Outage Probability of Multiple-Input Single-Output (MISO) Systems with Delayed Feedback[†]

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Abstract

We investigate the effect of feedback delay on the outage probability of multiple-input single-output (MISO) fading channels. Channel state information at the transmitter (CSIT) is a delayed version of the channel state information available at the receiver (CSIR). We consider two cases of CSIR: (a) perfect CSIR and (b) CSI estimated at the receiver using training symbols. With perfect CSIR, under a short-term power constraint, we determine: (a) the outage probability for beamforming with imperfect CSIT (BF-IC) analytically, and (b) the optimal spatial power allocation (OSPA) scheme that minimizes outage numerically. Results show that, for delayed CSIT, BF-IC is close to optimal for low SNR and uniform spatial power allocation (USPA) is close to optimal at high SNR. Similarly, under a long-term power constraint, we show that BF-IC is close to optimal for low SNR and USPA is close to optimal at high SNR. With imperfect CSIR, we obtain an upper bound on the outage probability with USPA and BF-IC. Results show that the loss in performance due to imperfection in CSIR is not significant, if the training power is chosen appropriately.

I. INTRODUCTION

Channel State Information is very crucial in determining the performance of any wireless system. The minimum outage probability of multiple-input single-output (MISO) channels with perfect channel state information at the receiver (CSIR) and no channel state information at the transmitter (CSIT) is derived in [1]. For reasonably low outage probabilities, uniform spatial power allocation (USPA) across the spatial dimension is the optimal strategy. Outage probability of MISO systems with perfect CSIT and CSIR is derived in [2]. It is shown that feeding back the CSI provides significant gain in the performance, and that beamforming to the direction of the channel is optimal and provides a constant SNR gain over no CSIT under short-term power constraint (i.e., transmit power is constant over each transmission interval). In the case of long-term average power constraint, it is also possible to adapt the transmission power level based on channel feedback (i.e., temporal power control). Outage can be reduced significantly by saving power when the channel is strong and using the saved power when the channel is worse. The optimum power allocation strategy to minimize the outage probability over fading channels and MISO fading channels is determined in [3] and [2] respectively.

In practice, the feedback channel resources are seldom perfect enough to provide instantaneous and noiseless feedback. Under the short term power constraint, and for two cases of imperfect

feedback namely mean feedback and covariance feedback, spatial schemes that a) minimize the outage probability are studied in [4], [5], and b) maximize the mutual information are studied in [6]. In [7], BER performance of spatial schemes in the presence of delayed feedback has been studied. Under a long-term power constraint, minimum outage probability with temporal power control for quantized CSIT has been studied in [8]. In practice, it is also not feasible to have a perfect estimate of CSIR. Usually, channel state information at the receiver is estimated using training symbols, and the resources used during the training period have to be accounted for. Outage probability with preamble based CSIR and quantized CSIT has been studied in [8]. In [9], maximizing mutual information in the presence of channel estimation error and delayed feedback has been studied.

In this paper, we focus on the effect of the delay in feedback on the performance from the point of view of outage probability. Using the delayed feedback model in [10], we solve the problem of minimum outage transmission over MISO channels under both short-term and long-term power constraints. Under a short-term power constraint, beamforming is optimal if the transmitter has perfect CSI. We analyze the loss in performance of beamforming due to the delay in the feedback and derive an analytical expression for the outage probability of beamforming with imperfect CSIT (BF-IC). Results show that BF-IC, which allocates total power in the direction of CSIT, is better at low SNR while USPA [1], which allocates equal power in all the directions and does not require any feedback, is better at high SNR. However, none of the above two strategies is optimal. The minimum outage transmission strategy for a given delay, optimal spatial power allocation (OSPA) is determined. OSPA involves beamforming along the spatial modes and optimal power allocation across the spatial modes. Numerical results show that BF-IC is very close to OSPA for low SNR while USPA is close to OSPA for high SNR. Since OSPA does not provide significant gain at any SNR, compared to the best of BF-IC and USPA, the cross-over SNR at which USPA becomes better than BF-IC is important and can be used to switch between BF-IC and USPA. We present the equation to determine this cross-over SNR and solve it numerically.

Under a long-term power constraint, with perfect CSIT, the optimal beamforming (to the channel direction) and temporal power control strategy is obtained in [2]. We numerically evaluate the outage probabilities for BF-IC and USPA with temporal power control. Again, BF-IC is better

at low SNR while USPA is better at high SNR. Finally, we extend the analysis with delayed feedback and perfect CSIR to the case of delayed feedback and imperfect CSIR. An upper bound on the outage probability of USPA and BF-IC with imperfect CSIR is obtained. The loss in performance due to the error in estimation of CSIR is shown to be negligible if the training power is chosen optimally.

The rest of the paper is organized as follows. In Section II, our system model is introduced. In Section III, under the short power constraint, outage probability with BF-IC and OSPA are determined and compared with USPA. In Sections IV and V, the long term power constraint and imperfect CSIR are considered. Finally, Sections VI and VII present the results and conclusions.

II. SYSTEM MODEL

The MISO system with M transmit antennas and 1 receive antenna is, as usual, modeled as

$$y = \mathbf{h}^H \mathbf{x} + z, \tag{1}$$

where $\mathbf{h} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$ is a $M \times 1$ independent, identically distributed (i.i.d) and zero-mean circularly symmetric complex Gaussian channel vector, $\mathbf{x}$ is a $M \times 1$ channel input vector and z is zero-mean unit-variance additive white Gaussian noise (AWGN). We use a block fading model, where the channel coefficients are assumed to be fixed within a given duration, known as coherence interval. We assume high correlation between successive time durations. Using the Gaussian channel vector model, the delay in the feedback is captured by the correlation coefficient ρ between CSIT and CSIR. The old channel and the actual channel can be related as follows [10]:

$$\mathbf{h} = \rho \mathbf{h}_{old} + \sqrt{1 - \rho^2} \mathbf{w}, \tag{2}$$

where $\mathbf{h}_{old}$ is the delayed CSIT, ρ is a correlation coefficient, and $\mathbf{w} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$ is independent of $\mathbf{h}_{old}$. The gap between no CSIT ($\rho = 0$) and perfect CSIT ($\rho = 1$) is bridged using ρ. Lower the delay in the feedback, higher the value of ρ.

III. SHORT-TERM POWER CONSTRAINT

Assuming a short-term power constraint [3], such that the transmit power is not a function of time, the mutual information is given by

$$I(\mathbf{x}; y/\mathbf{h}, \mathbf{h}_{old}) = \log(1 + P\mathbf{h}^H \mathbf{Q}\mathbf{h}), \tag{3}$$

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where $\mathbf{Q}$ is the input covariance matrix such that $Tr(\mathbf{Q}) = 1$ and P is the transmit power.

Consider the two extreme cases: zero feedback ($\rho = 0$) and instantaneous feedback ($\rho = 1$). For $\rho = 0$, where the transmitter does not have any knowledge of the channel state information, the diversity strategy with the power distributed equally among the M orthogonal independent transmit directions, i.e., USPA is optimal [1], i.e., we have $\mathbf{Q} = \dfrac{\mathbf{I}}{M}$, and

$$P_{out\mathbf{USPA}}(M, R, P) = \Gamma_M \left(\frac{e^R - 1}{P/M} \right),\tag{4}$$

where $P_{out\mathbf{USPA}}(M, R, P)$ is the outage probability (as defined in [1]) for a $M \times 1$ system using USPA corresponding to a transmit power constraint P and rate R (in nats/transmission), and $\Gamma_M(\cdot)$ is the incomplete Gamma function defined as $\Gamma_M(x) = \dfrac{1}{(M-1)!} \displaystyle\int_0^x t^{M-1} e^{-t} dt$.

For $\rho = 1$, where the transmitter has perfect CSI, beamforming is optimal [2], i.e., $\mathbf{x} = \dfrac{\mathbf{h}}{\sqrt{\mathbf{h}^H \mathbf{h}}} s$, where s is a scalar i.i.d. Gaussian input, $\mathbf{Q} = \dfrac{\mathbf{h}\mathbf{h}^H}{\mathbf{h}^H \mathbf{h}}$, and

$$P_{out\mathbf{BF}}(M, R, P, \rho = 1) = \Gamma_M \left(\frac{e^R - 1}{P} \right).\tag{5}$$

Outage performance for $\rho = 1$ is $10\log_{10}M$ dB better than the performance for $\rho = 0$. For $0 < \rho < 1$, where we do not have perfect CSIT, we evaluate the outage performance of beamforming using the imperfect CSIT in Section III-A. We also determine the optimal spatial power allocation strategy that minimizes the outage probability in Section III-B and compare it with beamforming using the imperfect CSIT.

A. Beamforming using imperfect CSIT (BF-IC)

In this section, the loss in performance due to the presence of the delay in the feedback is analyzed and an expression for the outage probability (equation (12)) is derived. This is a simple extension of beamforming from perfect CSIT to the imperfect CSIT case, where beamforming is performed using the imperfect CSIT assuming that it is the actual channel. Therefore, we have $\mathbf{x} = \dfrac{\mathbf{h}_{old}}{\sqrt{\mathbf{h}_{old}^H \mathbf{h}_{old}}} s$, where s is a scalar i.i.d. Gaussian input, and

$$\mathbf{Q} = \frac{\mathbf{h}_{old} \mathbf{h}_{old}^H}{\mathbf{h}_{old}^H \mathbf{h}_{old}}.\tag{6}$$

Substituting (6) in (3) and denoting the feedback SNR $\mathbf{h}_{old}^H \mathbf{h}_{old}$ by γ, we get

$$I(\mathbf{x}; y/\mathbf{h}, \mathbf{h}_{old}) = \log\left(1 + P\frac{\mathbf{h}^H \mathbf{h}_{old} \mathbf{h}_{old}^H \mathbf{h}}{\gamma}\right). \tag{7}$$

Now, we derive the outage probability for the specific model described in equation (2). Note that γ is Gamma distributed with the pdf given by

$$f_\Gamma(\gamma) = \frac{\gamma^{M-1} e^{-\gamma}}{(M-1)!}. \tag{8}$$

The expression for the mutual information for a given $\mathbf{h}_{old}$ can be simplified as follows.

$$\frac{\mathbf{h}^H \mathbf{h}_{old} \mathbf{h}_{old}^H \mathbf{h}}{\gamma} = \frac{|\mathbf{h}^H \mathbf{h}_{old}|^2}{\gamma} = \frac{|(\rho \mathbf{h}_{old} + \sqrt{1 - \rho^2}\mathbf{w})^H \mathbf{h}_{old}|^2}{\gamma}$$

$$= \frac{(1 - \rho^2)}{2}\left|\sqrt{\frac{2\rho^2}{(1-\rho^2)}}\gamma + \sqrt{2}\frac{\mathbf{w}^H \mathbf{h}_{old}}{\sqrt{\gamma}}\right|^2. \tag{9}$$

Hence, the mutual information given $\mathbf{h}_{old}$ can be simplified as

$$I(\mathbf{x}; y/\mathbf{h}, \mathbf{h}_{old}) = \log\left(1 + P\frac{(1 - \rho^2)}{2}A\right), \tag{10}$$

where $A = \left|\sqrt{\delta} + \sqrt{2}\frac{\mathbf{w}^H \mathbf{h}_{old}}{\sqrt{\gamma}}\right|^2$, $\delta = 2\mu\gamma$ and $\mu = \frac{\rho^2}{1-\rho^2}$. Note that $\frac{\mathbf{w}^H \mathbf{h}_{old}}{\sqrt{\gamma}}$ given $\mathbf{h}_{old}$ is a zero mean complex Gaussian random variable with variance $||\frac{\mathbf{h}_{old}^H}{\sqrt{\gamma}}||^2 = 1$. Thus, A given γ is a non-central chi-square (nc-χ^2) random variable with two degrees of freedom and parameter δ. Observe that the distribution of mutual information given $\mathbf{h}_{old}$ depends on only $\gamma = |\mathbf{h}_{old}|^2$. Therefore we have the following expression for the outage probability for a given γ.

$$\Pr(\text{outage}/\gamma) = \Pr\left(\log\left(1 + P\frac{(1 - \rho^2)}{2}A\right) < R\right)$$

$$= F_{(\text{nc-}\chi^2, 2, \delta)}(2\beta), \tag{11}$$

where $\beta = \frac{e^R - 1}{P}(\mu + 1)$, and $F_{(\text{nc-}\chi^2, 2, \delta)}(\cdot)$ is the CDF of a non-central chi-square random variable with two degrees of freedom and parameter δ. The overall probability of outage can be simplified as

$$P_{out\mathbf{BF\text{-}IC}}(M, R, P, \rho) = \int_0^\infty f_\Gamma(\gamma)\Pr(\text{outage}/\gamma)d\gamma$$

$$= \frac{1}{(1 + \mu)^{M-1}}\sum_{i=0}^{M-1}\binom{M-1}{i}\mu^i \Gamma_{(i+1)}\left(\frac{e^R - 1}{P}\right). \tag{12}$$

The derivation of equation (12) is shown in the appendix . Note that $(1+\mu)^{M-1} = \sum_{i=0}^{M-1} \binom{M-1}{i} \mu^i$. Therefore, the result (12) can be interpreted as the weighted average of $\Gamma_K \left(\frac{e^R-1}{P} \right)$, which is the outage probability of a $K \times 1$ MISO system with perfect CSIT, where K varies from 1 to M. Therefore, at high SNR, we expect the outage probability with BF-IC to be dominated by the first term ($K = 1$), which decays as $\frac{1}{\text{SNR}}$.

The asymptotic diversity gain at infinite SNR, defined as

$$d = - \lim_{\text{SNR} \to \infty} \frac{\log P_{out}}{\log SNR}, \tag{13}$$

can be quantified. From (12), using the approximation $\Gamma_M(x) \simeq \frac{x^M}{M!}$ for very small x, we can show that the asymptotic diversity gain of the BF-IC scheme is 1 for imperfect CSIT, i.e.,

$$\text{Diversity Gain } d = \begin{cases} 1 & \text{for } 0 \leq \rho < 1 \\ M & \text{for } \rho = 1 \end{cases}. \tag{14}$$

This result can be explained intuitively as follows. At very high SNR, the outage probability is dominated by the error in the CSIT rather than channel being in deep fade. However, for USPA, the asymptotic diversity gain is M independent of ρ. Therefore, USPA is always better than BF-IC at high SNR. The cross-over SNR $SNR_{cross}(\rho, R, M)$ can be obtained by equating the outage probabilities of the two schemes: (4) and (12). Although there is no closed form expression for cross-over SNR, it can be computed numerically. By comparing the operating SNR with the cross-over SNR, one can switch between BF-IC and USPA.

B. Optimal Spatial Power Allocation (OSPA)

We have seen that neither beamforming nor uniform spatial power allocation is the optimal strategy for any given ρ ($0 < \rho < 1$). We find the optimal spatial power allocation strategy that minimizes the outage probability. Our results show that OSPA allocates a fraction λ of the power along the spatial mode corresponding to the imperfect CSIT with the remaining power being equally distributed among the other orthogonal spatial modes.

The overall outage probability is minimized by minimizing $P_{out}(\mathbf{h}_{old})$, outage probability given $\mathbf{h}_{old}$, for each realization of $\mathbf{h}_{old}$. The outage probability for a given $\mathbf{h}_{old}$ is given by

$$P_{out}(\mathbf{h}_{old}) = \Pr \left(\mathbf{h}^H \mathbf{Q} \mathbf{h} < \frac{e^R - 1}{P} \right). \tag{15}$$

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Using (2), $P_{out}(\mathbf{h}_{old})$ can be simplified as

$$\Pr\left(((\sqrt{\mu}\mathbf{h}_{old} + \mathbf{w})\mathbf{Q}((\sqrt{\mu}\mathbf{h}_{old} + \mathbf{w})^H < \beta\right), \tag{16}$$

where $\beta = \frac{e^R-1}{P}(\mu+1)$ and $\mu = \frac{\rho^2}{1-\rho^2}$. The outage probability given by (16) is equivalent to the outage probability of a MISO channel with a mean feedback of $\sqrt{\mu}\mathbf{h}_{old}$, which is minimized without any loss of generality by minimizing over the fraction of the power spent in the direction of the mean feedback [4], [5]. Rest of the power is spent equally in the M-1 orthogonal beams.

Since $\mathbf{Q}$ is positive semi-definite, we have the eigenvalue decomposition (EVD) $\mathbf{Q} = \mathbf{V}\widetilde{\mathbf{Q}}\mathbf{V}^H$, where $\widetilde{\mathbf{Q}} = \text{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_M\}$ is a diagonal matrix with $\lambda_i \geq 0$ representing the power allocated to the direction indicated by the corresponding column vector of the unitary matrix $\mathbf{V}$. It has been shown in [4] that the unitary matrix $\mathbf{V}$ that minimizes the outage probability (16) is of the form $\mathbf{V} = [\frac{\mathbf{h}_{old}}{\sqrt{\gamma}}, \mathbf{v}_2, \mathbf{v}_3, \ldots, \mathbf{v}_M]$, where $\{\mathbf{v}_i\}, 2 \leq i \leq M$ is an arbitrary set of $(M-1)$ orthonormal vectors that are orthogonal to $\mathbf{h}_{old}$. Hence, we have $\mathbf{d} = \mathbf{V}^H\mathbf{h}_{old} = [\sqrt{\gamma}, 0, 0, \ldots, 0]^T$ and $\mathbf{g} = \mathbf{V}^H\mathbf{w} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$. Thus (16) is simplified as

$$P_{out}(\mathbf{h}_{old}) = \Pr\left((\mathbf{g} + \sqrt{\mu}\mathbf{d})^{\mathbf{H}}\widetilde{\mathbf{Q}}(\mathbf{g} + \sqrt{\mu}\mathbf{d}) < \beta\right). \tag{17}$$

$$\text{Let } \frac{\xi}{2} = (\mathbf{g} + \sqrt{\mu}\mathbf{d})^{\mathbf{H}}\widetilde{\mathbf{Q}}(\mathbf{g} + \sqrt{\mu}\mathbf{d})$$

$$= \mathbf{g}^{\mathbf{H}}\widetilde{\mathbf{Q}}\mathbf{g} + \mathbf{g}^{\mathbf{H}}\widetilde{\mathbf{Q}}\sqrt{\mu}\mathbf{d} + \sqrt{\mu}\mathbf{d}^{\mathbf{H}}\widetilde{\mathbf{Q}}\mathbf{g} + \sqrt{\mu}\mathbf{d}^{\mathbf{H}}\widetilde{\mathbf{Q}}\sqrt{\mu}\mathbf{d}$$

$$= \sum_{i=1}^{M} \lambda_i \mid g_i \mid^2 + 2\lambda_1\sqrt{\mu\gamma}\text{Re}(g_1)) + \lambda_1\mu\gamma \tag{18}$$

$$= \lambda_1\{[Re(g_1) + \sqrt{\mu\gamma}]^2 + [Im(g_1)]^2\} + \sum_{i=2}^{M} \lambda_i \mid g_i \mid^2.$$

Observe that ξ is symmetric over λ_i, $i = 2$ to M. Hence, there is no reason to prefer any one λ_i over others. Therefore, λ_i's should be equal for $i = 2$ to M. This observation allows the random variable ξ to be expressed in terms of λ_1 alone, using which the outage probability is determined easily in terms of the CDF of a single random variable. This is not explicitly used in the expressions for outage probability in [4] (see equation (10) in [4]). Further simplification of the outage expression based on this observation is presented below. Denote λ_1 by λ for convenience.

$$\text{Tr}(\widetilde{\mathbf{Q}}) = 1 \Rightarrow \lambda_i = \frac{1-\lambda}{M-1}, \text{ for i} = 2 \text{ to } M \tag{19}$$

$$\Rightarrow \xi = \lambda A + \frac{1-\lambda}{M-1} B, \tag{20}$$

where A $= \{[\sqrt{2}Re(g_1) + \sqrt{2\mu\gamma}]^2 + [\sqrt{2}Im(g_1)]^2\}$ is Non-Central Chi-Square distributed with 2 degrees of freedom and non-centrality parameter $\delta = 2\mu\gamma$ and B $= 2\sum_{i=2}^{M} |g_i|^2$ is Central Chi-Square distributed with 2(M-1) degrees of freedom. Observe that ξ depends only on $\gamma = \mathbf{h}_{old}^H \mathbf{h}_{old}$. Therefore, we denote the outage probability for a given γ and λ as $P_{out}(\gamma, \lambda)$, given by

$$P_{out}(\gamma, \lambda) = Pr(\xi < 2\beta) = F_\xi(2\beta), \tag{21}$$

where $F_\xi(.)$ represents the CDF of ξ. To complete the solution, it remains only to find the optimal value of λ for each γ. Consider the two extreme cases: $\rho = 0$ and $\rho = 1$.

For $\rho = 0$, $\mu = 0$ and $\xi = 2\sum_{i=1}^{M} \lambda_i |g_i|^2$ is symmetric over $\lambda_i, i = 1$ to M, i.e., All the directions are identical, and hence, equal power is spent in each direction. Therefore, $\lambda_{opt}(\gamma) = \frac{1}{M}$, for $\rho = 0$. As ρ tends to 1, μ tends to ∞. Therefore, the co-efficient of λ_1 becomes large compared to the coefficients of the other λ_i's, and hence, it is optimal to spend all the power in that direction. Therefore, $\lambda_{opt}(\gamma) = 1$ for $\rho = 1$.

Consider the case of $0 < \rho < 1$. When $\gamma = 0$, $\delta = 2\mu\gamma = 0$. In this case, we get

$$\lambda_{opt}(\gamma = 0) = \frac{1}{M} \quad \text{for any } \rho. \tag{22}$$

As $\gamma \to \infty$, $\delta = 2\mu\gamma \to \infty$. In this case, we get $\lambda_{opt}(\gamma \to \infty) = 1$ for any ρ. Therefore, for $0 < \rho < 1$, we expect $\lambda_{opt}(\gamma)$ to start from $\frac{1}{M}$ at $\gamma = 0$ and approach 1 as γ increases.

The minimum outage probability for a given γ is given by $P_{out}(\gamma, \lambda_{opt}(\gamma)) = \min_\lambda P_{out}(\gamma, \lambda)$, where $\lambda_{opt}(\gamma)$ is the solution of

$$\frac{\partial P_{out}(\gamma, \lambda)}{\partial \lambda} = 0 \text{ in the range from } \frac{1}{M} \text{ to } 1. \tag{23}$$

Expressing $P_{out}(\gamma, \lambda)$ as

$$P_{out}(\gamma, \lambda) = \Pr\left(\lambda A + \frac{1-\lambda}{M-1} B < 2\beta\right)$$
$$= \int_0^{\frac{2\beta}{\lambda}} f_A(a) F_B\left(\frac{(2\beta - \lambda a)(M-1)}{1-\lambda}\right) da, \tag{24}$$

equation (23) can be simplified as

$$\int_0^{\frac{2\beta}{\lambda}} f_A(a) \exp\left(\frac{(M-1)\lambda a}{2(1-\lambda)}\right) (2\beta - \lambda a)^{(M-2)}(2\beta - a) da = 0. \tag{25}$$

Although a closed form expression for $\lambda_{opt}(\gamma)$ does not appear to be available, it can be determined numerically by a one-dimensional numerical search over the range. The overall outage probability can then be determined by averaging $P_{out}(\gamma, \lambda_{opt}(\gamma))$ over γ.

IV. LONG-TERM POWER CONSTRAINT

Achieving minimum outage probability under a long-term power constraint involves power allocation in both spatial and temporal domains. For a given feedback SNR γ, and a corresponding fixed power allocation policy, the problem of minimizing the outage probability can be formulated as

$$\min_{\mathbf{Q}} \Pr\left(\mathbf{h}^H \mathbf{Q} \mathbf{h} < \frac{e^R - 1}{Pp(\gamma)}\right). \tag{26}$$

This is equivalent to minimizing $P_{out}(\gamma, p(\gamma), \lambda)$, given by

$$P_{out}(\gamma, p(\gamma), \lambda) = \Pr\left(\lambda A + \frac{1-\lambda}{M-1} B < \frac{2\beta}{p(\gamma)}\right) = F_\xi\left(\frac{2\beta}{p(\gamma)}\right) \tag{27}$$

over λ, fraction of the power spent in the direction of the imperfect feedback. $\lambda_{opt}(\gamma, p(\gamma))$ is the solution of $\dfrac{\partial P_{out}(\gamma, p(\gamma), \lambda)}{\partial \lambda} = 0$ and will range from $\frac{1}{M}$ to 1. The optimal temporal power control policy $p(\gamma)$ minimizes

$$E_\gamma[P_{out}(\gamma, p(\gamma), \lambda_{opt}(\gamma, p(\gamma)))] = \int_0^\infty f_\Gamma(\gamma) P_{out}(\gamma, p(\gamma), \lambda_{opt}(\gamma, p(\gamma))) d\gamma,$$

subject to the power constraint:

$$\int_0^\infty f_\Gamma(\gamma) p(\gamma) d\gamma = 1. \tag{28}$$

However, finding optimal $p(\gamma)$ and the corresponding $\lambda_{opt}(\gamma, p(\gamma))$ is difficult, since we do not have closed form expression for $\lambda_{opt}(\gamma, p(\gamma))$. Therefore, based on the intuition from the results for the short-term power constraint, the suboptimal schemes BF-IC with temporal power control and USPA with temporal power control are considered and analyzed.

In USPA, the power is distributed equally among the orthogonal independent transmit directions, i.e, $\lambda = \frac{1}{M}$ or $\mathbf{Q} = \frac{\mathbf{I}}{\mathbf{M}}$. Therefore, the outage probability for a given γ and the corresponding $p(\gamma)$ in equation (27) is simplified as

$$P_{out}(\gamma, p(\gamma)) = \Pr\left(A + B < \frac{2M\beta}{p(\gamma)}\right) = F_{(nc-\chi^2, 2M, \delta)}\left(\frac{2M\beta}{p(\gamma)}\right). \tag{29}$$

From calculus of variations [11] (using Theorem 4.2.1 in [11]), the temporal power control function that minimizes the outage probability with USPA can be shown to satisfy:

$$k_1 = \left(\frac{2M\beta}{p^2(\gamma)}\right) f_{(\text{nc-}\chi^2,2M,\delta)}\left(\frac{2M\beta}{p(\gamma)}\right), \tag{30}$$

where k_1 is a constant chosen such that $p(\gamma)$ satisfies the power constraint (28) and is non-negative. Finally, $p(\gamma)$ is determined numerically from equations (28) and (30).

In BF-IC, the spatial power allocation scheme is fixed such that the power is spent in only one direction corresponding to the imperfect CSIT, i.e., $\lambda = 1$. Therefore, we have

$$P_{out}(\gamma, p(\gamma)) = \Pr\left(A < \frac{2\beta}{p(\gamma)}\right) = F_{(\text{nc-}\chi^2,2,\delta)}\left(\frac{2\beta}{p(\gamma)}\right). \tag{31}$$

Again, using calculus of variations, the termporal power control function that minimizes the outage probability with BF-IC can be shown to satisfy

$$k_2 = \left(\frac{2\beta}{p^2(\gamma)}\right) f_{(\text{nc-}\chi^2,2,\delta)}\left(\frac{2\beta}{p(\gamma)}\right), \tag{32}$$

where k_2 is a constant, chosen such that $p(\gamma)$ satisfies the power constraint (28) and is non-negative. $p(\gamma)$ can be obtained numerically as before using equations (28) and (32).

V. EFFECT OF IMPERFECT CSIR

We assume the training and MMSE channel estimation model as in [8]. M training symbols are transmitted at the start of each T symbol block with the ith training symbol being transmitted only from the ith antenna. The MMSE estimate of CSI ($\hat{\mathbf{h}}$) is:

$$\hat{\mathbf{h}} = \frac{\sqrt{P_t/M}}{P_t/M + 1}\left(\sqrt{\frac{P_t}{M}}\mathbf{h} + \mathbf{n}\right), \tag{33}$$

where P_t is the total power used for training, and $\mathbf{n}$ is the additive white Gaussian noise vector corresponding to the M training symbols. Let P_d be the power used during data transmission period per symbol. P_t and P_d are related by the equation: $P_t + P_d(T - M) = PT$. Let σ_E^2 denote the estimation error variance, i.e., $Cov(\mathbf{e}) = \sigma_E^2 \mathbf{I}_{M \times M}$, where $\mathbf{e} = \mathbf{h} - \hat{\mathbf{h}}$. It can be shown that $\sigma_E^2 = \frac{M}{P_t + M}$. The CSIR is $\hat{\mathbf{h}}$. The CSIT, which is a delayed version of the CSIR is $\hat{\mathbf{h}}_{old}$, which is the MMSE estimate of $\mathbf{h}_{old}$. Using (2) and (33), the correlation coefficient ρ_e between the CSIT ($\hat{\mathbf{h}}_{old}$) and CSIR ($\hat{\mathbf{h}}$) can be obtained as $\rho_e = \frac{P_t}{P_t + M}\rho$. Observe that ρ_e can at most be ρ (for very large training power) and is less than ρ for moderate values of training power.

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Given the MMSE estimate of the CSI ($\hat{\mathbf{h}}$) at the receiver, the mutual information of the BF-IC scheme after accounting for the training period can be lower bounded using the result in [12]. A similar mutual information lower bound can be obtained for the USPA scheme using the results in [9], [8]. This lower bound on the mutual information is given by:

$$I(\mathbf{x}; y|\hat{\mathbf{h}}, \hat{\mathbf{h}}_{old}) \geq \frac{T - M}{T} \log \left(1 + \frac{P_d}{1 + \sigma_E^2 P_d} \hat{\mathbf{h}}^H \mathbf{Q} \hat{\mathbf{h}} \right). \tag{34}$$

Defining $\hat{\mathbf{h}}_{sc} = \frac{1}{\sqrt{(1-\sigma_E^2)}} \hat{\mathbf{h}}$ such that $Cov(\hat{\mathbf{h}}_{sc}) = \mathbf{I}_{M \times M}$, the lower bound on the mutual information (34) can be written as

$$I(\mathbf{x}; y|\hat{\mathbf{h}}, \hat{\mathbf{h}}_{old}) \geq \frac{T - M}{T} \log \left(1 + P' \hat{\mathbf{h}}_{sc}^H \mathbf{Q} \hat{\mathbf{h}}_{sc} \right),$$

$$\text{where } P' = P_d \frac{1 - \sigma_E^2}{1 + \sigma_E^2 P_d}. \tag{35}$$

Substituting the value of σ_E^2 obtained for the training model, we get $P' = \frac{P_d P_t}{P_t + M P_d + M}$.

For USPA, $\mathbf{Q} = \frac{\mathbf{I}_{M \times M}}{M}$. Clearly, the lower bound on mutual information above becomes equivalent to a system with perfect CSIR, but with different values of average SNR (P') and rate (R'). Therefore, the outage probability is upper bounded as follows:

$$P_{out\mathbf{USPA}}(M, R, P') \leq \Pr \left(\log \left(1 + P' \frac{\hat{\mathbf{h}}_{sc}^H \hat{\mathbf{h}}_{sc}}{M} \right) < R \frac{T}{T - M} \right) = \Gamma_M \left(\frac{e^{R'} - 1}{P'/M} \right), \tag{36}$$

$$\text{where } P' = \frac{P_d P_t}{P_t + M P_d + M}, R' = R \frac{T}{T - M} \tag{37}$$

The asymptotic diversity gain of USPA with imperfect CSIR remains M. Furthermore, the SNR gap between the perfect and imperfect CSIR cases can be significantly reduced by choosing value of P_t or P_d that maximizes P' under the constraint $P_t + P_d(T - M) = PT$ [8].

In BF-IC, the transmit covariance matrix is $\mathbf{Q} = \frac{\hat{\mathbf{h}}_{old} \hat{\mathbf{h}}_{old}^H}{\hat{\mathbf{h}}_{old}^H \hat{\mathbf{h}}_{old}} = \frac{\hat{\mathbf{h}}_{old,sc} \hat{\mathbf{h}}_{old,sc}^H}{\hat{\mathbf{h}}_{old,sc}^H \hat{\mathbf{h}}_{old,sc}}$, where $\hat{\mathbf{h}}_{old,sc}$ is a scaled version of $\hat{\mathbf{h}}_{old}$ with identity covariance matrix. Again, the lower bound on the system with imperfect CSIR is equivalent to the system with perfect CSIR with the parameters: average SNR (P') and rate (R') given by (37) and ρ_e. Following the simplifications as in Section III-A and the appendix, we get

$$P_{out\mathbf{BF-IC}}(M, R', P', \rho) \leq \frac{1}{(1 + \mu')^{M-1}} \sum_{i=0}^{M-1} \binom{M-1}{i} (\mu')^i \Gamma_{(i+1)} \left(\frac{e^{R'} - 1}{P'} \right), \tag{38}$$

where $\mu' = \frac{\rho_e^2}{1 - \rho_e^2}$, and P', R' are given by equation (37).

VI. Results & Observations

The rate of transmission (R) is chosen to be 2 nats/s/Hz throughout this section. Fig. 1 shows the performance of USPA (4) and BF-IC (12) for different values of feedback delay captured by ρ. BF-IC is better at lower SNRs and worse at high SNRs when compared to USPA for any $\rho < 1$. Fig. 2 shows the diversity gain of USPA and BF-IC for different number of transmit antennas (M) for $\rho = 0.999$. USPA does not require feedback and has a diversity gain of M, where as at high SNR, the outage probability with BF-IC is dominated by the error in CSIT. Thus, the diversity gain of BF-IC scheme is equal to 1, for any non zero delay in the feedback and any number of transmit antennas. Hence, USPA outperforms BF-IC at high SNR for all values of $\rho < 1$. Cross-over SNR is defined as the SNR after which USPA outperforms BF-IC. It can be seen from Fig. 1 that the cross-over SNR is a monotonically increasing function of ρ.

Fig. 3 shows $\lambda_{opt}(\gamma)$, the fraction of power spent in the direction of imperfect CSIT, as a function γ for the OSPA scheme. Observe that $\lambda_{opt}(\gamma)$ is larger for higher values of ρ implying that when the quality of feedback is higher, more power is spent in the direction of feedback. Fig. 4 compares the outage probability of BF-IC and USPA with OSPA for $\rho = 0.9$. We observe that (a) OSPA provides negligible gain in performance, (b) OSPA is computationally complex as it requires the transmitter to compute the optimal value of λ for each value of feedback SNR and adapt the power in the spatial modes correspondingly, and (c) OSPA requires an estimate of ρ to determine $\lambda_{opt}(\gamma)$ and any mismatch between the estimated value and the actual value will hurt the performance. On the other hand, USPA and BF-IC do not require any estimate of ρ and are very simple. Therefore, we suggest switching between BF-IC and USPA by comparing the operating average SNR with the cross-over SNR. For a given average SNR, it is also possible to choose between USPA and BF-IC based on the instantaneous feedback SNR γ (instead of switching based on the average SNR irrespective of γ). Equations (4) and (11) are the outage probabilties of USPA and BF-IC for a given γ. However, we know that switching based on average SNR is already very close to the performance of OSPA. Therefore, the possible improvement due to switching based on instantaneous SNR (instead of average SNR) is very small.

Fig. 4 also shows the performance of BF-IC and USPA with the corresponding optimal

temporal power control. As in the case of the short-term power constraint, temporal power control with BF-IC is better for low SNR and temporal power control with USPA is better at high SNR. The cross-over SNR is slightly lower with temporal power control.

Fig. 5 compares the outage probability of BF-IC and USPA for perfect CSIR with BF-IC and USPA for imperfect CSIR for $\rho = 0.9$. Both $M = 2$ and $M = 4$ are considered. T is chosen to be 100. Outage probability for two cases: (a) Preamble power same as data power ($P_d = P_t$) and (b) optimized preamble power is considered. Note that optimal power chosen for USPA is used as it is for BF-IC. The results suggest that the loss in performance due to imperfect CSIR is not significant for both USPA and BF-IC, if the power is chosen appropriately.

VII. SUMMARY

The problem of minimum outage transmission for a MISO system with M transmit antennas with delayed feedback is considered. The delay in the feedback is captured by ρ, the correlation coefficient between delayed CSIT and perfect CSIR. For a short-term power constraint, we derive an analytic expression for the outage probability of beamforming using imperfect CSIT, where the power is spent in only one direction corresponding to the imperfect CSI available with the transmitter and compare it with that of USPA, where the power is distributed equally among the M orthogonal and independent transmit directions. We also determine the optimal transmit strategy, i.e., OSPA, that minimizes the outage probability numerically. OSPA involves allocating a fraction of the power in the direction of the imperfect CSIT and the rest of the power is equally distributed among the $M - 1$ orthogonal and independent transmit directions. Results show that, for any $\rho < 1$, BF-IC is better at low SNR and worse at high SNR when compared to USPA. Furthermore, the asymptotic diversity gain for BF-IC is equal to 1 for any $\rho < 1$, independent of the number of transmit antennas. BF-IC is close to optimal at low SNR, while USPA is close to optimal at high SNR, i.e., OSPA does not improve the outage probability significantly compared to switching between BF-IC and USPA depending on the average SNR. The cross-over SNR can be determined numerically by equating the outage probabilities of BF-IC and USPA schemes. For the long-term power constraint, where the transmit power is varied with time based on the feedback SNR, we numerically evaluate the outage probabilities and show again that BF-IC is better at low SNR, while USPA is better at high SNR. Finally, we show that the performance loss due to imperfect CSIR is minimal if the training power is chosen appropriately.

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DERIVATION OF EQUATION (12)

$$F_{(\text{nc-}\chi^2,2M,\delta)}(y) = \sum_{k=0}^{\infty} \frac{\left(\frac{\delta}{2}\right)^k e^{-\frac{\delta}{2}}}{k!} F_{\chi^2,2M+2k}(y), \tag{39}$$

where $F_{\chi^2,2M+2k}(.)$ is the cdf of a central χ^2 random variable with $2M+2k$ degrees of freedom.

Using (39), (11) and substituting $\delta = 2\mu\gamma$, $P_{out\textbf{BF-IC}}(M,R,P,\rho)$ is simplified as

$$\begin{aligned}
P_{out\textbf{BF-IC}}(M,R,P,\rho) &= \int_0^{\infty} f_\Gamma(\gamma) \sum_{k=0}^{\infty} \frac{(\mu\gamma)^k e^{-\mu\gamma}}{k!} F_{\chi^2,2+2k}(2\beta) d\gamma \\
&= \sum_{k=0}^{\infty} \frac{\mu^k}{k!} F_{\chi^2,2+2k}(2\beta) \int_0^{\infty} f_\Gamma(\gamma) \gamma^k e^{-\mu\gamma} d\gamma \\
&= \sum_{k=0}^{\infty} \frac{\mu^k}{k!} \int_0^{\beta} \frac{x^k e^{-x}}{k!} dx \frac{(M+k-1)!}{(M-1)!(1+\mu)^{(M+k)}} \\
&= \frac{1}{(1+\mu)^M} \int_0^{\beta} e^{-x} g(x) dx,
\end{aligned} \tag{40}$$

$$\text{where } g(x) = \sum_{k=0}^{\infty} \frac{\binom{M+k-1}{k}}{k!} \left(\frac{\mu x}{1+\mu}\right)^k. \tag{41}$$

LEMMA. For any $m,n > 0$,

$$\binom{m+n}{m} = \binom{m+n}{n} = \sum_{i=0}^{min(m,n)} \binom{m}{i}\binom{n}{i}. \tag{42}$$

Proof: Using symmetry in m and n, we assume $m < n$ without any loss of generality. Observe that the L.H.S is the number of ways to chose m objects out of $m+n$. This can also be calculated by separating the $m+n$ objects in to 2 sets with sizes m and n and choosing i objects from the first set and choosing $n-i$ objects from the second set and varying i from 0 to m. Therefore,

$$\binom{m+n}{m} = \sum_{i=0}^{m} \binom{m}{i}\binom{n}{n-i} = \sum_{i=0}^{min(m,n)} \binom{m}{i}\binom{n}{i}. \square$$

Using the above lemma, $g(x)$ can be simplified as

$$\begin{aligned}
g(x) &= \sum_{k=0}^{\infty} \sum_{i=0}^{min(k,M-1)} \binom{M-1}{i}\binom{k}{i} \frac{1}{k!} \left(\frac{\mu x}{1+\mu}\right)^k \\
&= \sum_{i=0}^{M-1} \sum_{(k-i)=0}^{\infty} \binom{M-1}{i} \frac{1}{i!(k-i)!} \left(\frac{\mu x}{1+\mu}\right)^k
\end{aligned}$$

$$= \sum_{i=0}^{M-1} \frac{\binom{M-1}{i}}{i!} \left(\frac{\mu x}{1+\mu}\right)^i \sum_{(k-i)=0}^{\infty} \frac{1}{(k-i)!} \left(\frac{\mu x}{1+\mu}\right)^{k-i}$$

$$= e^{\left(\frac{\mu x}{1+\mu}\right)} \sum_{i=0}^{M-1} \frac{\binom{M-1}{i}}{i!} \left(\frac{\mu x}{1+\mu}\right)^i.$$

After substituting for $g(x)$ in (40), we get

$$P_{out\textbf{BF-IC}}(M, R, P, \rho) = \frac{1}{(1+\mu)^M} \sum_{i=0}^{M-1} \frac{\binom{M-1}{i}}{i!} \mu^i \int_0^\beta e^{-\left(\frac{x}{1+\mu}\right)} \left(\frac{x}{1+\mu}\right)^i dx$$

$$= \frac{1}{(1+\mu)^{M-1}} \sum_{i=0}^{M-1} \binom{(M-1)}{i} \mu^i \Gamma_{(i+1)} \left(\frac{e^R - 1}{P}\right).$$

(43)

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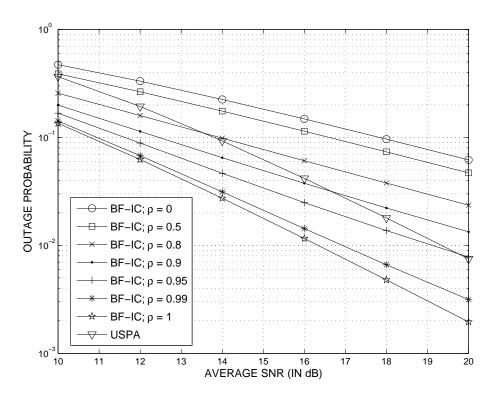
Fig. 1. Outage probabilities for Beamforming using imperfect CSIT (BF-IC) for various values of ρ, and uniform spatial power allocation (USPA) for $M = 2$ and $R = 2$ nats/s/Hz. Cross-over SNR is the SNR at which USPA and BF-IC have the same outage probability.
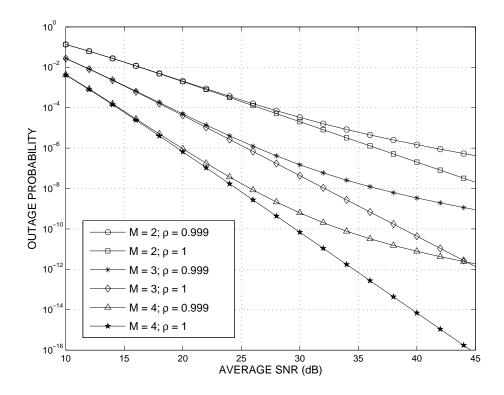
Fig. 2. Outage probability with BF-IC for various values of M for $\rho = 0.999$ and beamforming for $\rho = 1$ and $R = 2$ nats/s/Hz
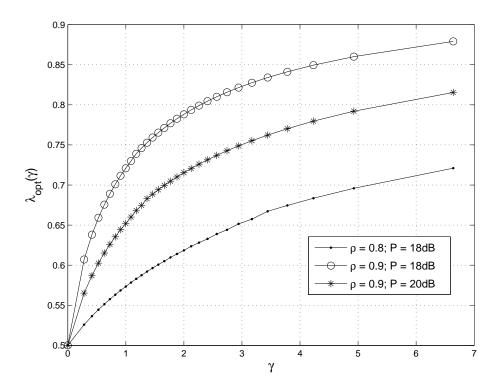
Fig. 3. Fraction of the power in the direction of imperfect CSIT $\lambda_{opt}(\gamma)$ for different values of ρ and P; $M = 2$ and $R = 2$ nats/s/Hz.
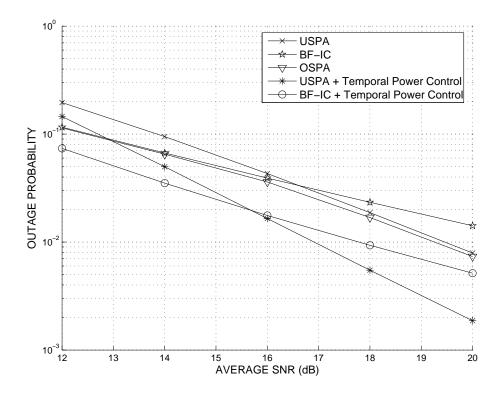
Fig. 4. Outage Probabilities for uniform spatial power allocation (USPA) and beamforming using imperfect CSIT (BF-IC) with and without temporal power control, and optimal spatial power allocation (OSPA) for $\rho = 0.9$; $M = 2$ and $R = 2$ nats/s/Hz.
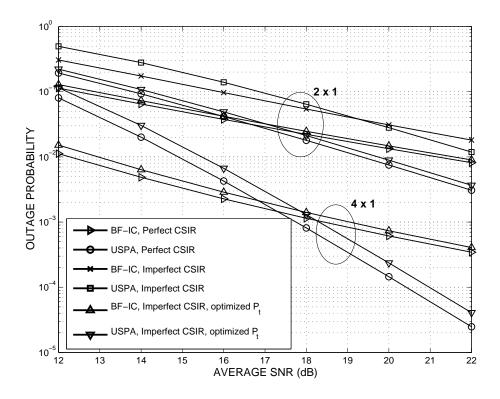
Fig. 5. Outage probabilities for Beamforming using imperfect CSIT (BF-IC) for $\rho = 0.9$, and uniform spatial power allocation (USPA) for $M = 2, 4$ and $R = 2$ nats/s/Hz. Outage for $M = 4$ with imperfect CSIR is plotted only for the optimized trianing power case.