Two Models for Noisy Feedback in MIMO Channels

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***Abstract*—Two distinct models of feedback, suited for FDD (Frequency Division Duplex) and TDD (Frequency Division Duplex) systems respectively, have been widely studied in the literature. In this paper, we compare these two models of feedback in terms of the diversity multiplexing tradeoff for varying amount of channel state information at the terminals. We find that, when all imperfections are accounted for, the maximum achievable diversity order in FDD systems matches the diversity order in TDD systems. TDD systems achieve better diversity order at higher multiplexing gains. In FDD systems, the maximum diversity order can be achieved with just a single bit of feedback. Additional bits of feedback (perfect or imperfect) do not affect the diversity order if the receiver does not know the channel state information.**

I. Introduction

Channel state information to the transmitters has been extensively studied in MIMO systems [1–11] to improve upon the diversity multiplexing tradeoff without feedback [12]. While earlier work often assumed noiseless feedback (possibly quantized), recent emphasis has been on studying the performance with noisy feedback [4–8] in single-user MIMO channels. Two distinct models of feedback have appeared. The first is that of quantized channel state information [1–5] which is more appropriate for asymmetric frequency-division duplex systems. The other is that of two-way training, suitable for symmetric time-division duplex systems and is the focus of study in [6–8]. In this paper, we study these two systems and provide comparisons.

In this paper, we consider the effect of varying amount of channel state information at the transmitter and the receiver for both models of feedback. It is known that assuming perfect channel state information at the receiver and a perfect feedback link gives us unbounded diversity order [3]. Here, we find that the diversity order is bounded if feedback link is imperfect and the receiver do not know channel state information. Thus, considering the noise in the system gives us bounded diversity order.

For the quantized feedback model, where the receiver knows the channel state perfectly and the feedback link is imperfect, we find that a diversity order of $mn(mn+2)$ can be achieved with power controlled feedback rather than $2mn$ without the power-controlled feedback in [5]. Further, we find that the receiver knowledge limits the diversity multiplexing performance and if the receiver needs to be trained, increasing the number of feedback levels beyond 2 (1 bit) does not improve the diversity. A maximum diversity order of $mn(mn + 1)$ is obtained at zero multiplexing when the receiver has to be trained with either perfect or imperfect power-controlled feedback.

For the two-way training model, we find that unbounded diversity order is obtained if either or both of the nodes have perfect channel state information. If none of the nodes have perfect channel state information and are trained, we are limited by diversity order of $mn(mn+1)$ for zero multiplexing gain which is the same as found in the case of quantized feedback. Thus, we see that the maximum diversity order of the quantized feedback model with just 1 bit of feedback is same as the diversity order of the two-way training model when all the imperfections in channel estimation are accounted. We further see that the matching performances above only holds for the zero multiplexing point while at general multiplexing, the two-way training model achieves higher diversity order than the quantized feedback model.

In TDD systems, we get unbounded diversity order if either of the nodes have perfect channel state information because the node with perfect channel state information can train the other well since the forward and the backward channels are the same. However, in FDD systems, this no longer holds since the forward and the backward channels are independent, and hence even if one of nodes has the channel state information of the link to it, it cannot help resolve the channel state at the other node.

The rest of the paper is outlined as follows. In Section II we give background on the two-way channel model and the diversity multiplexing tradeoff. In Section III, we summarize the diversity multiplexing tradeoff when there is no feedback link [12]. In Section IV and V, we present the diversity multiplexing tradeoff results for the two channel models. In Section VI, we give numerical results. Section VII concludes the paper.

II. Preliminaries

A. Two-way Channel Model

For the single-user channel, we will assume that there are m transmit antennas at the source and n receive antennas at the destination. The channel input output relation is given by

$$Y = HX + W \tag{1}$$

where the elements of H and W are assumed to be i.i.d. $CN(0,1)$. The transmitter is assumed to be power-limited, such that the long-term power is upper bounded, i.e, $\mathbb{E}\left[X^2\right] \leq \mathsf{SNR}$. Furthermore, the channel H is assumed to be fixed during a fading block of L consecutive channel uses, and changes from one block to another.

Since our focus will be studying feedback over noisy channels, we assume that the same multiple antennas at the transmitter and receiver are available to send feedback on an orthogonal channel. For the feedback path, the receiver will act as a transmitter and the transmitter as a receiver. As a result, the feedback source (which is the data destination) will have n transmit antennas and the feedback destination (which is the data source) will be assumed to have m receive antennas. Furthermore, a block fading channel model is assumed

$$Y_f = H_f X_f + W_f \tag{2}$$

where H_f is the MIMO fading channel for the feedback link, normalized much like the forward link. The feedback transmissions are also assumed to be power-limited, that is the reverse link has a power budget of $\mathbb{E}\left[X_f^2\right] \leq \mathsf{SNR}$.

In this paper, we consider two types of systems: TDD and FDD. In TDD systems, the channels in the forward and the backward directions are symmetric and hence $H = H_f^{\dagger}$ and training can take place from the receiver to the transmitter. On the other hand, in FDD systems, the forward and the feedback path are asymmetric and hence H and H_f are considered independent of each other. Also, the feedback path is used to send a quantized feedback power level.

B. Diversity Multiplexing Tradeoff

In this paper, we will only consider single rate transmission where the rate of the codebooks does not depend on the feedback index and is known to the receiver. Therefore, regardless of the feedback at the transmitter, the receiver attempts to decode the received codeword from the same codebook. Outage occurs when the transmission power is less than the power needed for successful (outage-free) transmission [12].

Note that all the index mappings, codebooks, rates, powers are dependent on SNR. The dependence of rate on SNR is explicitly given by $R = r\log\mathsf{SNR}$.[1]

The outage probability is the probability of outage and is formally defined in [6, 12, 13]. The system has diversity order $d(r)$ if the outage probability is $\doteq \mathsf{SNR}^{-d(r)}$ for a given multiplexing gain r. The diversity order $d(r)$ describes the achievable diversity multiplexing tradeoff.

We will now define a function $G(r,p)$ which will be used later throughout the text. This function signifies the diversity multiplexing tradeoff for a coherent system where there is no feedback and the power constraint is SNR^p and the rate is $\doteq r\log(\mathsf{SNR})$.

[1] We adopt the notation of [12] to denote $\doteq$ to represent exponential equality. We similarly use $\dot{<}, \dot{>}, \dot{\leq}, \dot{\geq}$ to denote exponential inequalities.

Definition 1. *Let $0 < r < p\min(m,n)$ and $p > 0$. Then, we define $G(r,p) \triangleq$*

$$\inf_{(\alpha_1,\cdots,\alpha_{\min(m,n)})\in A} \sum_{i=1}^{\min(m,n)} (2i-1+\max(m,n)-\min(m,n))\alpha_i$$

where $A \triangleq$

$$\{(\alpha_1^{\min(m,n)})|\alpha_1 \geq \ldots \alpha_{\min(m,n)} \geq 0, \sum_{i=0}^{\min(m,n)} (p-\alpha_i)^+ < r\}.$$

Note that $G(r,p)$ is a piecewise linear curve connecting the points $(r, G(r,p))$= $(kp, p(m-k)(n-k))$, $k = 0, 1, \ldots, \min(m,n)$ for fixed m, n and $p > 0$. This follows directly from Lemma 2 of [3]. Further, $G(r,p) = pG(\frac{r}{p}, 1)$.

C. Summary of results

In this paper, we will describe the diversity multiplexing tradeoff for the following cases: (1) only one-way links, (2) two-way channel with FDD-based quantized feedback reverse link, and (3) two-way channel with TDD-based reverse link. In all the three cases, we describe the effect of knowledge of channel state information at the transmitter and the receiver including imperfect knowledge due to the effect of noise in the channel while communicating the training or feedback symbols. Note that, in this paper, we ignore the time spent in training although it can be easily incorporated by seeing that $\min(m,n)$ timeslots are spent in training the receiver and the transmitter along the lines of [12], and replacing r with $\frac{L}{L-\tau}r$ where τ is the time slots used for the training. Moreover, as pointed out in [12], all the antennas at the transmitter and receiver may not be needed for general multiplexing when L is comparable to the number of antennas. The assumption of $L - \tau \geq m + n$ is taken in this paper so that the outage probability is of the same order as the error probability as in [12]. Further, $r < \min(m,n)$ will be assumed throughout the paper. All the results in the paper are summarized in Table I where the FDD results are parameterized by multiplexing (r) and levels of feedback (K). For notation, R represents models with perfect knowledge of channel at the receiver while $\widehat{\mathrm{R}}$ represents those on which the receiver is trained on a noisy channel. Further, $\mathrm{T_q}$ represents perfect quantized feedback while T_c represents channel knowledge using a training signal from the receiver on a perfect feedback channel. $\widehat{\mathrm{T}}_{\mathrm{q}}$ and $\widehat{\mathrm{T}}_c$ represents that there is an error in the feedback signal due to noise.

III. No feedback channel

In this section, we will consider that there is no feedback link from the receiver to the transmitter and all the transmissions are one-way from the transmitter to the receiver. We consider the cases of perfect channel state information and no channel state information at the receiver.

TABLE I

SUMMARY OF DIVERSITY MULTIPLEXING TRADEOFFS (IGNORING TRAINING AND FEEDBACK OVERHEADS).

Case	Main Characteristic	D-M Tradeoff
CSIR	Perfect Information at R	$d_{\text{CSIR}} = G(r, 1)$
$\text{CSI}\widehat{\text{R}}$	No information at transmitter or receiver	$d_{\text{CSI}\widehat{\text{R}}} = G(r, 1)$
CSIRT_q	Perfect quantized CSI at transmitter Perfect CSI at receiver	$d_{\text{CSIRT}_\text{q}}(r, K) = G(r, 1 + d_{\text{CSIRT}_\text{q}}(r, K - 1))$, $d_{\text{CSIRT}_\text{q}}(r, 0) = 0$
$\text{CSI}\widehat{\text{R}}\text{T}_\text{q}$	Perfect quantized CSI at transmitter	$d_{\text{CSI}\widehat{\text{R}}\text{T}_\text{q}}(r, K) = G(r, 1 + G(r, 1))$
$\text{CSIR}\widehat{\text{T}}_\text{q}$	Noisy quantized CSI at transmitter Perfect CSI at receiver	$d_{\text{CSIR}\widehat{\text{T}}_\text{q}}(r, K) = \min\Big(d_{\text{CSIRT}_\text{q}}(r, K), \max_{q_j \le 1 + d_{\text{CSIRT}_\text{q}}(r,j)}$ $\min_{i=1}^{K-1}(mn((q_i)^+ - (q_{i-1})^+) + d_{\text{CSIRT}_\text{q}}(r, i))\Big)$
$\text{CSI}\widehat{\text{R}}\widehat{\text{T}}_\text{q}$	No Genie-aided information with quantized feedback	$d_{\text{CSI}\widehat{\text{R}}\widehat{\text{T}}_q}(r, K) = G(r, 1 + G(r, 1))$
CSIRT_c	Noise-free training channel to transmitter Perfect CSI at receiver	$d_{\text{CSIRT}_c} = \infty$
$\text{CSI}\widehat{\text{R}}\text{T}_\text{c}$	Noiseless channel to transmitter	$d_{\text{CSI}\widehat{\text{R}}\text{T}_c} = \infty$
$\text{CSIR}\widehat{\text{T}}_\text{c}$	Noisy channel to transmitter Perfect CSI at receiver	$d_{\text{CSIR}\widehat{\text{T}}_\text{c}} = \infty$
$\text{CSI}\widehat{\text{R}}\widehat{\text{T}}_\text{c}$	No Genie-aided information with symmetric two-way channel	$d_{\text{CSI}\widehat{\text{R}}\widehat{\text{T}}_c} = mn + G(r, mn)$

A. CSIR: Perfect Channel State Information at the receiver

Suppose that the receiver has a perfect channel estimate H while the transmitter does not know H. The transmitter sends signal at rate R assuming that it will be possible for the decoder to decode. The fading blocks in which the receiver is not able to decode results in outage. The diversity multiplexing tradeoff in this case is given as follows.

Theorem 1 ([12])**.** *The diversity multiplexing tradeoff curve for the case of perfect CSIR is given by* $d_{CSIR} = G(r, 1)$ *for* $r < \min(m, n)$.

B. CSI$\widehat{R}$: Estimated CSIR

Suppose that both the receiver and the transmitter have no channel state information. In this case, the transmitter first trains the receiver at a power level of SNR and then sends data at rate R. Using this training based scheme, the diversity multiplexing tradeoff is given as follows.

Theorem 2 ([12])**.** *The diversity multiplexing tradeoff curve for the case of estimated CSIR is given by* $d_{CSI\widehat{R}} = G(r, 1)$ *for* $r < \min(m, n)$.

We note in this section that the channel state information at the receiver does not affect the diversity order calculations, i.e., the receiver with no knowledge of the channel state information can be trained to give the same diversity multiplexing tradeoff as the receiver with perfect knowledge of channel state information at the receiver.

IV. FDD SYSTEMS: QUANTIZED FEEDBACK CHANNEL

In this section, we will consider that there is a feedback channel on which quantized feedback coded as power levels can be sent. There are K power levels on the feedback link. $K = 1$ represents no feedback, and hence we will consider $K > 1$ in this section. We will consider two cases: (1) when the receiver has perfect channel state information and (2) when it has no channel state information and is trained by the transmitter. For both of these cases, we also consider two cases representing whether the feedback signal transmitted from the receiver is received perfectly or is corrupted by feedback channel noise. A non-power controlled feedback scheme was used in [5] to get an increase in diversity order with imperfect feedback while, in this paper, we extend the results to a power controlled feedback scheme.

A. CSIRT$_q$: Perfect CSIR with quantized CSIT

Suppose that the receiver has perfect channel state information, and the feedback link to the transmitter is also perfect and thus the feedback signal transmitted from the receiver is received perfectly at the transmitter. In this case, the receiver first decides a feedback index based on the channel and hence decides the power level. The receiver then sends this power level to the transmitter which is received perfectly and hence the transmitter decides on a power level based on what it decoded. Using this scheme, the following diversity multiplexing tradeoff can be obtained.

Theorem 3 ([3])**.** *Suppose that* $K \ge 1$ *and* $r < \min(m, n)$. *Then, the diversity multiplexing tradeoff for the case of perfect CSIR with noiseless quantized feedback is given as* $d_{CSIRT_q} = B^*_{m,n,K}(r)$, *where* $B^*_{m,n,K}(r)$ *is defined recursively as* $B^*_{m,n,K}(r) = G(r, 1 + B^*_{m,n,K-1}(r))$, $B^*_{m,n,0}(r) = 0$.

B. CSI$\widehat{R}$T$_q$: Estimated CSIR with perfect CSIT$_q$

In this subsection, we consider that the receiver does not know channel state information while the feedback link from the receiver to the transmitter is perfect. The transmission scheme in this scenario follows three rounds. In the first round, the transmitter sends a training signal to the receiver at power level of SNR. In the second round, the receiver uses the estimated channel state information to choose a feedback index for the transmitter which is received perfectly at the transmitter. In the third round, the transmitter chooses a power level based on what it decoded to first train the receiver and then transmits data at the same power level. Using this mechanism, the following diversity multiplexing tradeoff can be obtained.

Theorem 4 ([13]). *Suppose that $K > 1$ and $r < \min(m, n)$. Then, the diversity multiplexing tradeoff is given by $d_{CSI\widehat{R}T_q} = G(r, 1 + G(r, 1))$.*

Corollary 1. *The maximum diversity order that can be achieved in this case for $r \to 0$ is $mn(1 + mn)$ which is $(mn)^2$ greater than the diversity order without feedback.*

C. CSIR$\widehat{T}_q$: Perfect CSIR with noisy CSIT$_q$

In this subsection, the receiver has perfect channel state information while the feedback link is not perfect. The signal transmitted from the receiver is not received perfectly at the transmitter. We consider that the feedback is also power controlled and MAP estimation is done to decode the power levels. In this scenario, the receiver calculates and transmits the power level to the transmitter which then sends data at a power level based on what it decoded. Using this mechanism, the following diversity multiplexing tradeoff can be obtained.

Theorem 5 ([13]). *Suppose that $K > 1$ and $r < \min(m, n)$. Then, the diversity multiplexing tradeoff is given by $d_{CSIR\widehat{T}_q} \doteq \min(B_{m,n,K}(r), \max_{q_j \le 1+B_{m,n,j}(r)} \min_{i=1}^{K-1}(mn((q_i)^+ - (q_{i-1})^+) + B_{m,n,i}(r)))$.*

Corollary 2. *The diversity multiplexing tradeoff with one bit of imperfect feedback is same as the diversity multiplexing tradeoff with one bit of perfect feedback. In other words, for $K = 2$, $d_{CSIR\widehat{T}_q} = G(r, 1 + G(r, 1))$.*

Corollary 3. *For $K \to \infty$ and $r \to 0$, the maximum diversity order that can be obtained in this case is $mn(mn + 2)$ which is $(mn)^2$ more as compared to feedback scheme in [5].*

D. CSI$\widehat{R}\widehat{T}_q$: Estimated CSIR with noisy CSIT$_q$

In this subsection, the receiver does not know the channel state information. Moreover, the feedback link from the receiver to the transmitter is imperfect. The transmission scheme in this scenario follows three rounds. In the first round, the transmitter sends a training signal to the receiver at power level of SNR. In the second round, the receiver uses the estimated channel state information to obtain a feedback index for the transmitter which is received imperfectly at the transmitter. In the third round, the transmitter uses the power level based on what it decoded to first train the receiver and then transmits data at the same power level. Using this mechanism, the following diversity multiplexing tradeoff can be obtained.

Theorem 6 ([13]). *Suppose that $K > 1$ and $r < \min(m, n)$. Then, the diversity multiplexing tradeoff is given by $d_{CSI\widehat{R}\widehat{T}_q} = G(r, 1 + G(r, 1))$.*

Corollary 4. *The diversity multiplexing tradeoff with imperfect feedback and receiver training is same as the diversity multiplexing tradeoff with one bit of perfect feedback with perfect knowledge of channel state information at the receiver.*

In this section, we note that the knowledge of channel state information at the receiver also matters for diversity order. The diversity order obtained after training the receiver is less than that when the channel state information is perfectly known to the receiver. We further note that if the receiver does not know the channel state information, the diversity order is the same, irrespective of the feedback link being perfect or noisy. Since the forward and the backward channels were independent, if one of the nodes know the channel of the link to it, it cannot train the other node to its corresponding channel well (since it does not know of the channel to the other node) thus resulting in bounded gains.

V. TDD SYSTEMS: SYMMETRIC CHANNEL

In this section, we will consider that there is a symmetric feedback channel on which training signals can be sent. We will consider two cases: (1) when the receiver has perfect channel state information and (2) when it has no channel state information and is trained by the transmitter. For both of these cases, we also consider two cases representing whether the training signal transmitted from the receiver is received perfectly or is corrupted by additive white noise in the feedback channel.

A. CSIRT$_c$: Perfect CSIR with CSIT obtained by perfect training

Suppose that the receiver knows the channel state information perfectly and the transmitter can receive training symbols perfectly. The receiver can train the transmitter perfectly and thus, we get infinite diversity order.

Theorem 7. *Suppose that $r < \min(m, n)$. Then, the diversity order for the case of perfect CSIR with noiseless continuous feedback signal is $d_{CSIRT_c} = \infty$.*

B. CSI$\widehat{R}$T$_c$: Estimated CSIR with perfect CSIT$_c$

Suppose that the feedback channel is perfect and neither the transmitter nor the receiver know any channel state information. In this scenario, a training symbol is sent from the receiver letting the transmitter know the exact channel. Let the channel realization be H, and the non-zero eigenvalues of $HH^\dagger$ be $\lambda_1, \cdots, \lambda_{\min(m,n)}$. Let $\lambda_i \doteq \mathsf{SNR}^{-\alpha_i}$. Then, the transmitter trains the receiver using power level SNR divided by the probability of $\alpha = (\alpha_1, \cdots, \alpha_{\min(m,n)})$ as in [8]. This is followed by sending the data at the same power level. This scheme achieves infinite diversity order in this case. Hence,

Theorem 8. *Suppose that $r < \min(m, n)$. Then, the diversity multiplexing tradeoff for the case of perfect CSIT$_c$ with receiver training is $d_{CSI\widehat{R}T_c} = \infty$.*

C. CSIR$\widehat{T}_c$: Perfect CSIR with estimated CSIT$_c$

In this subsection, we consider that the receiver has perfect channel state information while the symmetric feedback link is imperfect and the received feedback signal is not the same as transmitted due to the action of noise. In this scenario, the receiver sends a power controlled symbol to the transmitter for training. Consider that the receiver quantizes the power levels into $\{0, 1, \cdots, K-1\}$. This power division is same as is done in [3]. When the channel is H, the receiver finds the eigenvalues of $HH^\dagger$ as $\lambda_i = i^{th}$ eigenvalue of $HH^\dagger$. Let $\lambda_i \doteq \mathsf{SNR}^{-\alpha_i}$ and let $\alpha_m = \min(\alpha_i)$. The receiver encodes the training symbol at power level $\mathsf{SNR}^{\alpha_m + \frac{i+1}{2K}}$ to communicate power level i. The transmitter estimates the power level by observing the received power and sends data at the power level as in Section IV-A. It can be seen that this scheme results in diversity multiplexing tradeoff equivalent to K levels of perfect feedback and hence as K grows large enough, we get infinite diversity.

Theorem 9. *Suppose that $r < \min(m, n)$. Then, the diversity order for the case of perfect CSIR with training the transmitter is $d_{CSIR\widehat{T}_c} = \infty$.*

D. CSI$\widehat{\widehat{RT}}_c$: Estimated CSIR with estimated CSIT$_c$

In this subsection, we consider that neither the receiver nor the transmitter know the exact channel. The feedback channel and the forward channel are symmetric and both are imperfect. In this scenario, the communication proceeds by the following two rounds. In the first round, the receiver trains the transmitter with a power level of SNR. In the second round, the transmitter uses the estimated channel to select a power level on which the receiver is trained and the data is transmitted.

Theorem 10. *Suppose that $r < \min(m, n)$. Then, the diversity multiplexing tradeoff is given by $d_{CSI\widehat{\widehat{RT}}_c} = mn + G(r, mn)$.*

Corollary 5. *The maximum diversity order as $r \to 0$ is $mn(1 + mn)$ which is same as the corresponding diversity order in the case of CSI$\widehat{\widehat{RT}}_q$. Thus, when all the noise in both the direct and the feedback channel are accounted for, the maximum diversity order is same in both the symmetric and asymmetric channel models.*

In this section, we note that if the transmitter or the receiver knows perfect channel state information, unbounded diversity gains are obtained. Since the forward and the backward channels were assumed the same, the node knowing the channel state perfectly can train the other using power controlled training and result in unbounded diversity order. However, if none of the nodes know the channel state information perfectly, we get bounded diversity order.

VI. Numerical Results

We now see the diversity multiplexing tradeoff for the different scenarios. In Figure 1, $m = n = 2$. For the quantized feedback model, one bit of feedback is assumed. We can see the seven diversity multiplexing tradeoffs in the figure, and the remaining three cases give infinite diversity. When the feedback link is perfect, the FDD system gives bounded diversity order while the TDD system gives unbounded diversity order. When all the imperfections are accounted for, the symmetric channel in TDD gives better diversity multiplexing performance than the asymmetric channel in FDD systems although the two meet as $r \to 0$.

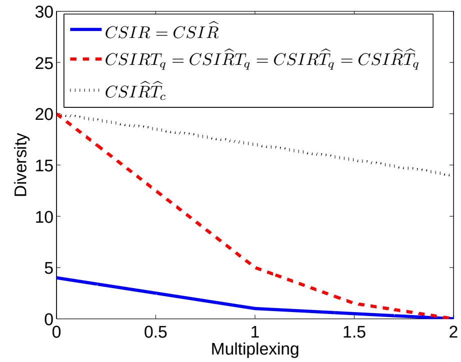

Fig. 1. The diversity multiplexing tradeoff for $m = n = 2$ and 1 bit of feedback.

VII. Conclusions

In this paper, we compare the FDD and TDD channel feedback models in the presence of errors. We find in TDD systems that if one of the nodes have perfect channel state information, the diversity order is unbounded which is not true in FDD systems. However when all the imperfections are accounted, FDD and TDD systems give same diversity order at arbitrarily low multiplexing gains while the TDD model achieves higher diversity order than FDD model for general multiplexing gains.

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