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Non-normality in combustion-acoustic interaction in diffusion flames: a critical revision†

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Perturbations in a non-normal system can grow transiently even if the system is linearly stable. If this transient growth is sufficiently large, it can trigger self-sustained oscillations from small initial disturbances. This has important practical consequences for combustion-acoustic oscillations, which are a continual problem in rocket and aircraft engines. Balasubramanian and Sujith (*Journal of Fluid Mechanics* 2008, 594, 29–57) modelled an infinite-rate chemistry diffusion flame in an acoustic duct and found that the transient growth in this system can amplify the initial energy by a factor, G_{max}, of order 10^5 to 10^7. However, recent investigations by L. Magri & M. P. Juniper have brought to light certain errors in that paper. When the errors are corrected, G_{max} is found to be of order 1 to 10, revealling that non-normality is not as influential as it was thought to be.

1. Results and discussion

In this note, we use the same model, discretization, and non-dimensionalization as Balasubramanian & Sujith (2008, 2013), labelled B&S for brevity, but include the corrected equations, which are listed below. It is implied that the following errata refer to B&S.

(*a*) The analytical steady solution, Z_{st} (appendix B, p. 54), obtained by separation of variables, is:

$$Z_{st} = X_i\,(1-\alpha) - Y_i\alpha - \frac{2}{\pi}\,(X_i+Y_i)\sum_{n=1}^{+\infty}\frac{\sin(n\pi\alpha)}{n\,(1+b_n)}\cos(n\pi y_c)\left(e^{a_{n1}x_c}+b_n e^{a_{n2}x_c}\right), \quad (1.1)$$

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where

$$a_{n1} \equiv \frac{Pe}{2} - \sqrt{\frac{Pe^2}{4} + n^2\pi^2}, \quad a_{n2} \equiv \frac{Pe}{2} + \sqrt{\frac{Pe^2}{4} + n^2\pi^2}, \tag{1.2}$$

$$b_n \equiv -\frac{a_{n1}}{a_{n2}} e^{\left(-2L_c\sqrt{\frac{Pe^2}{4}+n^2\pi^2}\right)}. \tag{1.3}$$

The non-dimensional coordinates of the combustion domain are x_c, y_c.

(*b*) The expressions for $C_m^{(n)}$ and W_{mk} (eq. (7), p. 36) are $2/L_c$ times the original terms due to Galerkin projection; and $W_{mk} = 1/L_c$ when $k = m$.

(*c*) The variable Y_1 in the right-hand side of the terms R_{nm} and J_{nm} (eq. (13), p. 37) is Y_i.

(*d*) $\dot{Q}_{av}$ (eqs. (18),(19), p. 39) is to be divided by 2 due to non-dimensionalization over the cross-sectional area.

(*e*) The multiplying factor ahead matrix M_1 (appendix B, p. 54) is $\frac{1}{(T_i+T_{ad})/2}$.

(*f*) The expression of the matrix B_{NN} (appendix B, p. 54) is $B_{NN} = -D + A_1 - A_2 + A_3 - A_4 + A_5$, where

$$A_5 = \frac{1}{(T_i + T_{ad})/2}[0\ 0\ 0\ \dots\ 0\ \sin(\pi x_f)\ \sin(2\pi x_f)\ \dots\ \sin(K\pi x_f)]^T[J_{00}\ \dots\ J_{0M}\ 0\ \dots\ 0]. \tag{1.4}$$

K is the number of Galerkin modes for acoustic discretization.

(*g*) The damping terms in the matrix S (appendix B, p. 55) are $+2\pi\xi_1, +4\pi\xi_2, \dots, +2K\pi\xi_K$.

(*h*) The numerator of the matrix A_4 (appendix B, p. 55) is 1 due to non-dimensionalization over the cross-sectional area of the duct.

Computations are performed by using 50×50 Galerkin modes in the flame domain, and 6 modes in the acoustic domain. When the number of Galerkin modes is increased to 70×70 in the flame and 12 in the acoustics, the eigenvalues and singular values change by less than 15%. The fixed parameters are the fuel mass ratio, $Y_i = 3.2$; the oxidizer mass ratio, $X_i = 3.2/7$; and the average temperature, $T_{av} = 1/0.685$. We set the damping coefficients to $c_1 = 0.013$ and $c_2 = 0.08$ in order to have marginally stable systems. The nonlinear behaviour of this thermo-acoustic system is not considered because it has been fully characterized by Illingworth, Waugh & Juniper (2013).

Figures 1a,b show the growth factor, G_{max}†, as a function of the Péclet number, Pe, and the non-dimensional half width of the fuel slot, α, respectively. In both cases, $1 < G_{max} \lesssim 10$. Furthermore, marginally stable but highly non-normal fluid-dynamic systems exhibit pseudospectra that protrude significantly into the unstable half-plane (Trefethen & Embree 2005). In this thermo-acoustic system, however, the pseudospectra around the most unstable eigenvalues are nearly concentric circles whose values decrease rapidly as the distance from the eigenvalue increases. This is a further demonstration that the system is only weakly non-normal. It is worth noting, however, that Juniper (2011) showed that even a small amount of non-normality can make a system somewhat more susceptible to triggering.

† We use the same norm as B&S, even though Chu's norm would be a more appropriate measure of the energy (Chu 1965)

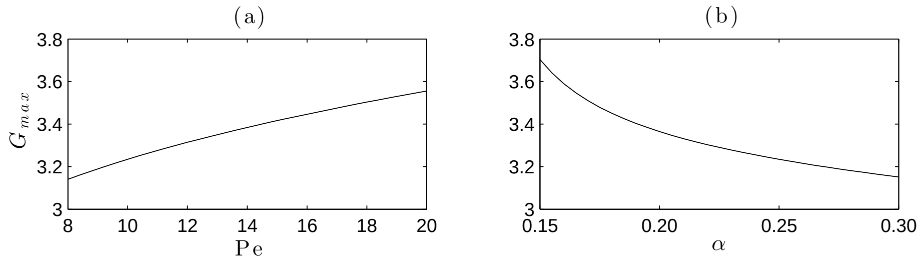

FIGURE 1. The growth factor, G_{max}, as a function of (a) the Péclet number, Pe, and (b) the fuel slot half width, α. Frame (a) has $\alpha = 0.25$, frame (b) has $Pe = 10$.

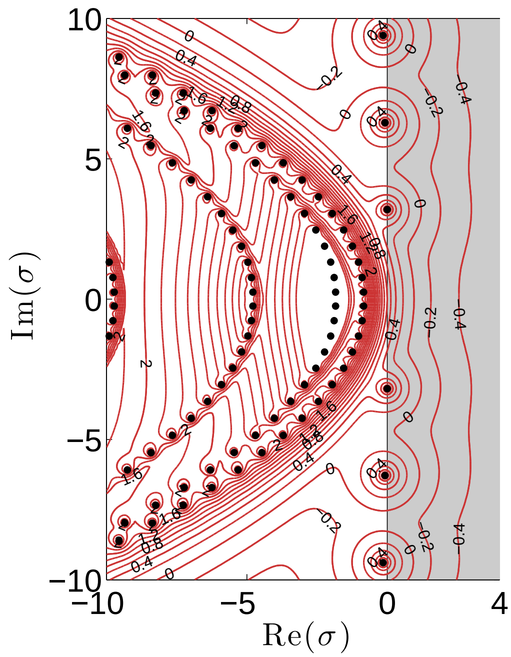

FIGURE 2. Logarithm of the pseudospectra, $\log_{10}(\epsilon)$. The parameters are the same as figure 1, with $\alpha = 0.25$ and $Pe = 10$. The dominant eigenvalue is $\sigma = -0.003 \pm 3.193\mathrm{i}$.

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