MULTI-CHANNEL EEG COMPRESSION BASED ON 3D DECOMPOSITIONS

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ABSTRACT

Various compression algorithms for multi-channel electroencephalograms (EEG) are proposed and compared. The multi-channel EEG is represented as a three-way tensor (or 3D volume) to exploit both spatial and temporal correlations efficiently. A general two-stage coding framework is developed for multi-channel EEG compression. In the first stage, we consider (i) wavelet-based volumetric coding; (ii) energy-based lossless compression of wavelet subbands; (iii) tensor decomposition based coding. In the second stage, the residual is quantized and coded. Through such two-stage approach, one can control the maximum error (worst-case distortion). Numerical results for a standard EEG data set show that tensor-based coding achieves lower worst-case error and comparable average error than the wavelet- and energy-based schemes.

Index Terms— arithmetic coding, three-way tensor, tensor decomposition, wavelet transform

1. INTRODUCTION

Electroencephalogram (EEG) is a recording of the electrical activity of the human brain, usually acquired by a number electrodes placed on the scalp. In the past decade, there has been tremendous growth in EEG based research activities, *e.g.,* automated EEG analysis for diagnosis of neurological diseases, and brain computer interfacing (BCI) [1]. In most applications, EEG recordings are done for an extended period, and long-term recordings often generate massive EEG data sets. Therefore, EEG compression plays an important role for efficient storage and transmission. The main challenges for EEG compression are as follows:

- the number of EEG channels can be large (e.g., 256),
- the sampling rate can be high (several kHz) in order to capture evoked potentials and high frequency oscillations.

Many techniques have been developed for compressing EEG (see, e.g., [2] and references therein). However, those methods often compress individual channel separately. EEG signals from adjacent channels are often strongly correlated (inter-channel correlation), and each individual channel has temporal correlations (intra-channel correlation). The single-channel EEG compression algorithms, when extended directly to multi-channel EEG, will be inefficient as the inter-channel correlations are not exploited. For multi-channel EEG, intra- and inter-channel correlations must be exploited together for efficient compression.

Multi-channel EEG compression is less intensively studied, and one could find only few instances in the literature; we categorize the algorithms into lossless [3, 4] and lossy [5] methods. All multi-channel compression schemes consider inter- and intra-channel correlation separately, and exploit them by different techniques. However, intra- and inter-channel correlations are often not independent and exploiting them in a singe step may improve efficiency. We explore ways to arrange the multi-channel EEG in suitable form, particularly, to exploit both types of correlations in a single step.

In our previous work [2], we introduced a pre-processing technique where single-channel EEG is arranged as a matrix before compression; this representation improved the Rate-Distortion (R-D) performance over conventional compression schemes. Extending our previous work, in [9] we explored several ways to arrange multi-channel EEG as matrices and tensors, and evaluated several matrix/tensor decomposition techniques based on their R-D performance. Here we present a more systematic study of multi-way (or volumetric) representations of multi-channel EEG for the purpose of compression; we consider several compression schemes that use such representations, more specifically, based on 3D wavelet transforms or tensor decompositions. Moreover, we develop a two-stage framework for compression: in the first stage we compress the EEG by means of volumetric coding, tensor decomposition and energy-based coding of significant subbands; in the second stage, we apply arithmetic coding to the time residual for the first two algorithms and wavelet-domain residual for third algorithm, after uniform quantization. Such two-stage compression scheme allows us to bound the maximum (worst-case)distortion. We discuss the compression performance of the three compression algorithms by means of an average and worst-case distortion measure.

This paper is structured as follows. In Section 2 we explain the multi-way (or volumetric data) representation of multi-channel EEG. We outline our compression algorithms

(a) t/dt/s volume   (b) s/s/t volume
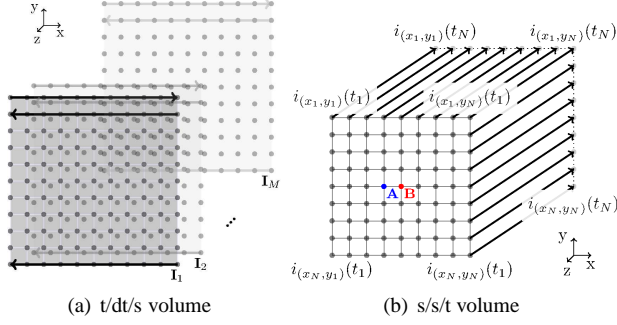
Fig. 1. Formation of 3D volume from multi-channel EEG. (a) The matrices $(I_1, I_2, \ldots, I_M)$ formed from single-channel EEG are stacked as volume ("t/dt/s volume"). (b) At any time instance, we form matrix from the multi-channel EEG (**A** & **B** denote samples from adjacent electrodes in EEG montage). N such matrices formed at subsequent time instances are then stacked along the z-direction to form a volume ("s/s/t volume").

in Section 3, and present our results in Section 4, followed by concluding remarks in Section 5.

2. TENSOR/VOLUMETRIC DATA FORMATION FROM EEG

Spatially adjacent channels of multi-channel EEG are strongly correlated, and each individual channel is strongly correlated across time. To exploit both spatial and temporal correlations simultaneously, we arrange multi-channel EEG as a 3D volume or three-way tensor. We consider two specific ways to extract a volumetric data from multi-channel EEG, where the three axes capture spatial and temporal variations in different form.

In Fig. 1(a) we illustrate the EEG volume formed according to our first method. The k-th slice I_k of the volume $\mathcal{I}$, extracted from channel k, can be written as:

$$\mathcal{I}_{t/dt/s}^{(k)} = \{I_k | k = 1, \ldots, M\} \tag{1}$$

$$= \begin{bmatrix} i_k(1) & i_k(2) & \cdots & i_k(N) \\ i_k(2N) & i_k(2N-1) & \cdots & i_k(N+1) \\ \vdots & \vdots & \ddots & \vdots \\ . & . & \cdots & i_k(N^2) \end{bmatrix}_{(N \times N)}.$$

From our previous studies [2, 6], we found that such arrangement leads to improved compression performance over conventional vector-based compression schemes. Next, the matrices associated with the single-channel EEG signals are stacked to form 3D volume, as shown in Fig. 1(a). Adjacent slices in the tensor correspond to adjacent EEG channels. We refer to this volume as "t/dt/s", where the x, y, and z directions reflect temporal (t), delayed (dt) temporal, and spatial (s) variations respectively.

We also consider an alternative method to form a tensor from multi-channel EEG. A matrix is formed from the multi-channel EEG at each time instance. We arrange the matrix such that its elements follow similar adjacency as the EEG montage; for the sake of brevity, we omit the details. We

stack the matrices from subsequent time instances to form a volume, as shown in Fig. 1(b). The $x - y$ plane reflects the spatial correlations, and the temporal correlations is along the z direction. We refer to this volumetric data as "s/s/t". The k-th slice of the volume may be written as:

$$\mathcal{I}_{s/s/t}^{(k)} = \{i_{(i,j)}(k) | k = 1, \ldots, N\} \tag{2}$$

$$= \begin{bmatrix} i_{(1,1)}(k) & i_{(1,2)}(k) & \cdots & i_{(1,N_2)}(k) \\ i_{(2,1)}(k) & i_{(2,2)}(k) & \cdots & i_{(2,N_2)}(k) \\ \vdots & \vdots & \ddots & \vdots \\ i_{(N_1,1)}(k) & i_{(N_1,2)}(k) & \cdots & i_{(N_1,N_2)}(k) \end{bmatrix}_{(N_1 \times N_2)},$$

where i and j refer to the position in the x-y plane, whereas the slice number k refers to the time index. The dimension of the $x - y$ plane is limited by the number of channels, and the slices in the $x - y$ plane may be square or rectangular.

3. COMPRESSION ALGORITHMS

We first perform lossy coding (Stage 1), followed by arithmetic coding of the quantized residuals (Stage 2). We consider three lossy compression algorithms (Stage 1): (i) 3D Wavelet volumetric coding, (ii) 3D Wavelet subband specific arithmetic coding, and (iii) tensor decomposition (PARAFAC) based coding. In the following we explain our three compression algorithms.

3.1. Wavelet-based Compression

3.1.1. Volumetric Coding Approach

Fig. 2 shows a diagram of the proposed two-stage coder for multi-channel EEG signals. We denote the EEG volume by $\mathcal{I}$ (both types of volumes, cf. Fig 1). In the first stage, we compress $\mathcal{I}$ with a scalable wavelet encoder based on successive bit-plane encoding, resulting in the compressed data $\mathcal{I}_{en}$; we use a bi-orthogonal wavelet transform (5/3 filters) as in our previous work [2]. The compressed data $\mathcal{I}_{en}$ is then decoded, yielding the reconstructed data $\mathcal{I}_l$. Next we quantize the residue $\varepsilon = \mathcal{I} - \mathcal{I}_l$, resulting in ε_q, which is compressed by arithmetic coding, leading to ε_{q-en}. Both are used by the decoder to approximate the original data. The compressed data $\mathcal{I}_{en}$ is first decoded, yielding the lossy reconstructed data $\mathcal{I}_l$. The data ε_{q-en} is passed through an arithmetic decoder and then dequantized, resulting in $\hat{\varepsilon}$. The latter is an approximation of the residual ε. Eventually, the data $\mathcal{I}$ is reconstructed as $\mathcal{I}_{nl} = \mathcal{I}_l + \hat{\varepsilon}$. The volume $\mathcal{I}_{nl}$ is at last rearranged to yield the reconstructed EEG signal(s). We can readily confirm the following relations:

$$\mathcal{I} = \mathcal{I}_l + \varepsilon \tag{3}$$
$$\mathcal{I}_{nl} = \mathcal{I}_l + \hat{\varepsilon}. \tag{4}$$

Therefore, it follows that $||\varepsilon - \hat{\varepsilon}||_\infty = ||\mathcal{I} - \mathcal{I}_{nl}||_\infty$, and hence $||\varepsilon - \hat{\varepsilon}||_\infty \leq \delta$ is equivalent to $||\mathcal{I} - \mathcal{I}_{nl}||_\infty \leq \delta$. The residual ε is uniformly quantized to generate quantization indices ε_q, with maximum error no larger than δ:
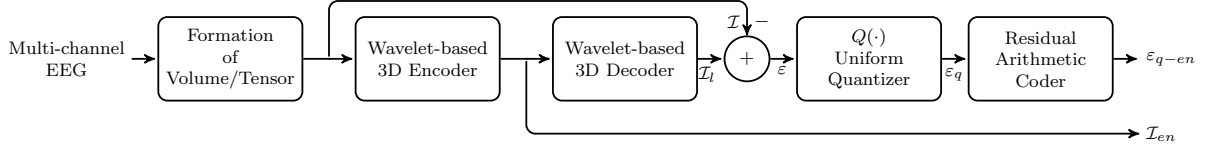
Fig. 2. Wavelet-based volumetric coding of multi-channel EEG

Table 1. Wavelet-based subband specific coding procedure

Step 1: **Initialization**

  (a) Form the volume from the multi-channel EEG,
$\mathcal{I} \leftarrow$ t/dt/s or s/s/t volume

  (b) Compute the Wavelet transform of the volume.
$\mathcal{I}_w = $ 3D-DWT$(\mathcal{I}, D)$
// D-level decomposition yields $7D + 1$ subband cubes

  (c) Determine the *relative energy density* (RED$_i$) of the subbands ($i = 1, \ldots, 7D + 1$)
$$\text{RED}(i) = \frac{\sum_j \mathcal{I}_w^i(j)^2}{N_i \cdot \sum_j \mathcal{I}_w(j)^2}$$
where N_i is the number of elements in i^{th} subband

  (d) Coding order $\mathbf{O} = $ descend(RED)

  (e) Set the threshold τ (% of total energy)

Step 2: **First-pass coding** - Coding of significant subbands until τ. Set *relative-energy* RE $= 0, i = 1$.

  while (RE $< \tau$)

    (a) Bitstream $\leftarrow AC(\mathcal{I}_w^{\mathbf{O}(i)})$ // Code the subband according to $\mathbf{O}$ by Arithmetic coding

    (b) RE $= $ RE $+ $ RED$_{\mathbf{O}(i)} \cdot N_{\mathbf{O}(i)}$
// update relative energy of the coded subband. $N(\mathbf{O}(i))$ is number of elements in the subband $\mathbf{O}(i)$

    (c) $i \leftarrow i + 1$

  end

Step 3: **Second-pass coding** - lossy coding of the remaining subbands ($\mathbf{O}(i + 1)$ to $\mathbf{O}(7D + 1)$)

  for ($j = i + 1 : 7D + 1$)

    (a) $\mathcal{I}_w^{\mathbf{O}(j)} = Q(\mathcal{I}_w^{\mathbf{O}(j)}, \delta)$
// Quantize the wavelet coefficients with quantizer step-size δ

    (b) Bitstream $\leftarrow AC(\mathcal{I}_w^{\mathbf{O}(j)})$

  end

$$\varepsilon_q = \begin{cases} \lfloor \frac{\varepsilon + \delta}{2\delta + 1} \rfloor, & \varepsilon > 0 \\ \lfloor \frac{\varepsilon - \delta}{2\delta + 1} \rfloor, & \varepsilon < 0 \end{cases}, \quad (5)$$

where $\lfloor \cdot \rfloor$ denotes the integer part of the argument. At the decoder end, the residual bitstream ε_{q-en} is decoded to yield ε_q, followed by a dequantizer defined to guarantee $||\varepsilon - \hat{\varepsilon}|| \leq \delta$:

$$\hat{\varepsilon} = (2\delta + 1)\varepsilon_q. \quad (6)$$

By adding the lossy reconstruction $\mathcal{I}_l$ and the dequantized residual $\hat{\varepsilon}$, we obtain the final near-lossless reconstruction $\mathcal{I}_{nl}$ with guarantee $||\mathcal{I} - \mathcal{I}_{nl}|| \leq \delta$. In words, the maximum distortion is therefore bounded to δ. The pre-processing step, *i.e.,* formation of tensor from multi-channel EEG, is the principle difference from the coders used in image compression [7].

3.1.2. *Subband Specific Arithmetic Coding (SAC)*

In this approach, we first order the wavelet subbands based on their relative energy density (RED). We use the same wavelets as in volumetric coding approach (cf. Section 3.1.1). In first stage, we compress the most significant wavelet subbands losslessly, followed by lossy compression of the subbands with smaller energy concentration. Specifically, in the first stage, we apply arithmetic coding to the subbands with highest RED, until a certain threshold τ (% of total energy) is reached. The remaining subbands are less significant in terms of their RED; we first quantize them (cf. (5)), and then apply arithmetic coding. This two-stage procedure results in lossy compression of the EEG signals. The pseudo-code of the algorithm is presented in Table 1. We code each subband separately using simple arithmetic coding where all the coefficients within the same subband are represented by a single probability model. It is noteworthy that in the second coding step, we quantize the wavelet subbands; this may lead to a substantial error in time domain. In other words, we cannot control the maximum distortion in time domain through this approach.
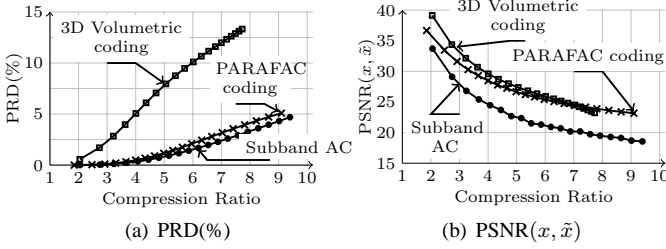
3.2. Tensor-based Compression

We apply parallel factor decomposition (PARAFAC) decomposition [8] to the three-way tensor $\mathcal{I}$, formed from multichannel EEG. In our previous study [9], we have shown that PARAFAC yielded the best compression performance among various other matrix and tensor decompositions. The PARAFAC based decomposition of a three-way tensor is given by:

$$\mathcal{I} = \sum_{i=1}^r \mathbf{a}_i \circ \mathbf{b}_i \circ \mathbf{c}_i + \mathcal{E}, \quad (7)$$

where $\mathcal{E}$ represents the residual tensor, and $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ represent the factors along the three modes, whereas $\circ$ stands for the outer-product along the particular mode. These three factors efficiently capture the major variations along the three modes. In the first-stage we encode the PARAFAC factors using a simple bit-plane coding scheme, and in the second stage, we apply arithmetic coding to the residuals after uniform quantization (5).

4. RESULTS

We test the performance of our compression algorithms on the EEG-Motor Mental Imagery datasets of physiobank database [10]. This EEG dataset consists of 64-channel recordings, recorded from healthy subjects at 80Hz sampling

Fig. 3. Compression performance of the wavelet-based volumetric coding, subband specific arithmetic coding and PARAFAC based coding for t/dt/s volume/three-way tensor.

rate and with 12 bit resolution. We analyze the performance of the algorithms based on compression ratio:

$$CR = \frac{L_{\text{orig}}}{L_{\text{comp}}}, \qquad (8)$$

where L_{orig} and L_{comp} are the bit length of original and reconstructed multi-channel EEG signals respectively. The quality of the reconstructed signal ($\tilde{x}$) is assessed using percent root-mean-square distortion (PRD (%)):

$$PRD\ (\%) = \sqrt{\frac{\sum_{i=1}^{N}(x(i) - \tilde{x}(i))^2}{\sum_{i=1}^{N} x(i)^2}} \times 100. \qquad (9)$$

We also use an alternative quantitative distortion measure, based on the maximum absolute difference between x and $\tilde{x}$:

$$PSNR(x, \tilde{x}) = 10 \log_{10}\left(\frac{2^Q - 1}{\max(|x - \tilde{x}|)}\right). \qquad (10)$$

We consider segments of 1024 samples from each channel, arranged in a suitable volume size, specifically, $32 \times 32 \times 64$ for t/dt/s volume, $8 \times 8 \times 1024$ for s/s/t volume. In all the three algorithms, we vary the quantization step-size from 0 (lossless) till 19 (lossy), and measured the CR and PRD(%) for each step-size. The energy threshold (τ) for the subband specific arithmetic coding is fixed to 50%; we obtained the best results for that value of the threshold. The results are summarized in Fig. 3. Since the results for t/dt/s and s/s/t are similar, we only show results for the t/dt/s volume construction.

It is clear from Fig. 3 that SAC outperforms both volumetric and PARAFAC coding with respect to PRD(%); with respect to PSNR, volumetric and PARAFAC coding perform similarly, but they clearly outperform SAC. The residual quantization is performed in time-domain for volumetric and PARAFAC coding, whereas in SAC, quantization is performed in wavelet domain. The idea behind SAC is to quantize the residual wavelet subbands with least energy. As we are quantizing wavelet coefficients in SAC, the maximum error in time domain cannot be controlled, and consequently, it is larger than the other two approaches considered here. Interestingly, it is promising that the average error (PRD) is smaller for SAC compared to other two approaches. However, the distortion may be large in very few samples, which may be tolerable in some specific applications.

5. CONCLUSION

We have presented novel compression schemes for multi-channel EEG. The main idea is to exploit the intra- and inter-channel correlations simultaneously by arranging the multi-channel EEG as a volume, and to represent that volume in different ways. Particularly, we considered volumetric coding, energy-based coding of wavelet subbands, and tensor based coding. Next we compressed the residual, which allows us to bound the worst-case distortion (in volumetric and PARAFAC coding). The tensor-based coding scheme yields smaller worst-case error than both subband specific coding and volumetric coding, yet the average error is only slightly larger than in subband specific coding and much smaller than in volumetric coding. Therefore, tensor-based coding is an attractive approach for multi-channel EEG compression. If larger worst-case distortion is tolerable, wavelet subband coding may also be a suitable option. In our future study, we planned to improve the worst-case error of the proposed wavelet subband specific coding by suitable threshold selection.

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