C_{tb}^{*} = 3.58$ $C_{tb}^{*} = 0.292$ $C_{a}^{*} = 0.292$ $C_{a}^{*} = 0.292$ $C_{a}^{*} = 0.292$	$ \begin{array}{rcl} \kappa_{h} &= 52.6 \\ \kappa_{b} &= 52.6 \\ \kappa_{t} &= 0.877 \\ \kappa_{tb} &= 52.6 \\ \kappa_{th} &= 0.877 \\ \kappa_{d} &= 0.877 \\ \kappa_{d} &= 0.877 \\ \kappa_{d} &= 0.877 \\ \kappa_{c} &= 25.5 \\ \end{array} $	

*The units of damping constants, giving rise to linear and nonlinear forces, are kN/m/s and kN/(m/s)³, respectively. †The units of spring constants, giving rise to linear and nonlinear forces, are kN/m and kN/m³, respectively.

Table 2	Parameter values	to tractor and	i new seat	suspension
				3030613101

		· · · · · · · · · · · · · · · · · · ·
Distance of back	tire from C.G. a	= 0.847 m
Distance of front	tire from C.G. b	= 1.185 m
Distance of seat	from C.G. c	= 0.768 m
Wavelength of re	= 4.57 m	
Radius of gyratic	= 1.0224 m	
Magnitude of im (field depression	= 0.05 m	
Phase angle bety back tire input	ween the front and	= 160°
Mass M	Damping constant C	Spring constant K (kN/m)
$M_s = 4.537$	$C_{st} = 0.116$	$K_{st} = 68.67$ $K_{sb} = 68.915$
$M_{ct} = 2667.24$	$C_{gf}^* = 2.374$ $C_{gr}^* = 4.434$	$K_{gf}^* = 553.28$ $K_{gr}^* = 496.38$

*Represents the parameter values for two (front or back as the case may be) tires

in *Table 2* along with the minimum body response parameters of the seat suspension, determined by computer simulation.

Occupant-tractor composite model

The composite model of the occupant-tractor, moving on an irregular terrain, is shown in *Figure 1*. The composite model is analysed (by computer simulation by using CSMP) for

- Steady-state responses (amplitude ratios, acceleration levels of body parts, seat and pitch response of chassis) to sinusoidal inputs applied at the tractor tires
- 2. Transient vertical vibration responses of the body parts and seat to trapezoidal type of pulse input applied at the tractor tires

Steady-state analysis. In deriving the dynamic model of the tractor-occupant for simulation and analysis, a number of simplifying assumptions are made:

- 1. The road or field profile is approximated to be sinusoidal shape and is of 0.05 m in amplitude.¹
- The vehicle is considered in one plane only, the longitudinal plane through the centre of gravity with wheels combined with the chassis mass.
- 3. Forces and couples due to wheel rotations and draught forces are ignored.
- Rotational (pitch) vibrations for the occupant body parts are considered to be the same as that of the tractor chassis.
- 5. Displacements are considered to be sufficiently small for the tires and spring motions (of the tractor) to be always within their linear range, whilst small angular displacements allowed the sine of angles to be replaced in the equations of motion by the angles in radians, i.e., $\sin \theta = \theta$.

The composite model of tractor occupant is thus subjected to sinusoidal vibrations due to the ground reaction forces (that the tractor would be subject to, at its speed range while traversing its terrain). While deriving the governing equations of motion, the pitch motion of the tractor chassis in addition to the vertical motion is included. The stiffness and damping characteristics of torso, thorax, diaphragm and abdomen are represented by nonlinear springs and nonlinear dashpots.⁹ The equation of motion for each mass consists of the inertia term and forces exerted on the mass by the springs and dashpots due to the relative motion of the connected masses. The governing second-order nonlinear coupled ordinary differential equations of the various masses of the composite model (shown in *Figure 1*) are put down as follows:

$$M_h \ddot{y}_1 + C_h (\dot{y}_1 - \dot{y}_2) + K_h (y_1 - y_2) = 0$$
 (1)

where (i) M_h , C_h , and K_h are, respectively, the mass, damping constant, and spring constant of head, (ii) y_1 and y_2 are the displacements of head and back respectively, (iii) \dot{y}_1 , \dot{y}_2 , and \ddot{y}_1 refer to velocities of head, back, and acceleration of head, respectively.

Back

$$M_{b}\ddot{y}_{2} + C_{h}(\dot{y}_{2} - \dot{y}_{1}) + C_{b}(\dot{y}_{2} - \dot{y}_{7}) + C_{tb}(\dot{y}_{2} - \dot{y}_{3}) + C_{tb}(\dot{y}_{2} - \dot{y}_{3})^{3} + K_{h}(y_{2} - y_{1}) + K_{tb}(y_{2} - y_{3}) + K_{tb}(y_{2} - y_{3})^{3} + K_{b}(y_{2} - y_{7}) = 0$$
(2)

where (i) M_b , C_b , K_b are, respectively, the mass, damping constant, and spring constant of the back, (ii) C_{tb} and K_{tb} are the damping constants and spring constant of tissue between the torso and back, (iii) y_3 , y_7 are the displacements of the torso and pelvis, respectively, (iv) \dot{y}_3 , \dot{y}_7 , and \ddot{y}_2 refer to velocities of the torso, pelvis, and acceleration of the back, respectively.

Torso

$$M_{t}\ddot{y}_{3} + C_{tb}(\dot{y}_{3} - \dot{y}_{2}) + C_{tb}(\dot{y}_{3} - \dot{y}_{2})^{3} + C_{t}(\dot{y}_{3} - \dot{y}_{4}) + C_{t}(\dot{y}_{3} - \dot{y}_{4})^{3} + K_{tb}(y_{3} - y_{2}) + K_{tb}(y_{3} - y_{2})^{3} + K_{t}(y_{3} - y_{4}) + K_{t}(y_{3} - y_{4})^{3} = 0$$
(3)

where (i) M_i , C_i , K_i are, respectively, the mass, damping constant, and spring constant of the torso, (ii) y_4 , \dot{y}_4 , and \ddot{y}_3 refer, respectively, to displacement and velocity of thorax and acceleration of torso.

Thorax

$$M_{ih}\ddot{y}_{4} + C_{i}(\dot{y}_{4} - \dot{y}_{3}) + C_{i}(\dot{y}_{4} - \dot{y}_{3})^{3} + C_{ih}(\dot{y}_{4} - \dot{y}_{5}) + C_{ih}(\dot{y}_{4} - \dot{y}_{5})^{3} + K_{i}(y_{4} - y_{3}) + K_{i}(y_{4} - y_{3})^{3} + K_{ih}(y_{4} - y_{5}) + K_{ih}(y_{4} - y_{5})^{3} = 0 \quad (4)$$

where (i) M_{th} , C_{th} , K_{th} are respectively the mass, damping constant and spring constant of thorax (ii) y_5 , \dot{y}_5 , \ddot{y}_4 refer respectively to displacement and velocity of a diaphragm and acceleration of thorax.

Diaphragm

$$\begin{split} M_{d}\ddot{y}_{5} + C_{th}(\dot{y}_{5} - \dot{y}_{4}) + C_{th}(\dot{y}_{5} - \dot{y}_{4})^{3} + C_{d}(\dot{y}_{5} - \dot{y}_{6}) \\ + C_{d}(\dot{y}_{5} - \dot{y}_{6})^{3} + K_{th}(y_{5} - y_{4}) + K_{th}(y_{5} - y_{4})^{3} \\ + K_{d}(y_{5} - y_{6}) + K_{d}(y_{5} - y_{6})^{3} = 0 \end{split}$$

where (i) M_d , C_d , K_d are, respectively, the mass, damping constant, and spring constant of diaphragm, (ii) y_6 , \dot{y}_5 , \ddot{y}_5 refer, respectively, to displacement and velocity of abdomen and acceleration of diaphragm.

Abdomen

$$M_{a}\ddot{y}_{6} + C_{d}(\dot{y}_{6} - \dot{y}_{5}) + C_{d}(\dot{y}_{6} - \dot{y}_{5})^{3} + C_{a}(\dot{y}_{6} - \dot{y}_{7}) + C_{a}(\dot{y}_{6} - \dot{y}_{7})^{3} + K_{d}(y_{6} - y_{5}) + K_{d}(y_{6} - y_{5})^{3} + K_{a}(y_{6} - y_{7}) + K_{a}(y_{6} - y_{7})^{3} = 0$$
(6)

where (i) M_a , C_a , K_a are, respectively, the mass, damping constant, and spring constant of abdomen, (ii) y_7 , \dot{y}_6 are, respectively, the displacement and velocity of pelvis and acceleration of abdomen.

Pelvis

$$\begin{split} M_{p} \ddot{y}_{7} + C_{a} (\dot{y}_{7} - \dot{y}_{6}) + C_{a} (\dot{y}_{7} - \dot{y}_{6})^{3} + C_{b} (\dot{y}_{7} - \dot{y}_{2}) \\ &+ C_{p} (\dot{y}_{7} - \dot{y}_{8}) + K_{a} (y_{7} - y_{6}) + K_{a} (y_{7} - y_{6})^{3} \\ &+ K_{p} (y_{7} - y_{8}) + K_{b} (y_{7} - y_{2}) = 0 \quad (7) \end{split}$$

where (i) M_p , C_p , K_p are, respectively, the mass, damping constant, and spring constant of pelvis, (ii) y_8 , \dot{y}_8 , and \ddot{y}_7 refer to the displacement and velocity of seat and acceleration of pelvis, respectively.

Seat

$$M_{s}\ddot{y}_{8} + C_{\rho}(\dot{y}_{8} - \dot{y}_{7}) + C_{st}(\dot{y}_{8} - \dot{y}_{9} + c\theta) + K_{st}(y_{8} + y_{9} - c\theta) + K_{sb}(y_{8} - y_{9} + c\theta) + K_{\rho}(y_{8} - y_{7}) = 0 \quad (8)$$

where (i) M_s , C_{st} , K_{st} , K_{sb} are, respectively, the seat mass, damping constant, and spring constants of semicircular leaf spring and compression spring of the new seat suspension, (ii) y_9 , \dot{y}_9 , \ddot{y}_8 refer, respectively, to the displacement, velocity of chassis, and acceleration of seat, (iii) c and θ are the distance of the seat from the centre of gravity of the tractor chassis in the longitudinal plane and chassis pitch, respectively. The opposing actions of springs K_{st} and K_{sb} on the seat (brought about by the lever mechanisms *DE* and *FG* schematically shown in *Figure 1*) are shown by their coefficients in equation (8).

Chassis

The vertical vibration equation of the chassis is

$$M_{cl}\ddot{y}_{9} + C_{sl}(\dot{y}_{9} - \dot{y}_{8} - c\theta) + C_{gf}(\dot{y}_{9} + b\theta) + C_{gr}(\dot{y}_{9} - a\dot{\theta}) + K_{sl}(y_{9} + y_{8} - c\theta) + K_{sb}(y_{9} - y_{8} - c\theta) + K_{gf}(y_{9} + b\theta) + K_{gr}(y_{9} - a\theta) = C_{gf}A\omega\cos\omega t + C_{gr}A\omega\cos(\omega t - \alpha) + K_{gf}A\sin\omega t + K_{gr}A\sin(\omega t - \alpha)$$
(9)

where (i) M_{ct} is the mass of the chassis, (ii) C_{gf} , K_{gf} are, respectively, the damping and spring constant of front tractor tires, (iii) C_{gr} , K_{gr} are, respectively, the damping and spring constants of rear tires, (iv) a, b, are, respectively, the distance of rear and front tires from the centre of gravity of the chassis in the longi-

tudinal plane, (v) A, ω are, respectively, the amplitude of input displacement and circular frequency of this displacement applied at the tire contact points to the ground, (vi) α is the phase angle between the input displacements applied at the front and rear tires, (vii) y_9 is the acceleration of the chassis. The pitch vibration equation of the chassis is

$$M_{ct}\rho^{2}\ddot{\theta} + bC_{gf}(\dot{y}_{9} + b\dot{\theta}) - aC_{gr}(\dot{y}_{9} - a\dot{\theta})$$

$$- cC_{st}(\dot{y}_{9} - \dot{y}_{8} - c\dot{\theta}) - cK_{st}(y_{9} + y_{8} - c\theta)$$

$$- cK_{sb}(y_{9} - y_{8} - c\theta) + bK_{gf}(y_{9} + b\theta)$$

$$- aK_{gr}(y_{9} - a\theta)$$

$$= bC_{gf}A\omega\cos\omega t - aC_{gr}A\omega\cos(\omega t - \alpha)$$

$$+ bK_{gf}A\sin\omega t - aK_{gr}A\sin(\omega t - \alpha) \qquad (10)$$

where (i) $\hat{\theta}$ and ρ are the angular acceleration and radius of gyration of the tractor chassis, respectively.

The above coupled nonlinear differential equations are solved by CSMP* simulation on the computer to give y_i , y_i and θ responses to steady-state sinusoidal forcing functional inputs at different frequencies of vibration. By dividing the amplitude responses of the body parts by input amplitude of vibration, the amplitude ratio of the various body parts and seat are computed. The parameter determination for the new seat suspension is carried out for minimizing the responses of the body parts in the 0.5-11-Hz frequency range. The parameters of the suspension, which give the maximum vibration responses of the body parts, are listed in *Table 2*.

Transient analysis. The relevance of this study is to choose the design parameters of seat suspension such that body parts do not suffer damage due to sudden high-amplitude relative displacements between them at the onset of vibrations, when the tractor is encountered with sudden obstructions for a short while. This is also supported by von Gierke,¹¹ who states: 'it is not the pressure per se, but the resulting relative displacement of adjacent tissue that leads to the stimulation of various receptors as well as to ultimate injury'. The obstructions or ground irregularities are idealized (as shown in Figure 2) by trapezoidal type of inputs with a maximum amplitude of 0.05 m. The two inputs in sequence, represent front tire and back tire displacement inputs, respectively. This type of input represents two obstacles (surface irregularities) of height 0.05 m, width 0.3 m each and separated by 1.58 m. When the front tire is on the peak of the first obstacle, the second obstacle will be located at 0.45 m (a + b - 1.58 =0.45 m) ahead of the back tire. The front tire is subjected to the first input irregularity and the back tire to the second after a time lag of 0.09 s, characterising the time taken by the tractor rear tire to reach the irregularity when it is moving at a speed of 18Km/hour. The distances between obstacles and speed of tractor

^{*}CSMP refers to continuous system modelling program.



Figure 2 Displacement input at the tractor tires for transient responses

are chosen such that it represents the most adverse type of input condition to which the tractor tires could be subjected.

The vibration displacement inputs at front and back tires, respectively Figure 2, are mathematically represented by x_1 and x_2 given by

$$x_{1} = \frac{10At}{T} [u(t) - u(t - 0.1T)] + A[u(t - 0.1T) - u(t - 0.9T)] + 10A(1 - \overline{T}) [u(t - 0.9T) - u(t - T)]$$
(11)
10A(t - 1.5T)

$$x_{2} = \frac{10A(t - 1.5T)}{T} [u(1.5T) - u(t - 1.6T)] + A[u(t - 1.6T) - u(t - 2.4T)] + \frac{10A}{T} [2.5T - t] [u(t - 2.4T)]$$

-u(t-2.5T)] (12)

where u(t) represents the unit step function, defined as,

$$u(t) = 0 \qquad \text{for } t < 0$$
$$= 1 \qquad \text{for } t \ge 0$$

and A = 0.05 m, T = 0.06 s.

The governing vibration equations of the composite model, for the trapezoidal type of displacement inputs at the tractor tires, remain same as equations (1) to (8) for body parts and tractor seat. The equations for the tractor chassis are modified as follows:

$$\begin{split} M_{ct}\ddot{y}_{9} + C_{st}(\dot{y}_{9} - \dot{y}_{8} - c\dot{\theta}) \\ &+ C_{gf}(\dot{y}_{9} + b\dot{\theta}) + C_{gr}(\dot{y}_{9} - a\dot{\theta}) \\ &+ K_{st}(y_{9} + y_{8} - c\theta) + K_{sb}(y_{9} - y_{8} - c\theta) \\ &+ K_{gf}(y_{9} + b\theta) + K_{gr}(y_{9} - a\theta) \\ &= C_{gf}\dot{x}_{1} + C_{gr}\dot{x}_{2} + K_{gf}x_{1} + K_{gr}x_{2} \end{split}$$
(13)

$$M_{ct}\rho^{2}\ddot{\theta} + bC_{gf}(\dot{y}_{9} + b\dot{\theta}) - aC_{gr}(\dot{y}_{9} - a\dot{\theta}) - cC_{st}(\dot{y}_{9} - \dot{y}_{8} - c\dot{\theta}) - cK_{st}(y_{9} + y_{8} - c\dot{\theta}) - cK_{sb}(y_{9} - y_{8} - c\dot{\theta}) + bK_{gf}(y_{9} + b\theta) - aK_{gr}(y_{9} - a\theta) = bC_{gf}\dot{x}_{1} - aC_{gr}\dot{x}_{2} + bK_{gf}x_{1} - aK_{gr}x_{2}$$
(14)

where \dot{x}_i and x_i represent the corresponding velocities and displacements at the tractor tires.

Equations (13) and (14) along with equations (1) to (8) are programmed on the computer using CSMP simulation and solved to give y_i , the transient amplitude responses of the body parts and seat and the relative displacements between adjacent body parts. The parameters of the new seat suspension are determined such that the relative displacements between body parts are minimized in the 0.5-11 Hz frequency range. The minimum response parameters of the seat suspension in the transient vibration analysis are found to be same as those, presented in the steady-state vibration analysis and they are listed in *Table 2*.

Results and discussion

Validation of model

Figure 3 shows the calculated head-to-pelvis acceleration ratio as a function of frequency. Superimposed thereon, are the experimental values given by Goldman and von Gierke¹² and Pradko *et al.*^{13,14} for sinusoidal inputs. The good agreement between the model calculated and the experimental values provides a measure of confidence in the parametric values of the model. The first resonant peak for each of the body parts occurs at approximately 3 Hz, in general agreement with the results of Coermann *et al.*¹⁵ and Roberts *et al.*¹⁶ in a review of results of others.

Steady-state vibration responses

Now, this validated composite model (with minimum response parameters) is used in further analysis to find the responses of the body parts to vertical sinusoidal vibrations in the frequency range of 0.5 to 11



Figure 3 Head-to-pelvis acceleration ratio

Appl. Math. Modelling, 1988, Vol. 12, February 67



Figure 4 Amplitude ratio response of head and seat for new seat suspension

Hz. The results represent responses of some body parts (subjected to maximum responses) and their comparison with results of other research workers and the ISO³ recommendations.

As shown in Figure 4, the head and tractor seat undergo the maximum amplitude ratio of 0.028 and 0.028 at 3 Hz and 11 Hz, respectively. The body part undergoes higher response than that of the tractor seat. Comparing the responses of head with the experimental results of Radke,¹ for head in a tractor with a rigid seat, it is found that the new type of seat suspension of the tractor is able to reduce the amplitude ratio of head by 99.25% in lower frequencies and by 99.4% at higher frequencies.

Figure 5 shows the responses of back, torso, thorax, and diaphragm; the maximum responses are equal to 0.0273, 0.029, 0.028, and 0.027, respectively, occurring at 3 Hz. Among all body parts torso is subjected to the highest amplitude ratio of 0.029 at 3 Hz. At higher frequencies (7 to 11 Hz), the average response of all body parts are observed to be of the order of 0.003.

Acceleration responses of the body parts and seat, in the frequency range of 0.5 to 11 Hz for the sinusoidal type of input at the tractor tires, are shown in *Figure* 6 and *Figure 7*. *Figure* 6(a) shows the steady-state acceleration responses of head and tractor seat. The maximum responses of the head and seat are of the order of 0.739 m/s² and 7.708 m/s², respectively, both occurring at 11 Hz. Comparison of the model computed head acceleration response to the head response (17.5 m/s²) reported by Dupuis *et al.*¹⁷, at 2.58 Hz, shows that the new type of tractor seat suspension reduces the acceleration response of head by 97.6%.

The maximum acceleration responses of back and



Figure 5 Amplitude ratio response of back, torso, thorax, and diaphragm

torso (from Figure 6(b)), are found to be equal to 0.729 m/s² and 0.60 m/s², respectively, occurring for both of them at 11 Hz. Figure 7(a) shows the acceleration responses of diaphragm and thorax. Among all body parts, pelvis undergoes the maximum acceleration (0.996 m/ s²) response. Superimposed on Figure 7(b) is the 7-h



Figure 6 Acceleration responses of (a) head and seat, and (b) back and torso



Figure 7 Acceleration responses of (a) diaphragm and thorax, and (b) pelvis and abdomen with 7-h exposure limit curve (from ISO³)

'exposure limit' curve prescribed by ISO. It is found (from above figure) that the highest acceleration responses (in the frequency range of 0.5 to 11 Hz) of body part (for vertical vibration) namely, pelvis fall below 7-h exposure limit curve, thereby, showing the effectiveness of the new seat suspension in improving ride comfort considerably.

The model computed pitch response of the chassis, in the frequency range of 0.5 to 11 Hz, is represented in *Figure 8* which shows that the maximum pitch response of the chassis is of the order of $0.641^{\circ}/\text{cm}$ of input amplitude, occurring at 4 Hz. Comparing this result with the experimental results of Mathews⁴ for a tractor with rigid seat, and a tractor with front axle suspension and rigid seat, it is found that pitch response reductions of 84.7% and 75.9%, respectively, are obtained by providing the new type of tractor seat suspension.

Transient vibration responses

The purpose of this study is to choose the design parameters of the new seat suspension such that the body parts do not suffer damage due to sudden highamplitude relative displacements between them at the onset of vibrations, when the tractor is encountered with sudden obstructions for a short while (idealised by the trapezoidal type of pulse input, as shown in *Figure 2*). The group of body parts, which give max-



Figure 8 Chassis pitch response for new type of tractor seat suspension

imum relative displacements between themselves, are only chosen, here, to represent their responses. *Figure* 9 represents the transient response of pelvis-seat combination. The maximum response of seat is found to be higher than the body part (pelvis). All the responses die down to zero at 2.15 s.

Figure 10(a) and (b) show the transient responses of head-back and back-torso combinations, respec-



Figure 9 Transient response of pelvis and seat

Appl. Math. Modelling, 1988, Vol. 12, February 69



Figure 10 Transient response of (a) head and back, and (b) back and torso

tively. It is found, among all body parts, torso is subjected to the highest amplitude of 0.47 mm occurring at 0.168 s, after the pulse input is applied at the tires and it dies down to zero at 2.16 s.

Figures 11(a) and (b) represent the transient responses of abdomen-diaphragm and abdomen-pelvis, respectively. It is found that, among all body parts, the maximum relative displacement takes place between pelvis and abdomen and its value is computed as 0.19 mm, occurring at 0.12 s. Taking the length between these parts as 17 cm¹⁸ the strain value is computed as 0.112% which is found to be far less than the



Figure 11 Transient responses of (a) abdomen and diaphragm, and (b) abdomen and pelvis

strain percentage or breaking index* (42%) indicated in von Gierke¹¹ for the same body parts.

Table 3 presents summary and comparisons between steady state and transient responses of tractor seat and occupant body parts.

Conclusions

A mathematical model of the tractor-occupant system, with a new seat suspension for vibration response, is developed and it is analysed by computer simulation for determination of seat suspension parameters which minimize human body responses. From the responses

*Breaking index = Breaking strength/Young's modules = Percentage increase in length required to breakage.¹¹

Table 3 Comparison of steady-state and transient maximum responses of body or tractor parts

Body tractor part	Steady-state		Transient				
		Absolute amplitude		Relative amplitude between adjacent body parts			
	Amplitude ratio	Acceleration (m/s²)	Amplitude (mm)	Time of occurence (s)	Relative amplitude (mm)	Time of occurence (s)	Body parts involved
Heat	0.028	0.739	0.42	0.168	0.02	0.168	Head-back
Back	0.0273	0.729	0.40	0.144	0.08	0.12	Back-torso
Torso	0.0290	0.600	0.47	0.168	0.04	0.144	Torso-thorax
Thorax	0.0280	0.618	0.45	0.168	0.02	0.168	Thorax- diaphragm
Diaphragm	0.027	0.56	0.43	0.168	0.03	0.192	Diaphragm- abdomen
Abdomen	0.026	0.549	0.40	0.168	0.19	0.12	Abdomen-pelvis
Pelvis	0.025	0.996	0.46	0144	0.11	0.12	Pelvis-back
Seat	0.028	7.078	1.15	0.168			

of the model presented in *Figures 4 to 11* and *Table 3*, following conclusions are made:

- As indicated by the steady state vibration responses to sinusoidal type of input at the tire, the body parts are subjected to higher responses in lower frequencies and lower responses at higher frequencies when compared with the responses of seat.
- 2. The transient vibration responses of the body parts, when the trapezoidal type of pulse input is applied at the tires, are lower than that of the seat.

The above conclusions show the necessity of designing the suspension parameters based on the principle of minimizing the amplitude ratios of body parts rather than minimizing the response of only the tractor seat. This agrees with the views expressed by Mathews.⁷

- From steady-state body parts response characteristics, it is found that provision of new tractor seat suspension reduces the maximum
 - (i) Amplitude ratio response of the body parts to 0.029 (occurring at 3 Hz).
 - (ii) Acceleration of body parts to 0.996 m/s^2 .
 - (iii) The pitch response of the chassis to 0.641°/cm of input amplitude. The acceleration response of the body parts (over the frequency range of 0.5 to 11 Hz) is found to be below the 7-h exposure limit curve prescribed by the ISO.
- 4. From transient (body parts') response characteristics, it is found that the maximum relative displacements between the adjacent body parts are of the order of 0.19 mm (representing low strain of 0.112% between abdomen and pelvis). This shows very effective vibration isolation characteristics of the new tractor seat suspension.
- 5. It is found from our study that among vibration isolation criteria, the acceleration intensity criterion is the most important than other criteria since all other criteria are satisfied automatically when acceleration criterion of exposure limit is satisfied. This conclusion supports the ISO specification of acceleration as the main criterion for isolation or minimization of vibration intensity.

In brief the new type of tractor seat suspension discussed in this paper reduces the maximum body (i) amplitude ratio to 0.029, (ii) acceleration intensity level to below 7-h exposure limit curve, (iii) transient amplitude to 0.47 mm, (iv) relative amplitude between adjacent body parts to 0.19 mm and (v) pitch response of the chassis to 0.641°/cm of input amplitude, thereby, providing maximum riding comfort to the tractor occupant. Thus the mathematical model analysis for vibration response of tractor-occupant system has given useful results which could possibly be used for better tractor seat suspension design.

References

- Radke, A. O. Vehicle vibration . . . man's environment, ASME Paper, 1957, 57-A, 1-8
- 2 von Gierke, H. E. Transmission of vibratory energy through human body tissue, Proc. First National Biophysics Conf. Yale University Press, New Haven, Conn., 1959, pp. 647-668
- 3 ISO. Guide for the Evaluation of Human Exposure to Whole-Body Vibration. International Standards Organization DIS 2631, 1972
- 4 Mathews, J. An analogue computer investigation of the potential improvement in tractor ride afforded by a flexible front axle. J. Agric. Eng. Res. 1967a, 12, 48-54
- 5 Tomlinson, J. and Kyle, R. The development of a dynamic model of the seated operator. National Institute of Agricultural Engineering Department Note, 1970, DN/TE/037/1445
- 6 Mathews, J. The measurement of tractor ride comfort; SAE Meeting. Milwaukee, Wis., Paper No. 730795, 1973, pp. 1-16
- 7 Mathews, J. Progress in application of ergonomics to agricultural engineering, Agricultural Engineering Symp. Silsoe, 1967b
- 8 von Gierke, H. E. Biodynamic models and their applications. J. Acoust. Soc. Am., 1971, 50, 1397-1413
- 9 Muksian, R. and Nash, C. D. A model for the response of seated humans to sinusoidal displacements of the seat. J. Biomech., 1974, 7, 209-215
- 10 Huang, B. K. Digital simulation analysis of biophysical systems. *IEEE Trans. Biomed. Eng.*, 1972, 19, 128-139
- 11 von Gierke, H. E. Response of body to mechanical forcesan overview. Ann. N.Y. Acad. Sci., 1968, 152,172-186
- 12 Goldman, B. E. and von Gierke, H. E. The Effects of Shock and Vibration on Man. Report No. 60-3, Naval Medical Research Institute, Bethesda, 1960
- 13 Pradko, F., Lee, R. and Kaluza, V. Theory of human vibration response. ASME, Paper No. 66-WA/BHF-15, 1966
- 14 Pradko, F., Lee, R. and Greene, J. D. Human vibration response theory. ASME Biomechanics Monograph, 1967, pp. 205-222
- 15 Coermann, R. R. et al. The passive dynamic mechanical properties of the human thorax-abdomen system and of the whole body system. J. Aerospace Medicine 1960, 31, 443–455
- 16 Roberts, V. L., Stech, E. L. and Terry, C. T. Review of mathematical models which describe human response to acceleration. ASME. Paper No. 66-WA/BHF-13, 1966
- 17 Dupuis, H., Draeger, J. and Hartung. E. Vibration transmission to different parts of the body by various locomotions. *Biomechanics*, ed. P. V. Komi, University Park Press, Baltimore, 1976, pp. 537-543
- 18 Contini, R. Body segment parameters. II, Artificial Limbs 1972, 16, 1-19