



Two-layered dynamic control for simultaneous set-point tracking and improved economic performance

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ABSTRACT

This work introduces a multi-objective optimization strategy to handle conflicting set-point tracking and economic objectives in a two-layer hierarchical control framework. A dynamic multi-objective real-time optimizer (DMO), incorporated in the upper layer, handles multiple control objectives with set-point tracking being the higher priority objective and computes optimal plant trajectories. This plant-wide trajectory information is communicated to the lower-layer model predictive control (MPC) operating at a faster sampling rate. The conventional weight-based and lexicographical method for the DMO are discussed. A new algorithm is conceptualized based on the lexicographical method to handle prioritized objectives. The proposed algorithm modifies the higher priority tracking objective and establishes improved economic performance compared to the conventional techniques, with minimal effect on the conflicting tracking objective, through a systematic choice of the preferred Pareto solution. The proposed algorithm's efficacy, within the hierarchical framework, is analyzed using two case studies: A polymerization reactor and a multi-unit reactor–separator system.

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1. Introduction

Staying competitive and profitable in the current market-driven scenario requires a control framework for optimal management of multiple conflicting objectives. Conventional control structures employ a single plant-wide objective based on optimizing economic targets for maximizing profit. However, performance targets such as product quality, production goals, and environmental norms cannot be ignored in the pursuit of profitable plant operation [1]. In fact, some plant-wide objectives such as product quality set-points may be higher priority objective(s), since the off-spec product is not sellable [2]. Hence, there is a need to develop multi-objective control strategies handling objectives, such as set-point tracking, in addition to the traditional economic targets of maximizing profits. However, conflicting behavior of objectives and associated computational demands of a multi-objective control problem are issues that need to be addressed while designing the controller.

Traditionally, a plant-wide economic objective is solved at a steady-state RTO (real-time optimization) layer based on the process-related decisions received from the planning and scheduling layer. The RTO uses a nonlinear model to compute steady-state set-points for the underlying MPC (model predictive

control) layer [3]. Although steady-state RTO is prevalent in the process industry, it has several drawbacks: Long-term transients limits how often optimization can be performed [4], it is unable to handle higher frequency disturbances or dynamic cost parameters [3], and model inconsistency with the MPC layer can potentially lead to infeasible solutions [5]. One type of approach that addresses these issues are single-layered approaches, where the RTO layer is integrated with the lower-layer MPC [6] or the economic cost function itself is solved in the MPC layer [7]. Idris and Engell [8] demonstrated the computational issues of implementing a single-layer economic-MPC on a pilot-scale plant. Findeisen and Allgöwer [9] discussed loss in performance and instability due to delay in computing the optimal solution. Although significant developments in computational power make integrated single-layer control attractive [10], concomitant developments in process systems and the increasing importance of process intensification in plants comprising of multiple units present significant challenges to the single-layer approach. Consequently, computation of the optimal trajectory within the stipulated control interval is rendered computationally intractable [5]. Since the focus of the current work is on multiple conflicting control objectives, it is imperative to design a control strategy to handle the computational demands of optimization-based control algorithms. Hence, inspired from earlier works on dynamic hierarchical control [4,5,11], we employ a two-layer control framework, with the multi-objective optimization (MOO) solved in the upper dynamic RTO layer, operating at a slower

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time-scale to handle the computational issues associated with the MOO problem.

Several applications of MO control have been discussed in the literature. For example, polymerization processes require managing multiple process targets, such as maximizing conversion, minimizing undesired products, and achieving the desired quality parameters of the final polymer [12–14]. Often, the multiple objectives are prioritized, with safety or product quality targets being at a higher priority than economic objective. Anilkumar et al. [15] used a lexicographic approach to handle multiple set-point tracking objectives based on priority. He et al. [16] used lexicographic optimization to handle a similar case of prioritized set-point tracking and economic objectives. A few other applications of multi-objective optimization include fed-batch bioreactor [17], solar refrigeration plant [18], path tracking control with predefined speed profile [19], and autonomous vehicle control to reach the destination with minimum energy and minimum time [20]. Haßkerl et al. [21] formulated a weight-based control scheme for a reactive distillation system comprising of an economic cost function and a regularization term that penalizes violation on product purity. The respective weight parameters were tuned meticulously to obtain the desired trade-off between the two objectives. Tian et al. [22] and Li et al. [23] implemented a single-layer multi-objective control, where the desired trade-off between the economic and the tracking objective is manipulated through user-defined tuning parameter. Thus, different aspects of multi-objective control comprising of tracking and economic objective were studied using single-layered control frameworks, which requires the optimal solution be available within a sampling instance. However, this becomes challenging as the complexity of the system increases, particularly for multi-objective control. Secondly, most of the earlier works either involved offline optimization or required user intervention to select the required optimal solution. The current work addresses these issues through a multi-layered control framework equipped to handle the computational rigor of multi-objective problems. Different optimization methodologies are discussed, which can be applied depending on the nature of the control problem.

The scope of this work includes a two-layer hierarchical control framework for managing priority-based multiple objectives, aimed at tracking a process parameter to its desired set-point and simultaneously maximizing the profit. An upper-layer dynamic multi-objective optimizer (DMO) computes the optimal plant trajectory for a lower-layer MPC. MPC is preferred in the design of the two-layer framework as it can deal with multiple variables and process constraints [3]. We evaluate various multi-objective optimization techniques by quantifying the performance requirement of the control objectives. A popular technique uses weight to scalarize the multiple objectives into a single objective function. However, tuning these weighing parameters is a demanding task because the objectives belong to different domains can often lead to ill-conditioned or highly nonlinear tuning behavior [15,24]. In contrast, the lexicographic approach can handle priority-based optimization [17,25]. However, the optimal tracking solution may pose a too stringent constraint to the subsequent economic optimization. Hence, a second strategy is to minimize the tracking error only at the terminal point of the horizon [15]. In principle, this is analogous to communicating the desired set-point, allowing for a compromise in the performance during the transients. To exploit the advantages of the two techniques, we propose a new algorithm, which computes multiple Pareto solutions through modifications in the tracking function. The preferred solution is chosen online from the Pareto-optimal set. This solution is determined based on the solution closest to the ideal solution of the objective. It is easy to understand that it is impossible to achieve the ideal solution of the multiple

objectives simultaneously due to their conflicting nature, and the preferred Pareto solution offers the best trade-off. The optimal trajectories are computed from the preferred solution and are communicated to a lower-layer MPC.

The performance of various approaches is evaluated using two case studies. The first investigation is a free radical polymerization reactor [26], which involves tracking of a quality parameter and maximizing the net profit as prioritized and conflicting objectives. The second case study is a “plant-wide” reactor–separator system, with a similar set of objectives.

2. Plant and control configuration

Consider a system described by the following model

$$\begin{aligned}\dot{x} &= f(x, u) \\ y &= g(x)\end{aligned}\quad (1)$$

where $x \in \mathfrak{R}^n$, $u \in \mathfrak{R}^m$ and $y \in \mathfrak{R}^p$ are the states, manipulated inputs and output variables, respectively. We assume the state feedback case and that the model is known without an error.

Fig. 1 shows a multi-layer hierarchical control structure considered in this work. The upper-layer receives criteria for product specifications (e.g., quality or composition), production goals, economic parameters (prices or costs), etc. from a decision-making/ scheduling layer. These requirements are inherently multi-objective, calling for a dynamic multi-objective optimizer (DMO). The DMO operates at a slower sampling rate, computes the optimal trajectory over a predefined future horizon, and provides the trajectory to a lower-layer MPC (Model Predictive Control). The MPC operates at a faster sampling rate and implements the trajectory received from the DMO.

2.1. Dynamic multi-objective optimizer (DMO)

The DMO forms a crucial part of the proposed approach to compute optimal plant trajectories in the presence of multiple conflicting objectives. Specifically, the control problem of tracking certain process variables to the desired set-point while simultaneously maximizing the profit is addressed. The implementation of multi-objective control online can be computationally demanding. Hence, a hierarchical control framework (Fig. 1) is adopted, wherein the computational demand is managed by operating the DMO at a lower sampling rate than the lower-layer MPC. The sampling time, $\Delta\bar{t}$, of the DMO is determined by the frequency of RTO-relevant changes, slow-scale interaction dynamics in the plant, and computational requirements in solving the multi-objective optimization online. The dynamic multi-objective optimization is formulated as

$$\min_{\bar{u}_k} \Phi = \begin{bmatrix} \phi_1(\bar{X}_k, \bar{U}_k) \\ \phi_2(\bar{X}_k, \bar{U}_k) \\ \vdots \\ \phi_\ell(\bar{X}_k, \bar{U}_k) \end{bmatrix}\quad (2)$$

$$\text{where } \bar{x}_{j+1} = \bar{x}_j + \int_0^{\Delta\bar{t}} f(x, u) dt \quad (2a)$$

$$\text{with } \bar{x}_k = x(\bar{k}, \Delta\bar{t}), \quad \bar{j} = [\bar{k}, \bar{k} + N_o] \quad (2b)$$

$$\bar{y}_j = g(\bar{x}_j) \quad (2c)$$

$$\text{subject to } \bar{y}_j^{\min} \leq \bar{y}_j \leq \bar{y}_j^{\max} \quad (2d)$$

$$\bar{u}_j^{\min} \leq \bar{u}_j \leq \bar{u}_j^{\max} \quad (2e)$$

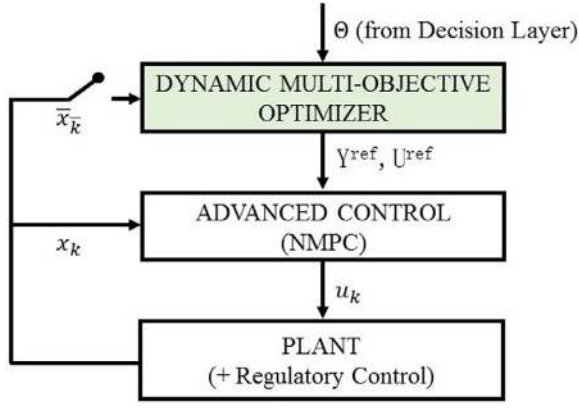


Fig. 1. Schematic of the dynamic hierarchical control structure.

In the above expressions, we assume ϕ_i to be arranged in order of their priorities, with ϕ_1 as the highest priority objective, without loss of generality. Each objective $\phi_i(\cdot, \cdot)$ is a scalar-valued function of states and inputs within the DMO horizon of size N_o . We use overbars to represent DMO-relevant variables. It is easy to see that we define

$$\bar{X}_{\bar{k}} = \begin{bmatrix} \bar{x}_{\bar{k}+1} \\ \bar{x}_{\bar{k}+2} \\ \vdots \\ \bar{x}_{\bar{k}+N_o} \end{bmatrix} \text{ and } \bar{U}_{\bar{k}} = \begin{bmatrix} \bar{u}_{\bar{k}+1} \\ \bar{u}_{\bar{k}+2} \\ \vdots \\ \bar{u}_{\bar{k}+M_o} \end{bmatrix} \quad (3)$$

as arrays of the state and input vectors of the DMO, respectively. Here, N_o and M_o are the prediction and control horizons of the multi-objective optimizer.

Various methods for handling priority-based multiple objectives in the dynamic optimization framework are discussed in Section 3. As seen in Fig. 1, the DMO operates infrequently and its state $\bar{x}_{\bar{k}}$ is initialized as the plant state $x(k, \Delta t)$ at time $t = \bar{k} \cdot \Delta t$, as seen in Eq. (2b). The DMO receives the plant state at $\bar{x}_{\bar{k}}$ and computes the optimal reference trajectories U^{ref} and Y^{ref} , which are then passed on to the lower-layer MPC. Optimization is carried out when either a decision is communicated from the scheduling layer or when disturbances in the process cause the plant trajectory to deviate substantially from Y^{ref} . Since we consider state feedback hierarchical control with no model-plant mismatch, the DMO becomes active at the end of its prediction horizon, after the implementation of the entire trajectory. Including the effect of disturbances on the existing multi-objective control problem can be an interesting work for the future.

2.2. Lower-layer NMPC

The upper-layer DMO communicates the computed optimal trajectories to the lower-layer NMPC. The NMPC operates at a faster interval than the DMO. The aim of NMPC is to track the communicated set-point trajectories, while meeting the system constraints. The objective function is given by:

$$\min_{\mathcal{U}_k} \Phi_k^{MPC} = \min_{\mathcal{U}_k} \left\{ \left\| Y_k^{ref} - \mathcal{Y}_k \right\|_{\mathcal{Y}}^2 + \left\| U_k^{ref} - \mathcal{U}_k \right\|_{\mathcal{U}}^2 + \left\| \Delta \mathcal{U}_k \right\|_{\mathcal{U}}^2 \right\} \quad (4)$$

subject to

$$x_{j+1} = x_j + \int_0^{\Delta t} f(x, u) dt \quad (4a)$$

$$y^{min} \leq y_j \leq y^{max} \quad (4b)$$

$$u^{min} \leq u_j \leq u^{max} \quad (4c)$$

In the above equations, $j = [k, k + P - 1]$, whereas

$$\mathcal{Y}_k = \begin{bmatrix} y_{k+1} \\ y_{k+2} \\ \vdots \\ y_{k+P} \end{bmatrix} \text{ and } \mathcal{U}_k = \begin{bmatrix} u_k \\ u_{k+1} \\ \vdots \\ u_{k+M-1} \end{bmatrix} \quad (5)$$

are vectors of P -step output predictions and M -step manipulated inputs. The lower-layer MPC operates at sampling time of Δt and Y^{ref} and U^{ref} are the reference trajectories communicated by the DMO. Any suitable approach, such as nonlinear MPC, sequentially linearized MPC, or linear MPC, may be used at this layer.

3. Dynamic multi-objective optimization algorithms

We now consider various algorithms to handle multi-objective dynamic optimization where the objective functions $\phi_i(\cdot, \cdot)$ are organized in order of their priority. Specifically, the multi-objective control problem of the current work is dealt by assigning higher priority to the tracking objective, $\phi_{track}(\cdot, \cdot)$, compared to the economic objective $\phi_{eco}(\cdot, \cdot)$, given as

$$\phi_{track} = \sum_{i=1}^{N_o} \left\| \bar{x}_{\bar{k}+i}^t - \theta_{spec} \right\|^2 \quad (6)$$

$$\phi_{eco} = \sum_{i=1}^{N_o} \varphi_{eco,i}(\bar{x}_{\bar{k}+i}, \bar{u}_{\bar{k}+i}, \theta_{eco}) \quad (7)$$

In the above, $\bar{x}^t \subset \bar{x}$ represent variables that must be tracked (typically quality or product specifications) to the desired specifications $\theta_{spec} \subset \Theta$. The parameters $\theta_{eco} \subset \Theta$ stand for a set of dynamic cost parameters. The parameters Θ may be provided to the DMO by the decision layer.

We first discuss the standard weight-based approach (Section 3.1), followed by the lexicographical approach in Section 3.2. Thereafter, we modify the endpoint-based lexicographical approach proposed by Anilkumar et al. [15] to the DMO in Section 3.3. Finally, to overcome the limitations of these approaches, we propose a Pareto-based lexicographical approach in Section 3.4.

3.1. Single objective weight-based method

The traditional weight-based approach augments the set of objectives with relative weight, and the multi-objective control problem is formulated as a single objective scheme. The mathematical representation of the same is given by:

$$\Phi_{wt}^* = \min_{\bar{U}_{\bar{k}}} \Phi_{wt} = \min_{\bar{U}_{\bar{k}}} (w_1 \phi_{track} + w_2 \phi_{eco}) \quad (8)$$

subject to constraints (2a)–(2e)

where, ϕ_{track} and ϕ_{eco} , are defined in Eqs. (6) and (7), and w_1, w_2 denote weighting parameters. Though the weight-based formulation is simpler to handle, the weighting parameters of the objective functions require meticulous tuning to arrive at the desired results [15,24]. This becomes challenging as processes engage objectives from different domains, and improper tuning of weights results in sub-optimal performance. Since the weights balance the trade-off between multiple objectives, the weight-based method is unable to handle priority-based multi-objective control [18].

3.2. Lexicographic method (conventional)

We briefly describe the standard lexicographic method in this section. An interested reader is referred to [15,17,25] for more details. This is a multi-layered optimization approach that begins with the optimization of the highest priority objective. The optimal value of the higher priority objective(s) is subsequently posed as constraints for the optimization of the lesser priority objectives. This procedure is continued until the optimization of the least priority objective, and the solution obtained thus becomes the optimal solution of the multi-objective problem.

Thus, the tracking objective (Eq. (6)) is first solved to minimize the tracking error over the entire horizon, N_o :

$$\phi_{track}^* = \min_{\bar{U}_k} \phi_{track}(\bar{X}_k, \bar{U}_k) \quad (9)$$

subject to constraints (2a)–(2e)

To exploit the available dynamic degrees of freedom, the solution ϕ_{track}^* so obtained is posed as a constraint, and the economic objective function is solved

$$\phi_{eco}^* = \min_{\bar{U}_k} \phi_{eco}(\bar{X}_k, \bar{U}_k) \quad (10)$$

$$\text{subject to } \phi_{track} \leq \phi_{track}^* \quad (10a)$$

and constraints (2a)–(2e)

3.3. Lexicographic method (terminal)

Despite the conflicting nature of the objectives, the stringent formulation of the conventional lexicographic method does not allow any compromise on the tracking objective to improve the economic cost function. Contrary to this methodology, Anilkumar et al. [15] proposed modifying the tracking objective Eq. (6) to optimize only the terminal point:

$$\phi_{N_o, track}^* = \min_{\bar{U}_k} \phi_{N_o, track}(\bar{X}_k, \bar{U}_k) = \min_{\bar{U}_k} \left\| \bar{x}_{k+N_o}^t - \theta_{spec} \right\|^2 \quad (11)$$

subject to constraints (2a)–(2e)

Anilkumar et al. [15] used this approach for the MPC layer, where multiple set-point tracking objectives are solved based on predefined priorities. This approach can be adapted for the DMO with a lower-priority economic objective. The conventional tracking formulation explained in the previous section with the modification in the lexicographic constraint is given by:

$$\phi_{eco}^* = \min_{\bar{U}_k} \phi_{eco}(\bar{X}_k, \bar{U}_k) \quad (12)$$

$$\text{subject to } \phi_{N_o, track} \leq \phi_{N_o, track}^* \quad (12a)$$

and constraints (2a)–(2e)

The principle behind this approach is to track the necessary variable to the set-point at the steady-state. A compromise in the transient tracking performance achieves improved economic cost due to conflicting interaction between the objectives.

3.4. Proposed Pareto-optimal lexicographic method

The algorithms discussed previously present the two extremes of the trade-off that could be achieved in the tracking objective to enhance the economic targets. We, therefore, propose a new approach that is aimed at finding an optimal trade-off in Pareto sense utilizing the lexicographic method as the base algorithm. Specifically, we propose a modification in the tracking objective that allows the computation of multiple Pareto solutions, each representing a different compromise between the objectives. The

best solution is chosen among the Pareto set. The set-point tracking objective to compute multiple trade-off solutions is modified as:

$$\phi_{\bar{n}, track} = \sum_{i=\bar{n}}^{N_o} \left\| \bar{x}_{k+i}^t - \theta_{spec} \right\|^2, \quad 1 \leq \bar{n} \leq N_o \quad (13)$$

With this modification, $\phi_{\bar{n}, track}$ is equal to ϕ_{track} of the conventional method for $\bar{n} = 1$ and equal to $\phi_{N_o, track}$ of the terminal-lexicographic method (Section 3.3) for $\bar{n} = N_o$. The economic objective formulation remains the same as in Eq. (7). The distinct values of \bar{n} generate different optimal solutions, which lead to different lexicographic constraints for the economic objective, generating multiple Pareto or non-inferior solutions.

We now describe the systematic implementation of the proposed algorithm to calculate the optimal plant trajectory. For a particular value of \bar{n} , the optimization for the tracking objective is given by:

$$\phi_{\bar{n}, track}^* = \min_{\bar{U}_k} \phi_{\bar{n}, track} \quad (14)$$

$$\text{subject to constraints (2a)–(2e)}$$

The optimal value $\phi_{\bar{n}, track}^*$ is retained as a lexicographic constraint while optimizing the economic objective:

$$\phi_{\bar{n}, eco}^* = \min_{\bar{U}_k} \phi_{eco}(\bar{X}_k, \bar{U}_k) \quad (15)$$

$$\text{subject to } \phi_{\bar{n}, track} \leq \phi_{\bar{n}, track}^* \quad (15)$$

and constraints (2a)–(2e)

As \bar{n} is varied for each lexicographic optimization of the economic objective, the algorithm computes a set of N_o Pareto-optimal solutions, each denoting distinct plant trajectories. Finally, we propose an algorithm for choosing the optimal trajectory from the set of Pareto trajectories, which leads to the best overall performance for the given objectives.

The implementation of the algorithm to choose the optimal trajectory is based on the standard vectorial representation of multi-objective problem formulation [25]. Recall that the above procedure yields a sequence of optimal input moves, $\bar{U}_{\bar{n}, track}^*$ (Eq. (14)) and $\bar{U}_{\bar{n}, eco}^*$ (Eq. (15)), for each value of $\bar{n} \in [1, N_o]$. The values of the tracking and economic objective functions obtained from the lexicographic optimization are normalized and expressed in a vector form as:

$$\Phi_{\bar{n}} = \begin{bmatrix} \tilde{\phi}_{\bar{n}, track} \\ \tilde{\phi}_{\bar{n}, eco} \end{bmatrix}^T, \quad 1 \leq \bar{n} \leq N_o \quad (16)$$

where,

$$\tilde{\phi}_{\bar{n}, track} = \frac{\min(\phi_{\bar{n}, track}) - \phi_{\bar{n}, track}}{\min(\phi_{\bar{n}, track}) - \max(\phi_{\bar{n}, track})} \quad (17a)$$

$$\phi_{\bar{n}, track} = \phi_{track}(\bar{X}_k, \bar{U}_{\bar{n}, track}^*) \quad (17b)$$

and max and min values represent the highest and lowest values of $\phi_{\bar{n}, track}$ among the N_o values. Similarly,

$$\tilde{\phi}_{\bar{n}, eco}^* = \frac{\max(\phi_{\bar{n}, eco}^*) - \phi_{\bar{n}, eco}^*}{\max(\phi_{\bar{n}, eco}^*) - \min(\phi_{\bar{n}, eco}^*)} \quad (18)$$

This ensures that all the $\tilde{\phi}$ values lie between 0 and 1. Furthermore, since $\min(\phi_{\bar{n}, track})$ and $\max(\phi_{\bar{n}, eco}^*)$ are the best tracking and economic objective values, respectively, the origin becomes the ideal or the desired solution (also called the utopian point). Thus $\Phi_{id} = (0, 0)$. Due to the conflicting nature of the objectives, the ideal point is not realizable, and hence the optimal solution is chosen as the closest Pareto point as determined by the

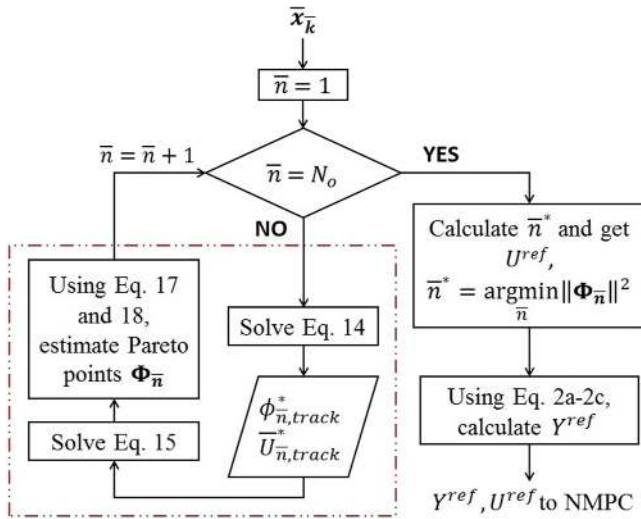


Fig. 2. Flowchart describing the procedure of the proposed algorithm in calculating the optimal trajectory for lower-layer NMPC.

squared-norm minimization:

$$\bar{n}^* = \underset{\bar{n}}{\operatorname{argmin}} \|\Phi_{\bar{n}} - \Phi_{id}\|^2 = \underset{\bar{n}}{\operatorname{argmin}} \|\Phi_{\bar{n}}\|^2 \quad (19)$$

In the above equation, the distances between the origin or the utopian point (Φ_{id}) and each of the Pareto points ($\Phi_{\bar{n}}$) are calculated. The Pareto point with minimum squared-norm value denotes the point closest to the utopian point. Hence, the solution corresponding to lexicographic optimization performed with $\bar{n} = \bar{n}^*$ is the preferred optimal solution, $U^{ref} = \bar{U}_{\bar{n}^*,eco}$.

The strict constraints of the conventional lexicographic method lead to the poor economic performance. We addressed this issue through the proposed algorithm, which generates different trade-off solutions at each optimization instant by varying the prediction window (or \bar{n}) of the priority objective. Eq. (19) chooses the best solution among the computed Pareto solutions. The procedure is summarized in Fig. 2. The optimal plant trajectory, Y^{ref} , is then calculated using the dynamic plant model (Eqs. (2a)–(2c)) and the optimal inputs $U^{ref} = \bar{U}_{\bar{n}^*,eco}$. The DMO then communicates the trajectories U^{ref} and Y^{ref} to the MPC layer. For a single layer control structure, the proposed algorithm may also be implemented at a multi-objective MPC layer.

4. Simulation studies

The proposed hierarchical control framework is demonstrated in two case studies: an isothermal polymerization reactor and a multi-unit reactor–separator system. In both examples, maintaining a product quality variable at its set-point is the priority objective while also aiming to maximize profit. The performance of the algorithms is analyzed and compared using the root mean squared error (RMSE) from the specification and the average profit.

$$\text{RMSE} = \sqrt{\frac{\sum_{j=1}^{T_f} (x_j^t - \theta_{spec})^2}{T_k}} \quad (20)$$

$$\text{Average profit} = \frac{\sum_{j=1}^{T_f} \varphi_{eco}(x_j, u_j, \theta_{eco})}{T_f}$$

These are the performance indicators of quality and economic objectives respectively. T_f denotes the total number of sample time steps considered for the simulation study. The simulations

Table 1
Performance comparison across different methods.

Algorithm	RMSE	Avg. profit
$w_1 = 0; w_2 = 1$	2933.9	619.62
$w_1 = 1; w_2 = 0$	1101.2	370.00
$w_1 = 1; w_2 = 1$	1105.4	563.93
$w_1 = 100; w_2 = 1$	1101.4	434.20
Lexicographical (conventional)	1105.1	449.06
Lexicographical (terminal)	1171.2	570.64
Proposed Pareto-optimal	1111.8	600.17
E-MPC (single layer)	1501.0	318.99

are performed on an intel core-i7 workstation with 16 GB RAM. The algorithm is coded on MATLAB 2018a, and the inbuilt optimization solver, *fmincon*, is used for the nonlinear optimization.

4.1. Case study 1: Free radical polymerization reactor

The proposed DMO strategy is investigated on an isothermal free radical polymerization reactor [27]. The dynamic model equations [26,27] are given by :

$$\begin{aligned} \frac{dC_M}{dt} &= -(k_p + k_{fm})C_M P_0 + \frac{F}{V}(C_{M,in} - C_M) \\ \frac{dC_I}{dt} &= -k_I C_I + \frac{F_I}{V} C_{I,in} - \frac{F}{V} C_I \\ \frac{dD_0}{dt} &= (0.5k_{\tau c} + k_{\tau d})P_0^2 + k_{fm}C_M P_0 - \frac{F}{V} D_0 \\ \frac{dD_1}{dt} &= M_m(k_p + k_{fm})C_M P_0 - \frac{F}{V} D_1 \end{aligned} \quad (21)$$

where, C_M and C_I are the concentration of monomer and the initiator, D_0 and D_1 are the molar and mass concentrations of the dead polymer chains, and

$$M_w = \frac{D_1}{D_0} \quad \sigma = 1 - \frac{C_M}{C_{M,in}}$$

are the number average molecular weight of the polymer chain (M_w) and net monomer conversion, respectively. The manipulated variables are F_I and F , which denote the initiator and monomer flowrates, respectively. The model parameter values are given in Maner et al. [27].

The priority objective is to track M_w to its set-point

$$\varphi_{track} = \|M_w - M_w^{ref}\|^2, \quad (22)$$

whereas the lesser priority economic objective is defined by the profit function [26,28]

$$\varphi_{eco} = FD_1 + 3500\sigma^{0.6} + 9 \times 10^{-4}M_w^{0.65} - aF_I^{0.5} \quad (23)$$

We consider the case where $M_w^{ref} = 2.5 \times 10^4$ and the cost parameter a varies dynamically as:

$$a = \begin{cases} 3500 & \text{if } 0.25 \text{ h} \leq t < 0.8 \text{ h} \\ 3000 & \text{if } 0.8 \text{ h} \leq t < 1.5 \text{ h} \\ 2500 & \text{if } 1.5 \text{ h} \leq t < 2 \text{ h} \end{cases} \quad (24)$$

The multi-objective optimization problem involving the set-point tracking and the economic objective is dealt with in the upper-layer DMO. The DMO manages the computational demand by operating at a slower sampling time, $\Delta \bar{t} = 0.03 \text{ h}$ (108 s). The sampling time of the lower-layer MPC is chosen as $\Delta t = 0.005 \text{ h}$ (18 s). The DMO predicts the control move for 0.3 h into the future (i.e., DMO prediction horizon, $N_o = 10$). The prediction and the control horizon for the NMPC are 30 and 5, respectively. The DMO is taken online at 0.25 h.

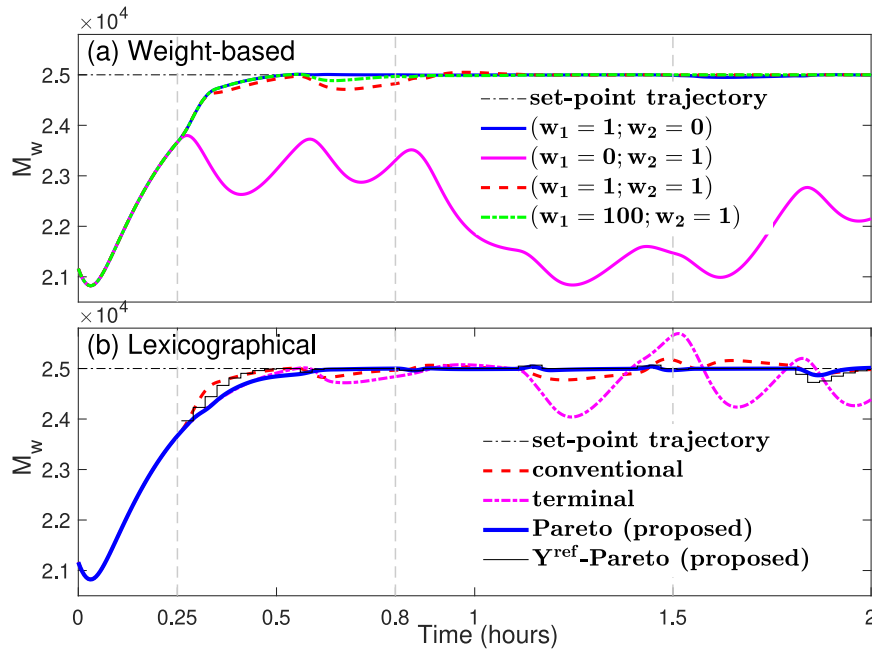


Fig. 3. Tracking performance of M_w implemented using (a) different weights for the weight-based method and (b) various lexicographical approaches. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

The performance of different DMO algorithms is quantified in Table 1. Initial investigations are carried out with the conventional weight-based method with a different set of weights. The tracking performance is the best for $w_1 = 1, w_2 = 0$, as indicated by the lowest RMSE value, which is expected as only the tracking objective is considered. This is confirmed by the trajectory of the tracking variable (solid-blue) in Fig. 3a. However, the average profit is the least across all methods. Conversely, for $w_1 = 0, w_2 = 1$, which presents the other end of the spectrum where only the economic objective is solved, the average profit is largest, albeit with the worst performance of the tracking function (solid-magenta line in Fig. 3a). The difficulty in choosing an optimal value of the relative weight is evident from the analysis of the performance indices reported for intermediate sets of weights. In comparison with the RMSE values obtained for $(w_1, w_2) = (100, 1)$ and $(1, 1)$, there is a slight improvement of 0.4% in the tracking performance. However, a 23% reduction in the profit is observed. Similarly, comparing the cases with $(100, 1)$ and $(1, 0)$, there is a 15% reduction in the profit even with the slightest change in the relative weight of the economic objective (from 0.01 to 0). This shows a highly nonlinear relationship between the weights and the objective function values, making it challenging to tune for the optimal weights.

Fig. 3b compares various lexicographical methods, which do not require any tuning parameters to compute the optimal trajectory. A comparison of the conventional lexicographical algorithm with the results obtained for the weight-based approach $(100, 1)$ shows a slightly lesser performance for the tracking objective with a minor improvement in profit (3.5%). Similarly, the terminal lexicographical methodology reports a 31% increase in the average profit, with some loss of set-point tracking performance (i.e., 6% increase in the RMSE). These results indicate that the two objectives show conflicting behavior. Hence, the proposed Pareto-optimal lexicographical approach targets some level of compromise on the tracking objective in the Pareto sense to improve the average profit. From Table 1, compared to the conventional lexicographical method, there is a slight reduction in the tracking performance (0.5% increase in RMSE), which is reasonably good

given that the conventional method implements strict optimization condition for set-point tracking. As seen in Fig. 3b, there is a negligible difference in the tracking performance of the proposed method (solid-blue line) compared to that of the conventional algorithm (dashed-red line). However, the average profit shows a 34% increase, which is an excellent trade-off obtained through a slight increase in the RMSE. A similar comparison with the weight-based approach $(100, 1)$ shows a 38% increase in profit with a minimal trade-off of the tracking objective (1% increase in RMSE). The multi-layer approach with different algorithms is also compared with the performance of economic MPC (E-MPC). Since an E-MPC handles a single economic objective, the tracking performance is affected as it conflicts with the performance of the economic objective. This results in a significantly higher RMSE value between the process set-point and the tracking variable.

Table 2 shows the best \bar{n}^* value and the corresponding values of $\Phi_{\bar{n}^*}$. It can be observed that $\phi_{\bar{n}, track} = 0$ for $\bar{n} = 1$ at all times. This is expected because ϕ_{track} is the highest priority objective. The first column lists the various time instances and the changes in the economic cost parameter (a , Eq. (24)) at $t = 0.25, 0.8, \text{ and } 1.5 \text{ h}$, triggers the DMO. The proposed algorithm implemented in the DMO allows trade-off in ϕ_{track} to achieve the best economic performance. At $t = 0.8 \text{ h}$, we can observe an interesting behavior where the best economic performance ($\phi_{\bar{n}, eco} = 0$) is obtained at $\bar{n}^* = 5$ with no trade-off from the tracking objective. Also, it can be seen at other DMO instances that a significant improvement in the profit is achieved with no or very less trade-off in ϕ_{track} . Since the computation of the Pareto front is computationally demanding, the proposed Pareto-based lexicographical approach requires a higher computation time than the other approaches. In the hierarchical control approach, the reference trajectories U^{ref} and Y^{ref} are communicated to the MPC after a time delay of δ seconds, where δ is an integral multiple of MPC-sampling-time Δt [5]. As the computational time of the proposed method is 27 s, a delay of $2\Delta t$ is assigned in this case. The remaining algorithms are implemented with a delay of one sampling time of the lower-layer MPC (i.e., $\delta = \Delta t = 18 \text{ s}$) as their computational delays are less than 18 s. This delay δ could

Table 2

Normalized optimal function and corresponding squared-norm values for different \bar{n} at every DMO online instance. The best \bar{n}^* for each time DMO online instance is shown in bold.

$\bar{n} \rightarrow$	\bar{n}^*		1	2	3	4	5	6	7	8	9	10
0.25	9	$\tilde{\phi}_{\bar{n},track}$	0	0	0.024	0.030	0.059	0.103	0.106	0.066	0.357	1
		$\phi_{\bar{n},eco}^*$	1	1	0.652	0.602	0.647	0.658	0.652	0.564	0	0.627
		$\ \Phi_{\bar{n}}\ ^2$	1	1	0.426	0.363	0.422	0.444	0.436	0.322	0.127	1.393
0.55	4	$\tilde{\phi}_{\bar{n},track}$	0	0.042	0.160	0.094	0.153	0.357	0.104	0.134	1	0.385
		$\phi_{\bar{n},eco}^*$	1	0.206	0.174	0.175	0.158	0.225	0.508	0.408	0.334	0
		$\ \Phi_{\bar{n}}\ ^2$	1	0.044	0.056	0.039	0.048	0.178	0.269	0.184	1.112	0.148
0.80	5	$\tilde{\phi}_{\bar{n},track}$	0	0	0	0	0	0	0.003	0.029	0.056	1
		$\phi_{\bar{n},eco}^*$	0.696	0.697	0.695	0.696	0	0.713	0.775	1	0.824	0.713
		$\ \Phi_{\bar{n}}\ ^2$	0.484	0.486	0.483	0.484	0	0.508	0.601	1.001	0.682	1.508
1.1	4	$\tilde{\phi}_{\bar{n},track}$	0	0.003	0.012	0.009	0.005	0.004	0.010	0.007	1	0.023
		$\phi_{\bar{n},eco}^*$	0.120	0.120	0.799	0	0.320	0.807	0.243	0.604	1	0.260
		$\ \Phi_{\bar{n}}\ ^2$	0.014	0.014	0.639	0	0.102	0.651	0.059	0.365	2	0.068
1.4	5	$\tilde{\phi}_{\bar{n},track}$	0	0.001	0.002	0.003	0.006	0.004	0.003	0.021	0.008	1
		$\phi_{\bar{n},eco}^*$	1	0.258	0.340	0.250	0	0.963	0.030	0.080	0.370	0.106
		$\ \Phi_{\bar{n}}\ ^2$	1	0.067	0.116	0.063	0	0.927	0.001	0.006	0.137	1.011
1.5	2	$\tilde{\phi}_{\bar{n},track}$	0	0	0.001	0.003	0	0	0.001	0.001	0.006	1
		$\phi_{\bar{n},eco}^*$	1	0	0.124	0.051	0.002	0.207	0.038	0.652	0.097	0.108
		$\ \Phi_{\bar{n}}\ ^2$	1	0	0.015	0.003	0	0.043	0.001	0.425	0.009	1.012
1.8	9	$\tilde{\phi}_{\bar{n},track}$	0	0.001	0.003	0.325	0.003	1	0.005	0.005	0.005	0.857
		$\phi_{\bar{n},eco}^*$	1	0.145	0.154	0.182	0.065	0.447	0.084	0.111	0	0.116
		$\ \Phi_{\bar{n}}\ ^2$	1	0.021	0.024	0.139	0.004	1.2	0.007	0.012	0	0.748

Table 3

Steady state parameter values of reactor–separator system.

Parameter	Value	Parameter	Value
F_{1in}	35 (kmol/h)	$M_{r,1}$	10 (kmol)
F_{2in}	35 (kmol/h)	$M_{r,2}$	25 (kmol)
$Q_{c,1}$	75 (kmol/h)	$M_{c,1}$	10 (kmol)
$Q_{c,2}$	75 (kmol/h)	$M_{c,2}$	50 (kmol)
F_b	50 (kmol/h)	x_{A0}	1
R	50 (kmol/h)	x_{B0}	0
T_0	360 (K)	T_w	300 (K)
k_{10}	8e10 (1/h)	E_1/R	9300 (1/K)
k_{20}	2e9 (1/h)	E_2/R	9000 (1/K)
α_A	18	ΔH_1	-4e4 (kJ/kmol)
α_B	9	ΔH_2	-4e4 (kJ/kmol)
α_C	1	c_p	700 (kJ/kmol K)
P	1 (atm)	c_{pw}	76 (kJ/kmol K)

Table 4

Variation in set-point and cost parameters with time.

Time frame	$x_{B,1}^{ref}$	β_B	β_W
0 h \leq t < 0.1 h	0.4347	7.5	1
0.1 h \leq t < 0.8 h	0.6	7.5	1
0.8 h \leq t < 1.5 h	0.6	4	0.2
1.5 h \leq t < 2.5 h	0.7	4	0.2
2.5 h \leq t < 3 h	0.7	5	0.5

increase depending on the complexity of the multi-objective optimization, which will be evident in the subsequent case study.

4.2. Case study 2: Reactor–separator system

The performance of the proposed DMO is analyzed in a multi-unit system consisting of two CSTRs followed by a separation column, as shown in Fig. 4. A series reaction ($A \xrightarrow{k_1} B \xrightarrow{k_2} C$) proceeds in the two reactors and the desired product B is obtained from the bottom stream F_b of the 5-stage distillation. The system is characterized through 18 states which includes 4 states in each of the reactors and 10 states in the separator (tray

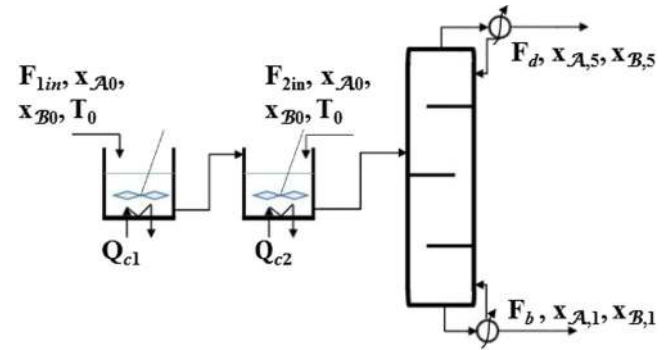


Fig. 4. Schematic of the reactor–separator system, comprising of two reactors followed by a five-stage distillation.

compositions of A and B). The model equations are given in [29] and the values of model parameters are tabulated in Table 3.

The tracking objective is to achieve the desired set-point of the product B in the reboiler outlet and simultaneously maximizing the overall profit:

$$\phi_{track} = \left\| x_{B,1} - x_{B,1}^{ref} \right\|^2 \quad (25a)$$

$$\phi_{eco} = \beta_B x_{B,1} F_b - \beta_W (Q_{c,1} + Q_{c,2}) \quad (25b)$$

The manipulated variables are the coolant flowrates ($Q_{c,1}$, $Q_{c,2}$) and F_b . For the lower-layer NMPC, the controlled variables are T_1 , T_2 , $x_{B,1}$. The variations in the cost parameters and the set-point are tabulated in Table 4. As explained earlier, re-optimization is carried out when these changes are communicated to the DMO.

The sampling time ($\Delta \bar{t}$) chosen for the optimizer is 0.05 h (180 s) whereas the lower-layer NMPC operates with a time (Δt) of 0.005 h (18 s). The prediction and the control horizon for the DMO and NMPC are (10, 3) and (40, 2) respectively. The DMO is taken online at 0.1 h and rest of the time instances where the cost parameters varies are represented as vertical gray lines in Fig. 5.

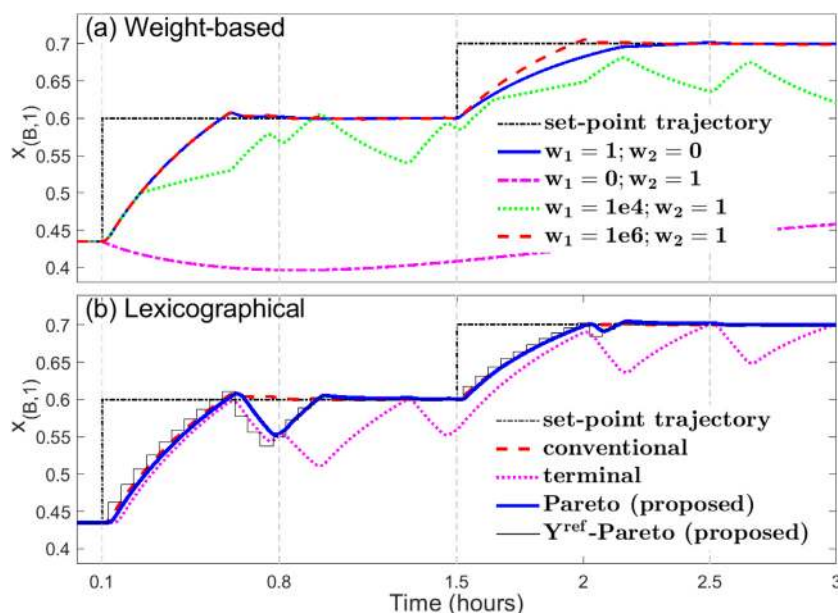


Fig. 5. Tracking performance of $x_{B,1}$ implemented using (a) different weights using the weight-based approach (b) lexicographic-based algorithms.

Table 5

Performance comparison across different methods.

Algorithm	RMSE	Avg. profit
$w_1 = 0; w_2 = 1$	0.2307	79.23
$w_1 = 1; w_2 = 0$	0.0428	-12.18
$w_1 = 100; w_2 = 1$	0.2307	79.23
$w_1 = 1e4; w_2 = 1$	0.0624	37.36
$w_1 = 1e6; w_2 = 1$	0.0412	21.91
Lexicographic (conventional)	0.0423	14.86
Lexicographic (terminal)	0.0605	30.19
Proposed Pareto-optimal	0.0451	24.51
E-MPC (single layer)	0.2293	88.94

The performance parameters of both the objectives are listed in Table 5. Understandably, due to the conflicting set of objectives, a weight-based approach with (1,0) produces good set-point tracking performance (Fig. 5a) with the least average profit. Conversely, simulation using the weight pairs (0,1) shows the highest profit and poor tracking performance. With the weights of the tracking objective tuned from 0 to 100, we observed no change in the tracking performance, implying the nonlinear tuning relationships. Also, it is required to explore higher tuning parameters, ($w_1 = 1e6$), for a satisfactory trade-off performance.

Further, the results of different lexicographic methods are analyzed. Fig. 5b shows that the conventional lexicographic method (solid-red line) shows excellent tracking performance, with a significant improvement in average profit compared to the case where only the higher priority objective is targeted ($w_1 = 1, w_2 = 0$). The lexicographic (terminal) tracks the set-point poorly (dotted-magenta line), though there is a 103% improvement in the average profit compared to the conventional approach. Comparing the conventional lexicographic method with the proposed Pareto-based lexicographic method, we observe that a little compromise in the tracking objective (6% increase in RMSE). However, there is a tremendous improvement in profit (65%). Visual inspection of the tracked variable in Fig. 5b shows minimal variation between the performance of the proposed method (solid-blue line) and the best possible tracked trajectory, which is that of the conventional lexicographic method (dashed-red line). The performance of the multi-layer controller

is also compared with the traditional E-MPC, which targets only economic performance. As seen in Table 5, we can observe that the E-MPC delivers the best economic performance with the highest value of average profit. However, the tracking performance is very poor, with very high RMSE value. This demonstrates the superior performance of the multi-layer framework for multi-objective control.

As explained previously, for every DMO online instance, the algorithm computes the trajectories corresponding to \bar{n}^* , chosen based on the least squared-norm values. The simulation time required for the weight-based optimization was approximately 6 s. In comparison, the three lexicographic methods required up to 28 s (conventional), 17 s (terminal) and 89 s (proposed). This includes the time required for computing Y^{ref} and U^{ref} , which are then communicated to the lower-layer MPC. For the proposed method, the delay is taken as $\delta = 5\Delta t$ (90 s). Similarly, for the conventional and terminal lexicographic methods, $\delta = 2\Delta t$ and $\delta = \Delta t$, respectively, whereas the trajectories from weight-based methods are communicated at the next sampling instance (i.e., $\delta = \Delta t$).

5. Conclusion

A dynamic hierarchical control was demonstrated in this work to handle multiple conflicting objectives comprising of set-point tracking and maximization of profit. Optimal plant trajectories were computed by the upper-layer DMO, which had a lower sampling rate. The trajectories were communicated to the lower-layer NMPC, which were operated at a higher sampling rate. A detailed procedure for implementing the multi-layer control framework was also explained with an exclusive focus on handling the computational demand of multi-objective optimization.

Different optimization algorithms were analyzed and evaluated in two case studies. The first simulation study was carried on an isothermal single unit polymerization process, which required optimal control action to track the average molecular weight of the polymer to the desired set-point as well as maximize the overall profit. The disadvantages of implementing the traditional weight-based approach were clearly demonstrated. It was shown that the nonlinear tuning behavior of the relative weights acts as a detrimental factor in computing optimal control action.

Lexicographic-based methods were introduced, which did not require explicit tuning parameters to compute optimal solutions for multi-objective optimization. A new algorithm was also proposed, and the performance benefits obtained for the economic objective with a slight trade-off of the tracking objective was explained.

The second study was conducted on a multi-unit system comprising of two CSTRs, followed by a separator connected in series. Evaluation of different optimization methodologies was carried out similar to the first case study. The advantages of the proposed method were apparent, particularly in comparison with the terminal point lexicographic approach due to the poor tracking performance of the latter. Finally, the notion behind fixing the simulation delay time is explained for each algorithm.

CRedit authorship contribution statement

Arvind Ravi: Conceptualization, Methodology, Investigation, Validation, Writing - original draft, Editing. **Niket S. Kaisare:** Conceptualization, Methodology, Writing - review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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