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THE ANOMALY STRUCTURE OF $(2,0)$ HETEROTIC WORLDSHEET SUPERGRAVITY WITH GAUGED R-INVARIANCE

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ABSTRACT

We present a superfield approach to the theory of $(2,0)$ worldsheet supergravity. The $(2,0)$ structure group is enlarged to Lorentz $\times U(1)$ and the anomaly structure of this extended theory is studied. We then modify the theory by adding the Lorentz Chern-Simons term and present a two dimensional Green-Schwarz mechanism. The resultant theory corresponds to superconformally invariant string theories outside the critical dimension.

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1 Introduction

It has been shown [1] that all four-dimensional string vacua with $N = 1$ spacetime supersymmetry are associated with new minimal supergravity which is a rather restrictive form of supergravity. One of the features of this supergravity is the existence of a local $U(1)$ R-invariance. It is of interest to ask whether this local $U(1)$ invariance is anomalous at the quantum level. This has been answered in the affirmative [2] but the calculation is extremely tedious and opaque. Two dimensional $(2, 0)$ supergravity has a similar $U(1)$ symmetry and hence is a good laboratory to study this R-invariance. In this paper, we have studied the theory of heterotic $(2, 0)$ supergravity and compute the superfield anomaly structure. This is done by choosing an algebraic gauge where supergraph calculations are simplified. We also provide a comprehensive discussion on the relevant supercurrents and their conservation laws. This also provides a framework to study various target space symmetries using techniques introduced in [3] after suitably generalising to the $(2, 0)$ case. Recently a four-dimensional superspace version of the Green-Schwarz mechanism [4] has been employed in [2] to cancel the anomaly. As is true about most calculations in four dimensional supergravity, explicit off-shell calculations are prohibitively complicated. Unlike the four dimensional case, we are able to make a supergraph calculation of the complete off-shell anomaly. In this paper, we also study a two dimensional Green-Schwarz mechanism. We first modify the theory by coupling the Chern-Simons term to a chiral scalar. This is done by modifying the chirality constraint on the scalar. We find that this not only provides us with the required Chern-Simons term but also a Green-Schwarz term(albeit with a fixed coefficient). We also introduce a separate Green-Schwarz term. We find that this cancels the anomaly for an appropriate choice of couplings for the Chern-Simons and the Green-Schwarz terms. It removes the restriction on the numbers of chiral scalars and fermions. Since the number of scalars correspond to the dimension of the target space, this may have some relevance to non-critical strings. This will be studied elsewhere. Again, this an interesting laboratory to study the superspace Green-Schwarz mechanism.

The paper is organised as follows. Section 2 deals with $(2,0)$ supergeometry. Section 3 gives the super-Weyl transformations. Section 4 discusses supergauge which is an algebraic(non-derivative) gauge suitable for supergraph calculations. Section 5, we give the matter Lagrangians and also derive the ghost Lagrangians. In section 6, we study the various conservation laws for the relevant supercurrents and interpret them. We then make a supergraph calculation of the one-loop effective action to determine the structure of the anomaly and find conditions for its cancellation. Finally, in section 7, we present a two dimensional Green-Schwarz mechanism which is used to cancel the anomaly. We briefly discuss how it could be related to non-critical strings.

2 (2,0) Supergeometry

In this section we define (2,0) superspace and discuss various aspects of (2,0) supergravity. The structure group of the superspace is chosen to be $SO(1,1)\times U(1)$. The (p,q) superalgebra is described in appendix B. Superspace is given by $z^M = (x^m, \theta^{+1}, \bar{\theta}^{+\bar{1}})$. One introduces a supervielbien $E_M{}^A$ and the corresponding one-form $E^A = dz^M\ E_M{}^A$ where $A = a, +1, +\bar{1}$ are tangent space indices(See appendix A, [5] and [6] for notation). The one-forms have the following U(1) weights.

$$w(E^a) = 0, w(E^{+1}) = i, w(E^{+\bar{1}}) = -i \tag{2.1}$$

We also introduce two Lie-Algebra valued one-form gauge connections $\Phi_B{}^A = dz^M \Phi_{MB}{}^A$ and $A_B{}^A = dz^M A_{MB}{}^A$ corresponding to the Lorentz and U(1) groups respectively. Since both the groups are abelian, it is convenient to write $\Phi_{MB}{}^A = \Phi_M \kappa_B{}^A$ and $A_{MB}{}^A = A_M \varepsilon_B{}^A$ where $\kappa_B{}^A = \text{diag}\ (+1, -1, \frac{1}{2}, \frac{1}{2})$ and $\varepsilon_B{}^A = \text{diag}\ (0, 0, +i, -i)$ Similarly the torsion is defined by $T^A = \mathcal{D}E^A$ where $\mathcal{D}$ represents the covariant exterior derivative. The curvatures for the

Lorentz and U(1) groups are defined by $R = d\Phi$ and $F = dA$ respectively. Covariant derivatives are given by

$$\nabla_M \Omega^A = \partial_M \Omega^A + (-)^{mb} \Omega^B \Phi_{MB}{}^A \; + (-)^{mb} \Omega^B A_{MB}{}^A \tag{2.2}$$

where Ω^A is an arbitrary form.

The torsions and curvatures are subject to the following Bianchi identities.

$$\begin{aligned} \mathcal{D}T^A &= E^B R_B{}^A + E^B F_B{}^A \\ \mathcal{D}R &= 0 \\ \mathcal{D}F &= 0 \end{aligned} \tag{2.3}$$

The supergravity multiplet thus introduced is highly reducible. This reducibility is fixed by the following choice of torsion constraints [7]. A geometric basis for these torsion constraints has been provided in [8].

$$\begin{aligned} T_{\alpha\beta}{}^c &= -2i\delta^{c+}\delta_{\alpha\beta} \\ T_{+\alpha}{}^a &= T_{\alpha\beta}{}^\gamma = 0 \\ T_{a\beta}{}^\gamma &= T_{ab}{}^c = 0 \end{aligned} \tag{2.4}$$

We solve the Bianchi identities subject to these torsion constraints. The results are

$$-\tfrac{1}{2i}\, R_{-+1} \;=\; F_{-+1} = T_{+-}{}^{+\bar{1}} \tag{2.5}$$

$$\nabla_{+1} R_{-+1} \;=\; 0 \tag{2.6}$$

$$\tfrac{1}{2}\, R_{+-} + iF_{+-} \;=\; -2i\nabla_{+\bar{1}} T_{+-}{}^{+\bar{1}} \tag{2.7}$$

$$T_{-\alpha}{}^a = R_{\alpha\beta} = F_{\alpha\beta} \;=\; R_{+\beta} = F_{+\beta} = 0 \tag{2.8}$$

where $\alpha, \beta = +1, +\bar{1}$ and we have left out the complex conjugate equations. So the independent non-vanishing torsion components are $T_{+-}{}^\alpha$. All curvatures can be determined in terms of these components.

Note that, we have not introduced in (2.4) any constraints on the U(1) curvature. This is unnecessary since the Bianchi identities lead us to restrictions on the curvature. For example, the choice of ${T_{++1}}^{+1} = 0$ as a constraint implies through the Bianchi identities that $F_{+1+1} = 0$. We would also like to point out that if one works with a structure group of only the Lorentz group, one finds that certain torsion components can be interpreted as gauge connections for U(1) group [9]. This is probably a generic feature of (p,q) supergravity.

3 Super-Weyl Transformations

Super-Weyl transformations are those local tangent space transformations on ${E_M}^A$, ${\Phi_{MB}}^A$ and ${A_{MB}}^A$ that leave the torsion constraints (2.4) invariant [10] and for which infinitesimally,

$$\delta {E_M}^a = l {E_M}^a, \tag{3.1}$$

where l is a superfield. For the chosen constraints we find the need to introduce another superfield h to parametrise these transformations. The infinitesimal super-Weyl transformations are

$$\begin{aligned}
\delta_w {E_M}^a &= l {E_M}^a, && (3.2)\\
\delta_w {E_M}^\alpha &= {E_M}^\beta \left(\tfrac{1}{2}\, l {\delta_\beta}^\alpha + h {\varepsilon_\beta}^\alpha \right) - i {E_M}^+ \delta^{\alpha\beta} \partial_\beta l, && (3.3)\\
\delta_w \phi_M &= N_a {E_M}^a \partial_a l + {E_M}^\alpha \partial_\alpha l, && (3.4)\\
\delta_w A_M &= -\partial_M h + (\tfrac{1}{2i}\,) {E_M}^+ \, [\partial_{+1}, \partial_{+\bar{1}}] l + {E_M}^\beta {\varepsilon_\beta}^\alpha \partial_\alpha l. && (3.5)
\end{aligned}$$

where $\alpha = +1, +\bar{1}$. Note that under $U(1)$ transformations

$$\begin{aligned}
\delta {E_M}^\alpha &= {E_M}^\beta {\varepsilon_\beta}^\alpha \lambda && (3.6)\\
\delta A_m &= -\partial_m \lambda && (3.7)
\end{aligned}$$

where λ is the $U(1)$ parameter. It follows that the transformations corresponding to h are identical to U(1) gauge transformations and will not be regarded as part of the super-Weyl transformations. So we obtain the true super-Weyl transformations by setting $h = 0$ in the above equations.

$$\delta_w E_a{}^M = -l E_a^M + i\delta_a{}^+ \delta^{\alpha\beta} \partial_\alpha l \ E_\beta{}^M, \tag{3.8}$$

$$\delta_w E_\alpha{}^M = -\tfrac{1}{2}\ l E_\alpha^M, \tag{3.9}$$

$$\delta_w \phi_a = -l\phi_a + i\delta_a{}^+ \delta^{\alpha\beta} \partial_\alpha l \ \phi_\beta + N_a \partial_a l, \tag{3.10}$$

$$\delta_w \phi_\alpha = -\tfrac{1}{2}\ l\phi_\alpha + \partial_\alpha l, \tag{3.11}$$

$$\delta_w A_a = -l A_a + \delta_a{}^+ \left\{ i\delta^{\alpha\beta} \partial_\alpha l \ A_\beta - \tfrac{1}{2}\ [\partial_{+1}, \partial_{+\bar{1}}] l \right\}, \tag{3.12}$$

$$\delta_w A_\alpha = -\tfrac{1}{2}\ l A_\alpha + \varepsilon_\alpha{}^\beta \partial_\beta l. \tag{3.13}$$

For the (2,0) theory with ungauged $U(1)$, the super-Weyl transformations were parametrised by a chiral superfield [11]. Here it has become a full unconstrained superfield. Note that $A_+|$ can be set to zero using the highest component of l, the super-Weyl parameter. There would be no Faddeev-Popov ghosts associated with this gauge fixing since it transforms algebraically without any derivatives.

4 Super Gauge

In this section we go into a gauge, which we shall refer to as super gauge. This gauge is convenient for supergraph calculations which we will use for calculating the anomaly. This gauge is obtained by fixing all algebraic (that is, non derivative) gauge transformations. Since these are algebraic transformations, there are no Faddeev-Popov ghosts associated with fixing them. Secondly, we introduce the superconformal gauge. This gauge is of interest because it can be shown that the anomaly is local in this gauge (see equation (6.41)). It is the counterpart of the conformal gauge in the bosonic string.

Supergauge:

We first determine the independent components of the vielbein to linear order. Expand the vielbein as

$$E_M{}^A = e_M{}^A + e_M{}^B \; h_B{}^A \tag{4.1}$$

where $e_M{}^A$ is the flat vielbein and compute the torsion and curvature tensors to linear order in $h_B{}^A$. The torsion constraints as well as the Bianchi identities, then imply that the $h_B{}^A$'s are not all independent. We find that the only independent $h's$ are $h_-{}^a, h_{+1}{}^a, h_{+1}{}^{+1}$ and the complex conjugate $h's$. All the other variations are determined by the following relations.

$$\begin{aligned}
h_+{}^+ &= \tfrac{1}{2i}\left(D_{+1}h_{+\bar{1}}{}^+ + D_{+\bar{1}}h_{+1}{}^+\right) + \left(h_{+1}{}^{+1} + h_{+\bar{1}}{}^{+\bar{1}}\right) && (4.2)\\
h_+{}^- &= \tfrac{1}{2i}\left(D_{+1}h_{+\bar{1}}{}^- + D_{+\bar{1}}h_{+1}{}^-\right) && (4.3)\\
h_+{}^{+1} &= \tfrac{1}{2i}\left(D_{+1}h_+{}^+ - D_+h_{+1}{}^+ + \Omega_{+\bar{1}}\right) && (4.4)\\
h_-{}^{+\bar{1}} &= \tfrac{1}{2i}\left(D_{+1}h_-{}^+ - D_-h_{+1}{}^+\right) && (4.5)\\
h_{+1}{}^{+\bar{1}} &= -\tfrac{1}{2i}\, D_{+1}h_{+1}{}^+ && (4.6)\\
\phi_{+1} &= D_{+1}h_-{}^- - D_-h_{+1}{}^- && (4.7)\\
D_{+1}h_{+1}{}^- &= 0 && (4.8)\\
D_{+1}h_{+1}{}^{+1} &= -\tfrac{1}{2}\,\phi_{+1} - iA_{+1} && (4.9)
\end{aligned}$$

where we have not given the complex conjugate equations.

We now give the transformations of $h_B{}^A$. We demand that $e_M{}^A$ not transform under any of the transformations. We obtain to lowest order in $h_B{}^A$,

$$\begin{aligned}
\delta h_b{}^A &= -D_B\xi^A + \kappa_b{}^A L + \delta_b{}^A l && (4.10)\\
\delta h_\beta{}^A &= -D_B\xi^A + 2i\xi^\gamma\delta_{\gamma\beta}\delta^{A,+} + \kappa_\beta{}^A L + \tfrac{1}{2}\,\delta_b{}^A l + \varepsilon_\beta{}^A\lambda && (4.11)
\end{aligned}$$

where $\xi, L, l,$ and λ are parameters corresoponding to supercoordinate, Lorentz, super-Weyl and U(1) respectively. We shall reduce the number of independent $h's$ by means of algebraic gauge fixing. We have that

$$\delta h_{+1}{}^+ = 2i\xi^{+\bar{1}} - D_{+1}\xi^+ \tag{4.12}$$

Then using the $\xi^{+\bar{1}}$ transformation, we choose

$$h_{+1}{}^{+} = 0 \tag{4.13}$$

This is obviously an algebraic gauge choice since no derivatives are involved in the above transformation law. In order that we preserve this gauge condition, we need

$$\xi^{+\bar{1}} = \frac{1}{2i}\left(D_{+1}\xi^{+}\right) \tag{4.14}$$

Since $h_{+1}{}^{-}$ is constrained as in equation (4.8) to be antichiral, we solve for it in terms of an unconstrained complex superfield V^{-}. Let

$$\begin{aligned} h_{+1}{}^{-} &= D_{+1}V^{-} && (4.15)\\ h_{+\bar{1}}{}^{-} &= D_{+\bar{1}}\bar{V}^{-} && (4.16)\\ V^{-} &= S^{-} + iU^{-} && (4.17) \end{aligned}$$

where $\bar{V}^{-} = (V^{-})^{*}$ and S^{-} and U^{-} are real superfields. We induce an extra gauge degree of freedom by introducing the prepotential V^{-}. This is given by

$$\delta V^{-} = 2\bar{K}^{-} \tag{4.18}$$

where $D_{+1}\bar{K}^{-} = 0$.

The transformation law for V^{-} under supercoordinate transformations is obtained from that of $h_{+1}{}^{-}$. We obtain

$$\delta V^{-} = -\xi^{-} + 2\bar{K}^{-} \tag{4.19}$$

This implies that S^{-} and U^{-} transform as

$$\begin{aligned} \delta S^{-} &= -\xi^{-} + (K + \bar{K}) && (4.20)\\ \delta U^{-} &= i(K - \bar{K}) && (4.21) \end{aligned}$$

Then the ξ^- transformation can be chosen so that

$$S^- = 0 \tag{4.22}$$

Again, note that this is an algebraic gauge choice. This gauge condition is preserved provided

$$\xi^- = (K^- + \bar{K}^-) \tag{4.23}$$

where $D_{+1}\bar{K}^- = 0$ and $(K)^* = \bar{K}^-$.

We have that $h_-{}^-$ and $(h_{+1}{}^{+1} + h_{+\bar{1}}{}^{+\bar{1}})$ transform as

$$\delta h_-{}^- = -D_-\xi^- - L + l \tag{4.24}$$

$$\delta(h_{+1}{}^{+1} + h_{+\bar{1}}{}^{+\bar{1}}) = -D_+\xi^+ + L + l \tag{4.25}$$

where we have used equation (4.14) and its complex conjugate equation. This implies that

$$\delta(h_-{}^- - h_{+1}{}^{+1} - h_{+\bar{1}}{}^{+\bar{1}}) = -2L - D_-\xi^- + D_+\xi^+ \tag{4.26}$$

Then we can use of the Lorentz transformations to choose,

$$h_-{}^- = h_{+1}{}^{+1} + h_{+\bar{1}}{}^{+\bar{1}} \tag{4.27}$$

$$\equiv \Sigma \tag{4.28}$$

To preserve this gauge condition, we make the following Lorentz transformation,

$$L = -\tfrac{1}{2}\left(D_-\xi^- - D_+\xi^+\right) \tag{4.29}$$

where ξ^- is restricted to be of the form in equation (4.23). At this point, let us recap the gauge choices we have made so far. They are

$$h_{+1}{}^+ = 0 \tag{4.30}$$

$$S^- = 0 \tag{4.31}$$

$$h_-{}^- = h_{+1}{}^{+1} + h_{+\bar{1}}{}^{+\bar{1}} \tag{4.32}$$

This implies that the independent non-vanishing components of the vielbein are ${h_{+1}}^{+1}$, ${h_{+\bar{1}}}^{+\bar{1}}$, ${h_-}^+$ and U^-. We shall make further gauge choices to reach supergauge.

The Bianchi identities imply that $F_{+1+1} = 0$. This implies that

$$D_{+1}A_{+1} = 0 \tag{4.33}$$

We solve for this in terms of a complex prepotential W.

$$A_{+1} = D_{+1}W \tag{4.34}$$

$$W = Q + iR \tag{4.35}$$

where Q and R are real superfields. The transformation law of W is derived from that of A_{+1} to be

$$\delta W = \lambda + il \tag{4.36}$$

We induce an extra gauge degree of freedom by introducing the prepotential W. This is given by

$$\delta W = -2i\bar{\sigma} \tag{4.37}$$

where $D_{+1}\bar{\sigma} = 0$. This implies that Q and R transform as

$$\delta Q = \lambda + i(\sigma - \bar{\sigma}) \tag{4.38}$$

$$\delta R = l - (\sigma + \bar{\sigma}) \tag{4.39}$$

Then, using the super-Weyl transformation l, we can choose

$$R = 0 \tag{4.40}$$

This gauge condition is preserved if

$$l = (\sigma + \bar{\sigma}) \tag{4.41}$$

Similarly, using the $U(1)$ transformation λ, we can choose

$$Q = 0 \tag{4.42}$$

This implies that

$$A_{+1} = 0 \tag{4.43}$$

This gauge condition is preserved provided we choose

$$\lambda = -i(\sigma - \bar{\sigma}) \tag{4.44}$$

We shall refer to the transformations (4.41) and (4.44) parametrised by σ as the residual $U(1)$-Weyl transformation. Finally, in this algebraic gauge, we are left with three independent components of the vielbein. They are $h_-{}^+$, U^- and Σ. We now give their transformations under the residual supercoordinate transformations.

$$\begin{aligned}
\delta h_-{}^+ &= -D_-\xi^+ \\
\delta U^- &= i(K^- - \bar{K}^-) \\
\delta\Sigma &= -\tfrac{1}{2}\left(D_-\xi^- + D_+\xi^+\right)
\end{aligned} \tag{4.45}$$

At this point, we would like to remark that the transformation law for U^- seems to indicate that U^- can be set to zero by means of an algebraic gauge fixing. This is not true since K^- , but not U^- is constrained. The derivatives in the transformation law are hidden in the constraints on K^-. This can be shown explicitly by

$$\delta U^-|_{\theta^{+1}\bar{\theta}^{+\bar{1}}} = i\partial_+(K^-| + \bar{K}^-|) \tag{4.46}$$

This shows that the transformation involves derivatives. Under the residual $U(1)$-Weyl transformations, we have that

$$\begin{aligned}
\delta h_-{}^+ &= 0 \\
\delta U^- &= 0 \\
\delta\Sigma &= (\sigma + \bar{\sigma})
\end{aligned} \tag{4.47}$$

It can be shown that the Bianchi identities subject to the torsion constraints implies that Σ is expressible in the following form.

$$\Sigma = \tfrac{1}{2}\,(\varphi + \bar{\varphi}) \tag{4.48}$$

where $D_{+1}\bar{\varphi} = 0$.

The proof is as follows. As a consequence of the Bianchi identities, we obtain(see equation (4.9))

$$\tfrac{1}{2}\,\phi_{+1} + iA_{+1} + D_{+1}{h_{+1}}^{+1} = 0 \tag{4.49}$$

In this gauge, we have

$$A_{+1} \;=\; 0 \tag{4.50}$$

$$\phi_{+1} \;=\; D_{+1}(\Sigma - iD_{-}U^{-}) \tag{4.51}$$

Substituting for A_{+1} and ϕ_{+1} into (4.49), we get

$$D_{+1}(\tfrac{1}{2}\,\Sigma + \tfrac{1}{2i}\,D_{-}U^{-} + {h_{+1}}^{+1}) = 0 \tag{4.52}$$

We solve this equation by introducing an antichiral field $\bar{\varphi}$ $(D_{+1}\bar{\varphi} = 0)$. This implies

$$(\tfrac{1}{2}\,\Sigma + \tfrac{1}{2i}\,D_{-}U^{-} + {h_{+1}}^{+1}) = \bar{\varphi} \tag{4.53}$$

The above equation and its complex conjugate imply

$$\Sigma \;=\; \tfrac{1}{2}\,(\varphi + \bar{\varphi}) \tag{4.54}$$

$${h_{+1}}^{+1} - {h_{+\bar{1}}}^{+\bar{1}} \;=\; iD_{-}U^{-} + (\bar{\varphi} - \varphi) \tag{4.55}$$

where we have used $\Sigma = {h_{+1}}^{+1} + {h_{+\bar{1}}}^{+\bar{1}}$. This completes the proof.

At this stage we could use the super-Weyl transformations to set $\varphi = 0$. We shall not do so since we would like to keep φ for calculating the anomaly.

<u>Superconformal Gauge:</u>

One can make further gauge choices after reaching the supergauge. These involve derivative transformations and have non-trivial Faddeev-Popov ghosts associated with them. We set $h_-{}^+ = 0$ and $U^- = 0$ using supercoordinate transformations (4.45). This will be given in the next section. The only non-zero prepotentials in this gauge is φ and its complex conjugate. This is the superconformal gauge. In this gauge we have

$$\partial_- = e^{\frac{1}{2}\,(\varphi+\bar{\varphi})} D_- \tag{4.56}$$

$$\partial_{+1} = e^{\frac{1}{4}\,(3\bar{\varphi}-\varphi)} D_{+1} \tag{4.57}$$

This can also be reached from the Lorentz-Weyl-U(1) gauge. In the Lorentz-Weyl-U(1) gauge, we have

$$\partial_- = e^{\Sigma-\Upsilon} D_-, \tag{4.58}$$

$$\partial_{+1} = e^{(\frac{1}{2}\,(\Sigma+\Upsilon)+i\Lambda)} D_{+1}, \tag{4.59}$$

$$A_{+1} = e^{(\frac{1}{2}\,(\Sigma+\Upsilon)-i\Lambda)} - iD_{+\bar{1}}(\Sigma + i\Lambda). \tag{4.60}$$

We set $\Upsilon = 0$ using Lorentz transformations. We then use a combination of the U(1) and super-Weyl to set $A_{+1} = 0$. This implies that $(\Sigma + i\Lambda)$ should be replaced by a antichiral superfield $\bar{\varphi}$ $(D_{+1}\bar{\varphi} = 0)$. Using the complex conjugate relation, we obtain that

$$\Sigma \longrightarrow \tfrac{1}{2}\,(\varphi + \bar{\varphi}) \tag{4.61}$$

$$\Lambda \longrightarrow -\tfrac{1}{2i}\,(\varphi - \bar{\varphi}) \tag{4.62}$$

Finally, the residual transformations which preserve this gauge are given by

$$\delta\varphi = \sigma \tag{4.63}$$

where $D_{+\bar{1}}\sigma = 0$. The residual transformations can be identified as follows. $\frac{1}{2}\,(\sigma + \bar{\sigma})|$ corresponds to weyl transformations and $-\frac{1}{2i}\,(\sigma - \bar{\sigma})|$ corresponds to axial U(1) transformations. This is easily seen from the lowest components of (4.41) and (4.44).

5 Matter and Ghost Lagrangians

In this section we give the matter and ghost Lagrangians. We introduce two types of matter, a complex chiral scalar Z^m and a complex chiral spinor Ψ^I with Lorentz charge $-1/2$. The Z^m are covariantly chiral,

$$\nabla_{+\bar{1}} Z^m = 0 \tag{5.1}$$

and Ψ^I_- are covariantly chiral,

$$\nabla_{+\bar{1}} \Psi^I_- = 0. \tag{5.2}$$

The subscript in Ψ_- refers to the $-1/2$ Lorentz weight of the chiral spinor. Their Lagrangians are (see [7])

$$S_Z = i \int dz E^{-1} \left(Z^m \nabla_- \bar{Z}^n - \bar{Z}^n \nabla_- Z^m \right) g_{mn} \tag{5.3}$$

$$S_\Psi = \int dz E^{-1} \left(\bar{\Psi}^I_- \Psi^J_- \right) \eta_{IJ} \tag{5.4}$$

where $m, n = 1, \ldots, D$ and $I, J = 1, \ldots, N$. Also $dz = dx d\theta^{+1} d\theta^{+\bar{1}}$ and $E \equiv sdet E_M{}^A$. We shall drop the subscript in Ψ_- for the rest of the discussion.

The pure gravity action cannot be written as an integral over the full superspace. It can be written as a chiral action which necessitates the introduction of a chiral density. Let ϵ be the chiral density.

$$S_{gr} = \int dx d\theta^{+1} \epsilon R_{-+\bar{1}} + \int dx d\theta^{+\bar{1}} \bar{\epsilon} R_{-+1} \tag{5.5}$$

The pure gravity action can be written in full superspace in the supergauge. It is given by

$$S_{gr} = \int dz E^{-1} A_- . \tag{5.6}$$

We now derive the ghost Lagrangian. As discussed in the previous section, we do not need to introduce Faddeev-Popov ghosts in order to reach the supergauge. However to reach the superconformal gauge from the supergauge, we have to introduce Faddeev-Popov ghosts.

We have that (see (4.45))

$$\delta h_-{}^+ = -D_- \xi^+ \tag{5.7}$$

We then use the ξ^+ transformation to choose

$$h_-{}^+ = 0 \tag{5.8}$$

The Jacobian for the transformation is provided by the following Lagrangian.

$$S_{gh1} = \int dz b_+ D_- c^+ \tag{5.9}$$

where c^+ is the ghost corresponding to ξ^+. and b_+ is the antighost.

We have (see (4.45))

$$\delta U^- = i(K^- - \bar{K}^-) \tag{5.10}$$

Using this transformation, we can choose

$$U^- = 0 \tag{5.11}$$

The Jacobian for the transformation is provided by the following Lagrangian.

$$S_{gh2} = \int dz b_{--}(c^- - \tilde{c}^-) \tag{5.12}$$

where $\tilde{c}^-$ is the ghost corresponding to $\bar{K}^-$ and hence satisfies the same constraint as $\bar{K}^-$. Similarly c^- is the ghost corresponding to K^- and hence satisfies the same constraint as K^-. So we have

$$D_{+\bar{1}} c^- = 0 \tag{5.13}$$

$$D_{+1} \tilde{c}^- = 0. \tag{5.14}$$

It can easily be seen that the above action reduces to the usual ghost Lagrangian for the bosonic string on eliminating auxiliary fields. The complete ghost Lagrangian is as follows.

$$S_{gh} = S_{gh1} + S_{gh2} \tag{5.15}$$

$$= \int dz \left(b_+ D_- c^+ + b_{--}(c^- - \tilde{c}^-)\right) \tag{5.16}$$

The ghosts and antighosts $c^+, c^-, \tilde{c}^-$ and b_{--} have anticommuting statistics. We assume that the ghost Lagrangian is valid outside the superconformal gauge. This enables us to calculate the ghost contribution to the anomaly. This is done by replacing flat derivatives by covariant ones and introducing the superdeterminant into the integration measure. We also assume that c^- is covariantly chiral and $\bar{c}^-$ is covariantly antichiral. Hence, outside of the superconformal gauge, the ghost action is

$$S = \int dz E^{-1} \left(b_+ \nabla_- c^+ + b_{--}(c^- - \tilde{c}^-)\right) \tag{5.17}$$

We shall now calculate quantum corrections to the effective action and determine which transformations are anomalous. We shall then determine the conditions under which the quantum theory is non-anomalous.

6 One Loop Effective Action

In this section, we present a supergraph calculation of the superdiffeomorphic anomaly. This will be done by computing the one loop effective action Γ_{eff} and varying it under supercoordinate transformations. We will be carrying out the calculation in the linearised approximation defined by

$$E_M{}^A = e_M{}^A + e_M{}^B \; h_B{}^A \tag{6.1}$$

where $e_M{}^A$ is the flat vielbein. We will also work in the supergauge defined in section 4. We find that the superdensity in the linearised approximation is

$$E^{-1} = 1 + Str(h_B{}^A) \tag{6.2}$$

where

$$Str(h_B{}^A) = h_+{}^+ + h_-{}^- - h_{+1}{}^{+1} - h_{+\bar{1}}{}^{+\bar{1}} \tag{6.3}$$

In the supergauge, we obtain

$$E^{-1} = 1 + \Sigma$$

$$= \quad 1 + \tfrac{1}{2}\,(\varphi + \bar{\varphi}) \tag{6.4}$$

We also have

$$\partial_A = D_A - h_A{}^B D_B \tag{6.5}$$

where D_A are the flat superspace derivatives.

The matter and ghost fields have to satisfy covariant constraints. We solve for them in terms of fields which are flat constraints. This is described in appendix C. We obtain

$$Z^m \quad = \quad Z_0^m - iU^-(D_- Z_0^m) \tag{6.6}$$

$$\Psi^I \quad = \quad \Psi_0^I - iU^-(D_-\Psi_0^I) - \tfrac{1}{2}\,\Sigma\,\Psi_0^I + \tfrac{1}{2i}\,(D_-U^-)\Psi_0^I \tag{6.7}$$

$$c^- \quad = \quad c_0^- - iU^-(D_- c_0^-) + \Sigma\, c_0^- + i(D_-U^-)c_0^- \tag{6.8}$$

where we introduced flat chiral fields Z_0^m, Ψ_0^I and c_0^-. The expansion for the other fields can be obtained by taking the complex conjugate of the above equations. c^+, b_+ and b_{--} are not constrained and hence do not have to be expanded in terms of flat fields.

We now substitute equations (6.4), (6.5) and equations (6.6) to (6.8), into the matter and ghost lagrangians to obtain the linearised coupling to (2,0) supergravity in the supergauge. We expand the actions as follows

$$S = S_0 + S_{int} \tag{6.9}$$

We obtain

$$S_{Z0} \quad = \quad \frac{i}{2}\int dz\left(Z_0^m D_-\bar{Z}_0^n - \bar{Z}_0^n D_- Z_0^m\right) g_{mn} \tag{6.10}$$

$$S_{Zint} \quad = \quad \int dz(\tfrac{1}{2}\; h_-{}^+ D_{+1} Z_0^m D_{+\bar{1}}\bar{Z}_0^n + 2U^- D_- Z_0^m D_-\bar{Z}_0^n) g_{mn} \tag{6.11}$$

$$S_{\Psi 0} \quad = \quad \int dz \bar{\Psi}_0^I \Psi_0^J \eta_{IJ} \tag{6.12}$$

$$S_{\Psi int} \quad = \quad i\int dz U^-(D_-\bar{\Psi}_0^I\Psi_0^J - \bar{\Psi}_0^I D_-\Psi_0^J)\eta_{IJ} \tag{6.13}$$

$$S_{gh0} \quad = \quad \int dz\left(b_+ D_- c^+ + b_{--}(c_0^- - \tilde{c}_0^-)\right) \tag{6.14}$$

$$
\begin{aligned}
S_{gh\ int} &= \int dz h_-{}^+ \left(D_+(c^+ b_+) - \tfrac{1}{2i} \left[(D_{+1} b_+)(D_{+\bar{1}} c^+) + (D_{+\bar{1}} b_+)(D_{+1} c^+) \right] \right) \\
&\quad - \int dz i U^- \left(2 b_{--} D_- (c_0^- + \tilde{c}_0^-) + D_-(b_{--})(c_0^- + \tilde{c}_0^-) \right) \qquad (6.15)
\end{aligned}
$$

where we have rescaled fields b_{--}, b_+ and c^+ to get rid of the dependence on Σ in $S_{gh\ int}$. This redefinition makes the ghost Lagrangian $U(1)$-Weyl invariant . It enables us to derive conserved currents which are identical to the ones derived by the Noether construction (see [12, page 490]). The currents are obtained by functionally varying the Lagrangian with respect to the gravity prepotentials. We define the currents as follows

$$
S_{int} = \int dz \left(J_+ h_-{}^+ + J_- \Sigma + J_{--} U^- \right) \qquad (6.16)
$$

We find that classically, the currents defined above are given by,

$$
\begin{aligned}
J_+ &= \tfrac{1}{2}\, D_{+1} Z_0^m D_{+\bar{1}} \bar{Z}_0^n g_{mn} \\
&\quad + \left\{ D_+(c^+ b_+) - \tfrac{1}{2i} \left[(D_{+1} b_+)(D_{+\bar{1}} c^+) + (D_{+\bar{1}} b_+)(D_{+1} c^+) \right] \right\} \qquad (6.17) \\
J_- &= 0 \qquad (6.18) \\
J_{--} &= 2(D_- Z_0^m D_- \bar{Z}_0^n) g_{mn} + i(D_- \bar{\Psi}_0^I \Psi_0^J - \bar{\Psi}_0^I D_- \Psi_0^J) \eta_{IJ} \\
&\quad - i \left(2 b_{--} D_- (c^- + \tilde{c}_0^-) + D_-(b_{--})(c^- + \tilde{c}_0^-) \right) \qquad (6.19)
\end{aligned}
$$

We obtain the conservation laws for the currents by demanding that the Lagrangian be invariant under residual supercoordinate transformations (4.45). First, on varying S_{int} under the ξ^+ transformation, we obtain

$$
\delta S_{int} = - \int dz (J_+ D_- \xi^+ + \tfrac{1}{2}\, J_- D_+ \xi^+) \qquad (6.20)
$$

which on integrating by parts gives the following conservation law.

$$
D_- J_+ + \tfrac{1}{2}\, D_+ J_- = 0 \qquad (6.21)
$$

On expanding the above equation in components, we obtain three conservation laws.

$$\partial_- J_{U(1)\ +} \ + \ \partial_+ J_{U(1)\ -} = 0 \tag{6.22}$$

$$\partial_- J_{susy\ +} \ + \ \partial_+ J_{susy\ -} = 0 \tag{6.23}$$

$$\partial_- T_{++} \ + \ \partial_+ T_{+-} = 0 \tag{6.24}$$

which correspond to the conservation of the U(1), supersymmetry and energy-momentum tensor respectively. Secondly, under the K^- transformation, we obtain

$$\delta S_{int} = \int dz \{ J_- - \tfrac{1}{2}\ D_-(K^- + \bar{K}^-) + J_{--} i(K^- - \bar{K}^-) \} \tag{6.25}$$

This does not lead to two conservation laws, one each for K^- and $\bar{K}^-$ since they are related by complex conjugation. It also implies that the corresponding conservation can only be obtained in components. This should be expected since this is the conservation law corresponding to the non-supersymmetric sector. Let $\xi_0^- \equiv (K^- + \bar{K}^-)|$. The ξ_0^- component gives

$$\tfrac{1}{2}\ D_- J_- |_{\theta^{+1}\bar{\theta}^{+\bar{1}}} \ + D_+ J_{--}| = 0 \tag{6.26}$$

which becomes

$$\partial_- T_{+-} + \partial_+ T_{--} = 0 \tag{6.27}$$

This is the conservation of the energy-momentum tensor.

Finally, invariance under residual $U(1)$-Weyl transformations (4.47) implies that $J_- = 0$. This implies that the conservation laws (6.21) and (6.26) can be simplified to

$$D_- J_+ \ = \ 0 \tag{6.28}$$

$$D_+ J_{--}| \ = \ 0 \tag{6.29}$$

We can see that this is true classically. We will show that this is not a symmetry of the quantum theory except for a specific choice of D and N. The breakdown of super-Weyl invariance will manifest in the breakdown of the above conservation law.

We now calculate quantum corrections to the effective action. We now make use of S_0 to calculate the propagators for the various fields. They are

$$\langle Z_0^m(z)\bar{Z}_0^n(z')\rangle = \frac{iD_{+\bar{1}}D_{+1}}{\Box}|_z\delta(z-z')g^{mn} \tag{6.30}$$

$$\langle \Psi_0^I(z)\bar{\Psi}_0^J(z')\rangle = \frac{D_-D_{+\bar{1}}D_{+1}}{\Box}|_z\delta(z-z')\eta^{IJ} \tag{6.31}$$

$$\langle b_+(z)c^+(z')\rangle = \frac{2iD_+}{\Box}|_z\delta(z-z') \tag{6.32}$$

$$\langle b_{--}(z)c_0^-(z')\rangle = -\frac{D_{+1}D_{+\bar{1}}D_-}{\Box}|_z\delta(z-z') \tag{6.33}$$

$$\langle b_{--}(z)\tilde{c}_0^-(z')\rangle = +\frac{D_{+\bar{1}}D_{+1}D_-}{\Box}|_z\delta(z-z') \tag{6.34}$$

where $\delta(z-z') \equiv -\delta(x-x')(\theta^{+1}-\theta^{+1\,\prime})(\bar{\theta}^{+\bar{1}}-\bar{\theta}^{+\bar{1}\,\prime})$. We find that the only non-local terms in the effective action involve $h_-{}^+$ and U^-. The graphs of interest are given in figure 1. All other graphs are local terms. Since we are interested in the non-local part of the one-loop effective action, we will only deal with the graphs given in figures 1a-1c. The graphs are regulated using 't Hooft's trick [13] where the divergent integrals are made to converge by an appropriate choice of poles in the propagator. The results are

$$\begin{aligned}\Gamma_{eff} &= \frac{(2-D)}{16\pi}\int dz\left\{(\frac{D_{+1}D_+}{D_-}h_-{}^+)D_{+\bar{1}}h_-{}^+\right\} \\ &+ \frac{(2D+N-26)}{48\pi}\int dz\left\{(\frac{D_-^3D_{+1}}{D_+}U^-)D_{+\bar{1}}U^-\right\}\end{aligned} \tag{6.35}$$

The effective action is not invariant under the residual supercoordinate transformations given in equation (4.45) but is invariant under residual $U(1)$-Weyl transformations(4.47). This implies that conservation laws (6.21) and (6.26) are no longer valid. Hence there is an anomaly. We would like to shift the anomaly to the super-Weyl transformations by adding local counterterms. The most general local counterterms are as follows

$$\begin{aligned}W_{c.t.} = \frac{-1}{16\pi}\int dz \quad \{ \quad &\alpha D_{+1}h_-{}^+D_{+\bar{1}}\bar{\varphi} \\ + \quad &\beta D_{+1}\varphi D_{+\bar{1}}h_-{}^+\end{aligned}$$

$$
\begin{aligned}
&+ \quad \gamma D_{+1} h_-{}^{+} D_- D_{+\bar{1}} U^- \\
&+ \quad \epsilon D_{+1} U^- D_- D_{+\bar{1}} h_-{}^{+} \\
&+ \quad \kappa \varphi D_-^2 U^- + \nu \bar{\varphi} D_-^2 U^- + \tau \bar{\varphi} D_- \varphi \}
\end{aligned} \tag{6.36}
$$

It can be shown that for arbitrary values of D and N, there is no choice of counterterms that can achieve this objective. However, if D and N satisfy the following equation

$$
D - N + 20 = 0, \tag{6.37}
$$

this is possible. The following choice of coefficients in the counterterms does the job.

$$
\begin{aligned}
\alpha = &\ \beta = -(D-2), \\
\gamma = &\ -\epsilon = -i(D-2), \\
\kappa = &\ \nu = 2(D-2) \\
\tau = &\ 2i(D-2)
\end{aligned} \tag{6.38}
$$

For this choice of counterterms, the effective action takes the following simple form.

$$
\Gamma'_{eff} = \Gamma_{eff} + W_{c.t.} \tag{6.39}
$$

$$
= \frac{2-D}{8\pi} \int dz R_{-+1} \frac{1}{\Box} R_{-+\bar{1}} \tag{6.40}
$$

where $\Box \equiv -2D_+D_-$.

This action is local in the superconformal gauge. It is given by

$$
\Gamma'_{eff} = \frac{2-D}{16\pi i} \int dz (\varphi D_- \bar{\varphi} + c.c) \tag{6.41}
$$

The above action is invariant under the residual supercoordinate transformations but is not invariant under the residual $U(1)$-Weyl transformations (4.47). This implies that J_- is no longer identically zero. The quantum correction is obtained by functionally varying the effective action with respect to Σ. We obtain that

$$
J_- = -4i \frac{(2-D)}{8\pi} \frac{D_-}{\Box} F_{+-} \tag{6.42}
$$

For example, this implies in components that the trace of the stress- tensor is given by

$$T_{+-} = -\tfrac{1}{2i}\,\frac{(2-D)}{8\pi} R_{+-}| \tag{6.43}$$

The breakdown of $U(1)$-Weyl invariance is given by

$$D_+ J_+ = \frac{(2-D)}{8\pi i} F_{+-} \tag{6.44}$$

We obtain $U(1)$-Weyl invariance provided

$$D = 2 \; and \; N = 22 \tag{6.45}$$

So the theory is superconformally invariant at the quantum level for the above choices for D and N. The results are in agreement with those in [14]. It is interesting to note that this agreement holds inspite of the fact that we have gauged the $U(1)$ unlike in [14]. This is due to the fact that ${T_{++1}}^{+1}$ behaves precisely like A_-. This behaviour has been noticed in the second reference of [7]. On gauging the $U(1)$ field, one finds that the additional torsion constraint ${T_{++1}}^{+1} = 0$ can be imposed.

At this point, we would like to note that the quantum corrections have been obtained in a gauge (albeit an algebraic one). Is it possible to do so without making any gauge choices? We find that the presence of too many auxiliary fields (we mean fields that do not occur in the Wess-Zumino gauge) make it prohibitively complicated to carry out the calculation without any gauge choices.

7 Green-Schwarz Mechanism

In this section, we present a (superspace) two dimensional Green-Schwarz mechanism. This is of interest because it will provide an interesting toy model wherein one can do explicit calculations. This may also have some relevance to non-critical strings as we will see later.

The role of the antisymmetric tensor is played by a scalar field and the Chern-Simons term is given by the gauge potential. Let Y be a complex scalar superfield. We introduce its field strength (which is a one-form) by

$$H = dY \tag{7.1}$$

where d is the exterior derivative. We impose the chirality constraint as follows

$$H_{+\bar{1}} = 0 \tag{7.2}$$

The constraint implies that

$$H_{+\bar{1}} = \nabla_{+\bar{1}} Y = 0 \tag{7.3}$$

which implies that Y is a chiral superfield. Similarly, Y^* is antichiral. The Bianchi identity $dH = 0$ does not give anything new. The action for a scalar field is given by

$$\begin{aligned} S_0[Y] &= \frac{-i\eta}{2} \int dz E^{-1} \left(Y^* H_- + c.c. \right) &(7.4)\\ &= \frac{-i\eta}{2} \int dz E^{-1} \left(Y^* \nabla_- Y + c.c. \right) &(7.5) \end{aligned}$$

where $\eta = \pm 1$.

The (Lorentz) Chern-Simons term is given by

$$\Omega_{CS} = \phi \tag{7.6}$$

where ϕ is the Lorentz connection. We couple the Chern-Simons term by modifying the field strength as follows

$$\tilde{H} = dH + \sigma \Omega_{CS} \tag{7.7}$$

where σ is real. The Bianchi identity now gives

$$d\tilde{H} = \sigma R \tag{7.8}$$

We still demand that the modified field strength satisfy the same constraint as before , that is,

$$\tilde{H}_{+\bar{1}} = 0. \tag{7.9}$$

The constraint gives

$$\nabla_{+\bar{1}} Y + \sigma \phi_{+\bar{1}} = 0, \tag{7.10}$$

which implies that Y is no longer chiral. The torsion constraints (2.4) subject to the Bianchi identities imply that $R_{+\bar{1}+\bar{1}} = 0$ which in turn implies

$$\nabla_{+\bar{1}} \phi_{+\bar{1}} = 0 \tag{7.11}$$

We solve this by means of the prepotential T.

$$\phi_{+\bar{1}} = \nabla_{+\bar{1}} T \tag{7.12}$$

We can now define a related field $\tilde{Y}$ which is chiral.

$$\tilde{Y} = Y + \sigma T \tag{7.13}$$

In the presence of the Chern-Simons term, we modify the action (7.4) by substituting the modified field strength in the place of the unmodified one.

$$\begin{aligned} S_{CS} &= \frac{-i\eta}{2} \int dz E^{-1} \left(Y^* \tilde{H}_- + c.c. \right) &(7.14)\\ &= \frac{-i\eta}{2} \int dz E^{-1} \left(Y^* (\nabla_- Y + \sigma \phi_-) + c.c. \right) &(7.15) \end{aligned}$$

As stated earlier, Y is not chiral. Hence it is of interest to expand the above action in terms of $\tilde{Y}$ which is chiral. We obtain

$$S_{CS} = S_0[\tilde{Y}] + \frac{i\eta\sigma}{2} \int dz E^{-1} \left(\tilde{Y} (\phi_- - 2\nabla_- T^*) + c.c. \right) + \cdots \tag{7.16}$$

where the ellipsis refers to pure gravity terms. Note that this action is not invariant under Lorentz transformations

$$\delta\tilde{Y} = 0 \tag{7.17}$$

$$\delta T = L \tag{7.18}$$

where L is the superfield which parametrises the Lorentz transformation . However, as usual with Chern-Simons terms, we try to vary $\tilde{Y}$ to make it invariant. This is impossible. The best one can do is to decompose S_{CS} as

$$\begin{aligned} S_{CS} &= \left(S_0[\tilde{Y}] + \frac{-i\eta\sigma}{4}\int dz[\tilde{Y}(\phi_- + \nabla_- T^*) + c.c.]\right) \\ &\quad + \frac{3i\eta\sigma}{4}\int dz[\tilde{Y}(\phi_- - \nabla_- T^*) + c.c.] \end{aligned} \tag{7.19}$$

By making $\tilde{Y}$ transform, we can make the first term Lorentz invariant. Now the second term is not Lorentz invariant. The modified transformation law for $\tilde{Y}$ which accomplishes this is

$$\delta\tilde{Y} = \tfrac{1}{2}\,\sigma L \tag{7.20}$$

The first term corresponds to the usual CS coupling while the second term is really the Green-Schwarz term (with a fixed coefficient). This will be obvious from the component Lagrangian which we give later. Hence these two terms will suffice to cancel the anomaly as we shall see later. As always, one can also add a Green-Schwarz term with arbitrary coefficient to the above Lagrangian. It is given by

$$S_{GS} = i\tau\int dz E^{-1}(Y(\phi_- - \nabla_- T^*) + c.c.) \tag{7.21}$$

We obtain

$$\begin{aligned} S &= S_{CS} + S_{GS} \\ &= S_0 + ia\int dz[\tilde{Y}(\phi_- + \nabla_- T^*) + c.c.] \\ &+ ib\int dz[\tilde{Y}(\phi_- - \nabla_- T^*) + c.c.] \end{aligned} \tag{7.22}$$

where $a = (\frac{-\eta\sigma}{4})$ and $b = (\frac{3\eta\sigma}{4} + \tau)$.

In order to compute the contribution of the above field to the effective action, we expand $\tilde{Y}$ in terms of a flat chiral Y_0, that is

$$D_{+\bar{1}} Y_0 = 0 \tag{7.23}$$

We also work in supergauge where (from the complex conjugate of (4.51))

$$\phi_{+\bar{1}} = D_{+\bar{1}}(\Sigma + iD_- U^-) \tag{7.24}$$

This gives us an expression for T

$$T = (\Sigma + iD_- U^-) \tag{7.25}$$

It is of interest to expand the action (7.22) in components. Let

$$y = \frac{1}{2}\,(Y_0 + Y_0^*)| \tag{7.26}$$

$$z = \frac{1}{2i}\,(Y_0 - Y_0^*)| \tag{7.27}$$

The action is given (in supergauge) by

$$\begin{aligned} S = \int d^2x(\quad & \{a(y\partial^c \phi_c|) - b(yR_{+-}|)\} \\ & -az\{\tfrac{1}{2i}\,[D_{+1}, D_{+\bar{1}}](\phi_- + D_-\Sigma)| + i\partial_+\partial_-^2 U^-|\} \\ & -bz\{\tfrac{1}{2i}\,[D_{+1}, D_{+\bar{1}}](\phi_- - D_-\Sigma)| - i\partial_+\partial_-^2 U^-|\}) + \cdots \end{aligned} \tag{7.28}$$

where the ellipsis refers to terms coming from S_0 and the fermionic partners of y and z. As expected y couples to the Lorentz curvature through the Green-Schwarz term. The other term involving y is the CS term (after integrating by parts).

There are two non-local contributions to the one-loop effective action from (7.22). One is the one-loop contribution from S_0 which can be trivially obtained from the results of the previous section. This is possible since the graphs corresponding to S_0 are the same as in figure (1a) with the Y_0 fields running through the loop. All one has to do is replace D by $(D+1)$ in Γ_{eff} (equation 6.35) in order to include this contribution to the effective action.

The Chern-Simons and Green-Schwarz terms make contributions to the one-loop effective action by means of tree diagrams given in figure 2. The propagator for Y_0 is given by

$$\langle Y_0(z) Y_0^*(z')\rangle = \eta \frac{iD_{+\bar{1}}D_{+1}}{\Box}|_z \delta(z-z') \tag{7.29}$$

The contribution of the Chern-Simons and Green-Schwarz terms as given by the above graphs is

$$\begin{aligned}\Gamma_{GS} &= \eta\frac{-(a+b)^2}{2}\int dz \left\{ (\frac{D_{+1}D_+}{D_-}h_-{}^+)D_{+\bar{1}}h_-{}^+ \right\} \\ &+ \eta\frac{(a-b)^2}{2}\int dz \left\{ (\frac{D_-^3 D_{+1}}{D_+}U^-)D_{+\bar{1}}U^- \right\}\end{aligned} \tag{7.30}$$

On including all the contributions to the non-local part of the effective action, we obtain

$$\Gamma_{tot} = \Gamma_{eff} + \Gamma_{GS} \tag{7.31}$$

$$\begin{aligned} &= (\frac{(1-D)}{16\pi} - \eta\frac{(a+b)^2}{2})\int dz \left\{ (\frac{D_{+1}D_+}{D_-}h_-{}^+)D_{+\bar{1}}h_-{}^+ \right\} \\ &+ (\frac{(2D+N-24)}{48\pi} + \eta\frac{(a-b)^2}{2})\int dz \left\{ (\frac{D_-^3 D_{+1}}{D_+}U^-)D_{+\bar{1}}U^- \right\}\end{aligned} \tag{7.32}$$

We repeat the analysis of the last section to make the one-loop effective action to be invariant under residual supercoordinate transformations (4.45) and residual $U(1)$-Weyl transformations (4.47). We shall attempt to do so by adding local counterterms as given in equation (6.36). This is not possible unless, we demand

$$ab = -\eta\frac{(D-N+21)}{96\pi} \tag{7.33}$$

Now, unlike in the previous section D and N are arbitrary. The Green-Schwarz mechanism makes it possible to cancel the supercoordinate(Lorentz) anomaly without fixing either D or N. Now by adding local terms we can make Γ_{tot} invariant under residual supercoordinate transformations. We obtain

$$\Gamma_{tot} = \{\frac{(1-D)}{8\pi} - \eta(a+b)^2\} \int dz R_{-+1} \frac{1}{\Box} R_{-+\bar{1}} \tag{7.34}$$

The above action is not invariant under residual $U(1)$-Weyl transformations (4.47). We can achieve this by choosing

$$D = 1 - 8\pi\eta(a+b)^2 \tag{7.35}$$

So by appropriately choosing a, b, we can make the theory superconformally invariant. The new couplings imply that D and N are not constrained anymore. What are the implications of this freedom? First, it can be seen (from the component Lagrangian (7.28)) that for $a = 0$ corresponds to the linear dilaton coupling proposed by Myers [15] and extended by others [16]. It can also be seen that for $D > 1$, that $\eta = -1$ which implies that Y has a timelike kinetic energy term. For $a, b \neq 0$, as in Myers [15], one expects shifts in the conformal dimensions of vertex operators involving the superfield Y. The new feature for arbitrary a, b is that the conformal dimension is different for the left and right handed sectors (that is the dimension is of the form (h, h') with $h \neq h'$). Its connection to non-critical strings would lie in being able to give the superfield Y a suitable interpretation. This is being studied.

8 Conclusion

In this paper, we have studied the extended $(2, 0)$ heterotic supergravity and obtained the anomaly structure. We find that the critical dimension 2 and we also need 22 chiral fermions. This number is not altered by the presence of the gauged $U(1)$. presented a Green-Schwarz mechanism in two dimensions. Interesting questions are its relevance to non-critical strings for arbitrary a, b(the coefficients of the Chern-Simons and Green-Schwarz terms). It would be interesting to study the symmetries of this theory using deformations of the stress-tensor as in [1]. Finally, it is of interest to know the effect of including non-trivial monopole configurations to the theory. This is being studied.

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Appendix

A Notation

In flat superspace, the coordinates are z^A where $A = (+, -, +1, +\bar{1})$. The flat covariant derivatives D_A are given by

$$
\begin{aligned}
D_+ &= \partial_+ && \text{(A.1)}\\
D_- &= \partial_- && \text{(A.2)}\\
D_{+1} &= \partial_{+1} + i\theta^{+1}\ \partial_+ && \text{(A.3)}\\
D_{+\bar{1}} &= \partial_{+\bar{1}} + i\bar{\theta}^{+\bar{1}}\ \partial_+ && \text{(A.4)}
\end{aligned}
$$

Chiral Superfields are defined by

$$D_{+\bar{1}}B = 0 \tag{A.5}$$

where B is a generic superfield. Chiral superfields have the following component expansion

$$B = B_0 + \theta^{+1}\ B_{+1} + i\theta^{+1}\bar{\theta}^{+\bar{1}}\ \partial_+ B_0 \tag{A.6}$$

<u>Complex conjugation:</u>

In heterotic superspace, we cannot define hermitian conjugation. We define complex conjugation using the following rules.

$$
\begin{aligned}
(\theta^{+1}\ )^* &= \bar{\theta}^{+\bar{1}} && \text{(A.7)}\\
(\overrightarrow{D}_{+1})^* &= -(\overleftarrow{D}_{+\bar{1}}) && \text{(A.8)}\\
(A\ B)^* &= B^*\ A^* && \text{(A.9)}\\
(D_{+1}A)^* &= -(A^*\overleftarrow{D}_{+\bar{1}}) && \text{(A.10)}
\end{aligned}
$$

where A and B are arbitrary superfields.

B (p,q) supersymmetry algebra

We are working with in the lightcone coordinates with metric $\eta_{+-} = -1$. P_m are the generators of translations, Q^I_+ and $Q^{I'}_-$ are the generators of supersymmetry. The indices m,n are vector indices, $+,-$ are spinor indices (in the Majorana-Weyl representation) and I, J (I', J') count the number of left(right) supersymmetry generators with $I, J = 1, \ldots, p$ $(I', J' = 1, \ldots, q)$. The (p,q) super-Poincare algebra [17] is then given by

$$[P_m, P_n] = 0 \tag{B.1}$$

$$\{Q^I_+, Q^J_+\} = 2P_+\delta^{IJ} \tag{B.2}$$

$$\{Q^{I'}_-, Q^{J'}_-\} = 2P_-\delta^{I'J'} \tag{B.3}$$

$$\{Q^I_+, Q^{J'}_-\} = 0 \tag{B.4}$$

$$\left[Q^I_\alpha, P_m\right] = 0 \tag{B.5}$$

The supersymmetry algebra is completed by including gauge groups. There exist two types of groups, those that act trivially on the Q's and those which act non-trivially on the Q's. We will restrict ourselves to the latter groups which will be referred to as R-symmetry groups. Let R_a represent the generators of the group. The algebra now includes

$$[R_a, R_b] = i f_{ab}{}^c R_c \tag{B.6}$$

$$\left[Q^I_+, R_a\right] = \tilde{R}_{a}{}^{I}{}_{J} Q^J_+ \tag{B.7}$$

$$\left[Q^{I'}_-, R_a\right] = \breve{R}_{a}{}^{I'}{}_{J'} Q^{J'}_- \tag{B.8}$$

$$[P_m, R_a] = 0 \tag{B.9}$$

where $f_{ab}{}^c$ is the structure constant of the group and $\tilde{R}$ $(\breve{R})$ is the matrix representation of T_a in the space indexed by I,J (I',J'). By demanding that equations (B.2),(B.3), and (B.4) be preserved we find that the following relation has to be satisfied.

$$\tilde{R}_{aL}^{\ \ I}\tilde{R}_{aK}^{\ \ J}\delta^{KL} = \delta^{IJ} \tag{B.10}$$

The largest possible group which preserves the $p \times p$ matrix is $O(p)$. This implies that the largest possible R-symmetry group is $O(p) \times O(q)$, where the group $O(p)$ ($O(q)$) acts trivially on the left (right) supersymmetry generators. We would like to emphasise that the R-symmetry groups are not your usual gauge groups but correspond to an extension of the structure group from SO(1,1) to SO(1,1) $\times$ O(p) $\times$ O(q). Specialising to the case of (2,0), the structure group is then SO(1,1)$\times$ O(2).

C Expanding covariantly chiral fields in terms of flat chiral fields :

In this appendix we shall solve for a (generic) covariantly chiral superfield B in terms of a flat chiral superfield B_0. Let B have Lorentz charge ‘q’. We express B as follows

$$B = B_0 + f(B_0, h_B{}^A) \tag{C.1}$$

where f satisfies $f(B_0, h = 0) = 0$ and $f(B_0 = 0, h) = 0$. We have

$$\nabla_{+\bar{1}} B = 0 \tag{C.2}$$

Expanding to linear order in $h_B{}^A$ and substitute (C.1) , we obtain

$$D_{+\bar{1}} f - h_{+\bar{1}}{}^A D_A B_0 + q\phi_{+\bar{1}} B_0 = 0 \tag{C.3}$$

We now solve for f as a bilinear in B_0 and $h_B{}^A$. We obtain

$$f(B_0, h_B{}^A) = \bar{V}^-(D_- B_0) - q h_-{}^- B_0 + q(D_-\bar{V}^-)B_0 - \tfrac{1}{2i}\ h_{+\bar{1}}{}^+(D_{+1}B_0) \tag{C.4}$$

where we have used $h_{+\bar{1}}{}^- = D_{+\bar{1}}\bar{V}^-$ and $\phi_{+\bar{1}} = D_{+\bar{1}} h_-{}^- - D_- h_{+\bar{1}}{}^-$. This gives

$$B = B_0 + \bar{V}^-(D_- B_0) - q h_-{}^- B_0 + q(D_-\bar{V}^-)B_0 - \tfrac{1}{2i}\ h_{+\bar{1}}{}^+(D_{+1}B_0) \tag{C.5}$$

This gives the expansion of a covariantly chiral superfield in terms of a flat chiral superfield to linear order in ${h_B}^A$. The above expansion would be modified if the superfield B carries a $U(1)$ charge.

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Figure Captions

Figure 1: Graphs leading to non-local terms in the one loop effective action.

Figure 2: Graphs obtained from the Chern-Simons and Green-Schwarz terms.