Transport Anomalies and Marginal Fermi-Liquid Effects at a Quantum Critical Point

[Phys. Rev. Lett. **85**, 4602 (2000)]

D. Belitz[1], T.R. Kirkpatrick[2], R. Narayanan[3], and Thomas Vojta[4]
[1] *Department of Physics and Materials Science Institute, University of Oregon, Eugene, OR 97403*
[2] *Institute for Physical Science and Technology and Department of Physics, University of Maryland, College Park, MD 20742*
[3] *Theoretical Physics, University of Oxford, OX3 1NP, UK*
[4] *Institut für Physik, TU Chemnitz, D-09107 Chemnitz, Germany*
(October 27, 2018)

The conductivity and the tunneling density of states of disordered itinerant electrons in the vicinity of a ferromagnetic transition at low temperature are discussed. Critical fluctuations lead to nonanalytic frequency and temperature dependences that are distinct from the usual long-time tail effects in a disordered Fermi liquid. The crossover between these two types of behavior is proposed as an experimental check of recent theories of the quantum ferromagnetic critical behavior. In addition, the quasiparticle properties at criticality are shown to be those of a marginal Fermi liquid.

Long-range spatial and temporal correlations are characteristic of systems near critical points, where critical soft modes lead to power-law, or scale invariant, behavior of correlation functions [1]. This holds primarily for the order parameter correlations, but correlation functions of other observables that couple to the order parameter show related effects at a critical point. Away from critical points, soft modes unrelated to a phase transition can induce scale invariance in entire regions of parameter space. Such 'generic scale invariance' is in general stronger in quantum systems than in classical ones [2].

An example of generic scale invariance in quantum systems are the 'weak-localization' effects in disordered metals, which consist of nonanalytic temperature, frequency, and wavenumber dependences of various observables [3]. For instance, in three-dimensions (3-D) the density of states (DOS) as a function of bias voltage has a square-root singularity at the Fermi level, the conductivity varies like the square root of temperature, etc. These nonanalyticities provide corrections to the leading disordered Fermi-liquid behavior in the metal. They arise from the diffusive nature of the basic fermionic excitations which couple to the various observables. In perturbation theory, integrals over diffusion poles lead to nonanalyticities [3]. Alternatively, these phenomena can be considered as corrections to scaling near a disordered Fermi-liquid fixed point [4]. In 2-D, the corresponding effects are stronger and destroy the Fermi liquid. Near a quantum critical point, critical fluctuations appear. In general, they have a different dispersion relation then diffusive modes, and the nonanalyticities induced by them can be strong enough to destroy the Fermi-liquid behavior even in 3-D. Non-Fermi liquid behavior induced by quantum critical points has been of much interest lately, in particular in connection with an antiferromagnetic quantum critical point that has been proposed to underly some of the behavior of high-T_c superconductors [5].

In this Letter we study the effects of a *ferromagnetic* (FM) quantum critical point on the transport, tunneling, and quasiparticle (QP) properties of disordered metals. The thermodynamic properties at this phase transition have been discussed elsewhere [6–8]. We are motivated by the following observations: (1) The quantum FM transition is the best studied of all quantum critical points [9]. Recent theoretical work [6] has shown that the 'weak-localization' effects mentioned above couple to the quantum critical behavior and produce long-range interactions between spin density fluctuations. The most important implication is that these effects fundamentally modify the mean-field critical behavior predicted by Hertz [9]. However, the feedback of the critical behavior on the fermionic degrees of freedom, and in particular on the 'weak-localization' effects, has been investigated only to the extent that it is needed to determine the exact critical behavior. For instance, the behavior of the resistivity across the quantum FM critical point is not known theoretically. (2) There are examples of itinerant ferromagnets, for instance MnSi, whose Curie temperatures under ambient conditions are very low due to doping or alloying with nonmagnetic materials. Such materials can be driven through the quantum ($T = 0$) transition by means of stress tuning [10]. The quantum critical point is thus readily accessible. While these experiments could in principle determine, e.g., the magnetization as a function of the distance from the critical point, and thus provide a direct check of the theory put forward in Ref. [6], in practice this is difficult to do. On the other hand, transport measurements can be easily performed [10].

Given points (1) and (2) above, our motivation for this Letter is to report theoretical results on observables that qualitatively depend on the quantum critical behavior, but are more easily accessible experimentally than a direct determination of the latter. Such observables include the resistivity, and the tunneling DOS. We will also discuss the dephasing time, which provides a finite-temperature cutoff for various singularities in disordered electron systems, and the QP inelastic lifetime, which is of fundamental theoretical interest [11].

We start by stating our results. We will consider the following observables: The electrical conductivity σ, the tunneling DOS N, the phase relaxation time $\tau_{\rm ph}$, and the QP lifetime $\tau_{\rm QP}$. Let t be the dimensionless distance from the quantum critical point at zero temperature, $T = 0$, Ω the frequency, and ϵ the distance in energy space from the Fermi surface. We will use units such that $\hbar = k_{\rm B} = 1$. Let the disorder strength be characterized by the elastic mean-free path ℓ in units of the inverse Fermi wavenumber $k_{\rm F}$. In 3-D we find at criticality, $t = 0$,

$$\sigma(T) = \sigma_0 \left[1 + c_\sigma \left(\frac{T}{T_{\rm F}}\, g(\ln(\frac{\epsilon_{\rm F}}{T}))\right)^{1/3} + O(\sqrt{T})\right], \quad (1a)$$

$$N(\epsilon) = N_{\rm F} \left[1 + c_N \left(\frac{\epsilon}{\epsilon_{\rm F}}\, g(\ln(\frac{\epsilon_{\rm F}}{\epsilon}))\right)^{1/3} + O(\sqrt{\epsilon})\right], \quad (1b)$$

$$\tau_{\rm ph}^{-1}(\epsilon) = \epsilon_{\rm F} \left[c_\tau^{\rm ph}\,(\epsilon/\epsilon_{\rm F})\, g(\ln(\frac{\epsilon_{\rm F}}{\epsilon})) + O(\epsilon)\right] \quad , \quad (1c)$$

$$\tau_{\rm QP}^{-1}(\epsilon) = \epsilon_{\rm F} \left[\, c_\tau^{\rm QP}\,(\epsilon/\epsilon_{\rm F})\, \ln[\ln(\epsilon/\epsilon_{\rm F})]/\ln(\epsilon/\epsilon_{\rm F}) + O(\epsilon \ln\ln\ln\epsilon/\ln\epsilon)\right] \quad . \quad (1d)$$

Here σ_0 and $N_{\rm F}$ are the disordered Fermi-liquid values of σ and N, respectively, $\epsilon_{\rm F}$ is the Fermi energy, and

$$g(x) = \sum_{n=0}^{\infty} [(c(3)\,x)^n/n!]\, e^{(n^2-n)\ln(2/3)/2} \quad , \quad (2a)$$

with $c(3) = O(1)$, provides logarithmic corrections to power-law scaling. In an asymptotic expansion for large x, the leading term is

$$g(x) \approx [2\ln(3/2)/\pi]^{-1/2}\; e^{[\ln(c(3)\,x)]^2/2\ln(3/2)} \quad , \quad (2b)$$

The c_σ, c_N, and c_τ depend on the disorder. For weak disorder, $k_{\rm F}\ell >> 1$, their leading terms all are of the form $c_i = \tilde{c}_i/k_{\rm F}\ell$, with the $\tilde{c}_i$ constants of $O(1)$. Equation (1a) also holds if T is replaced by Ω, and Eqs. (1b), (1c), (1d) also hold if ϵ is replaced by T.

The terms shown are the exact leading temperature and energy dependences, within the framework of a perturbative renormalization group (RG), of these observables at criticality. At zero temperature, $T = 0$, and zero frequency or energy, $\epsilon = 0$, we find

$$\sigma(t) = \sigma_0 \left[1 + d_\sigma\, t\, g(\ln(1/t))\ln(1/t) + O(t)\right] \quad , \quad (3a)$$

$$N(t) = N_{\rm F} \left[1 + d_N\, t\, g(\ln(1/t))\ln(1/t) + O(t)\right] \quad . \quad (3b)$$

For weak disorder, $d_i = O(1)/k_{\rm F}\ell$. The scattering rates $1/\tau$, of course, vanish at zero energy and temperature. For sufficiently small values of ϵ at nonzero t their behavior is given by

$$\tau_{\rm ph}^{-1}(t,\epsilon) \propto \tau_{\rm QP}^{-1}(t,\epsilon) \propto (\epsilon/t)^{3/2} \quad . \quad (3c)$$

These results can be summarized by means of the following generalized homogeneity laws, valid for all $2 < D < 4$,

$$\sigma(t,T,\Omega) = \text{const.} \times t\, g(\ln(1/t))\,\ln b + F_\sigma(tb^{-2}f(b), \Omega f(b), Tf(b), ub^{-(D-2)}) \quad , \quad (4a)$$

$$\Delta N(t,\epsilon,T) = \text{const.} \times t\, g(\ln(1/t))\,\ln b + b^{-(D-2)}\, F_N(\epsilon f(b), Tf(b)) \quad , \quad (4b)$$

$$\tau_{\rm ph}^{-1}(t,\epsilon,T) = b^{-D}\, F_\tau(tb^{-2}f(b), \epsilon f(b), Tf(b)). \quad (4c)$$

Here $\Delta N = N - N_{\rm F}$, b is an arbitrary length scaling factor, $f(b) = b^D g(\ln b)$, and F_σ, F_N, and F_τ are scaling functions. The quasiparticle relaxation rate is given as a ratio,

$$\tau_{\rm QP}^{-1} = \epsilon H''(\epsilon)/H'(\epsilon) \quad , \quad (4d)$$

where H' and H'' are the real and imaginary part, respectively, of a causal function $H(\zeta = \epsilon + i0)$ of complex frequency ζ that obeys a homogeneity law

$$H(t,\zeta,T) = \text{const.} \times g(\ln b) + F_H(tb^{-2}f(b), \zeta f(b), Tf(b)) \quad . \quad (4e)$$

For completeness and later reference we also note that the specific heat coefficient, $\gamma = \lim_{T\to 0} C_V/T$, is given by $H(t,0,T)$ and thus obeys a homogeneity law [6–8],

$$\gamma(t,T) = \text{const.} \times g(\ln b) + F_\gamma(tb^{-2}f(b), Tf(b)) \; . \quad (4f)$$

By choosing b appropriately, Eqs. (1) and (3) are recovered from Eqs. (4). To obtain Eq. (3c), one also needs to use the fact that at the disordered Fermi-liquid fixed point the relaxation rates are proportional to $\epsilon^{3/2}$ [12].

We will first explain the significance of these results, and then sketch their origin. We first consider the dependence of the conductivity, or the DOS, on T or Ω or ϵ. In disordered metals away from any quantum critical point one observes the well-known $T^{1/2}$, $\epsilon^{1/2}$, or $\Omega^{1/2}$ behavior in 3-D [3]. This is one example of the weak-localization effects mentioned above, and it is due to the diffusive nature of the electrons. In a diffusive process, frequency scales like a wavenumber squared, $\Omega \sim b^{-2}$, and in a quantum problem, $T \sim \Omega$. Accordingly, the dynamical exponent z, defined by $\Omega \sim b^{-z}$, is $z = 2$ in such a system. Near a quantum critical point, this behavior changes to one characterized by the dynamical critical exponent characteristic of the phase transition. For the quantum FM transition in disordered system, according to a recent theory the dynamical exponent is $z = D$ [6], which is reflected in Eqs. (1) and (4), in contrast to Hertz's theory which yielded $z = 4$ [9]. Similarly, the correlation length exponent ν, defined by $t \sim b^{-1/\nu}$, is $\nu = 1/(D-2)$ according to Ref. [6], while it is $\nu = 1/2$

in Hertz's theory. The former value is reflected in Eqs. (3). Measuring ν and z directly at the quantum critical point would be hard, while an observation of the conductivity or the tunneling DOS on T, Ω or t should be much easier. Observing the crossover from the usual $T^{1/2}$ weak-localization nonanalyticity to the $T^{1/3}$ behavior predicted by the present theory may therefore be the easiest way to experimentally probe the dynamical critical behavior at the quantum FM transition. Similarly, the linear dependence of σ and N on t at $T=0$ reflects the value of ν. Also notice that $d\sigma/dt \propto g(\ln(1/t))\ln t$ at $T=0$, see Eq. (3a) [13].

The behavior of the relaxation times $\tau_{\rm ph}$ and $\tau_{\rm QP}$ is very remarkable from a theoretical point of view. In Landau Fermi-liquid theory, the relaxation rate is proportional to ϵ^2 [14], and in a disordered Fermi liquid one has $\tau_{\rm ph} \propto \tau_{\rm QP} \propto \epsilon^{3/2}$ [12]. The limiting case between Fermi-liquid and non-Fermi liquid behavior is given by a 'marginal Fermi liquid' [15] with a relaxation rate proportional to ϵ with logarithmic corrections. From Eqs. (1c,1d) we see that itinerant electrons at a quantum FM critical point provide a realization of a marginal Fermi liquid. The logarithmic T-dependence of the specific heat coefficient is consistent with this.

We now explain the origin of the scaling behavior discussed above. Microscopically, one should consider a theory of interacting electrons in the presence of quenched disorder and introduce an order parameter field that couples to the electron spin density. One then obtains a theory for the fermionic degrees of freedom that is missing the spin-density channel of the electron-electron interaction, coupled to a Landau-Ginzburg-Wilson (LGW) action for the order parameter field. This program has recently been carried out [7]. While this theory, and its RG analysis [8], are necessary for a complete justification of our results, most of them can be obtained from simple scaling arguments in conjunction with low-order perturbation theory. This we will show here, starting with a scaling analysis.

For scaling purposes, we note that the observables we are interested in can all be expressed in terms of fermionic correlation functions. It is therefore reasonable to assume that the scale dimensions of these observables and the underlying fermionic fields with respect to the magnetic fixed point are the same as with respect to the disordered Fermi-liquid fixed point identified in Ref. [4]. We will proceed under this assumption and initially focus on power laws, ignoring all logarithmic corrections. All comparisons of Eqs. (5) with Eqs. (1)-(4) are thus to be understood as up to logarithms.

The conductivity is a charge current correlation whose scale dimension with respect to the quantum magnetic fixed point is expected to be zero. However, σ will depend on the critical dynamics, since the paramagnon propagator enters the calculation of σ in perturbation theory [16]. The corrections to the Boltzmann conductivity will further depend on the leading irrelevant operator, which we denote by u. This is again related to diffusive electron dynamics, and one therefore expects the scale dimension of u to be the same as in disordered Fermi-liquid theory, namely, $[u] = -(D-2)$ [4]. Standard scaling arguments then suggest a generalized homogeneity law

$$\sigma(t,T,\Omega) = F_\sigma(tb^{1/\nu}, Tb^z, \Omega b^z, ub^{-(D-2)}) \quad . \qquad (5a)$$

With $z = D$ and $\nu = 1/(D-2)$ from Ref. [6] this is Eq. (4a). By putting $b = T^{-1/3}$, and using that $F_\sigma(0,1,0,x)$ is an analytic function of x, we obtain Eq. (1a).

Similarly, the leading correction ΔN to the DOS is given by an integral over a four-fermion correlation function whose diffusive dynamics lead to $\Delta N \sim b^{-(D-2)}$. This yields Eq. (4b),

$$\Delta N(t,\epsilon,T) = b^{-(D-2)}\, F_N(tb^{1/\nu}, \epsilon b^z, Tb^z) \quad . \qquad (5b)$$

Both the dephasing rate and the QP relaxation rate are dimensionally frequencies. Effectively, their scale dimensions are given by the critical time scale, $[\tau^{-1}] = D$ [8]. This implies Eqs. (4c,4d,4e),

$$\tau(\epsilon,T) = b^D\, \tau(\epsilon b^D, Tb^D) \quad . \qquad (5c)$$

Finally, the specific heat coefficient is expected to have zero scale dimension, as in Ref. [4]. We thus have Eq. (4f),

$$\gamma(t,T) = \gamma(tb^{1/\nu}, Tb^z) \quad . \qquad (5d)$$

The logarithmic corrections to scaling in Eqs. (1)-(4) can be understood in terms of Wegner's general classification [17]. The first kind is due to resonance conditions between scale dimensions, which lead to simple logarithms. The second kind is due to marginal operators, which can lead to complicated functions of logarithms. Here, both mechanisms are operative. The scale dimension of the DOS correction, $[\Delta N] = D-2$, is the same as that of the relevant operator t, $[t] = 1/\nu = D-2$. Further, the correction to the conductivity is proportional to the irrelevant variable u, and $[tu] = [\sigma] = 0$. These 'resonances' lead to the simple logarithms in Eqs. (4a) and (4b).

The more complicated logarithms embodied in the function $g(\ln b)$ are due to an effectively marginal operator that is due to the presence of two frequency scales in the problem, namely the critical one with $z = D$, and a diffusive one with $z = 2$. A full renormalization group analysis of this problem will be presented elsewhere [8], here we resort to low-order perturbation theory to make the result plausible. Perturbation theory is also useful for making our general results plausible. Consider the DOS as an example. To lowest order in both the disorder and the spin-triplet interaction amplitude K_t, the DOS correction in a disordered Fermi liquid is well known to have the form [3]

$$\Delta N(\epsilon) \propto G^2 \int d\mathbf{p}\, {\rm Re} \int_{i\Omega\to\epsilon+i0}^{\infty} d\omega\ K_t\ [\mathcal{D}(\mathbf{p},\omega)]^2 \ . \qquad (6a)$$

Here $\mathcal{D}$ is the basic diffusion propagator or 'diffuson',

$$\mathcal{D}(\mathbf{p},\omega) = 1/(\mathbf{p}^2 + GH|\omega|) \quad , \tag{6b}$$

with $G \propto 1/k_{\rm F}\ell$ the disorder parameter, and H proportional to the specific heat coefficient γ, see Eqs. (4). In a theory that keeps K_t to all orders and thus is capable of describing magnetism, K_t gets replaced by the paramagnon propagator $\mathcal{M}$, which has the structure [6]

$$\mathcal{M}(\mathbf{p},\omega) = 1/[t + a_{d-2}|\mathbf{p}|^{d-2} + a_2\mathbf{p}^2 + a_\omega|\omega|/\mathbf{p}^2] \ . \tag{6c}$$

The correction to N thus reads

$$\Delta N(\epsilon) \propto G^2 \int d\mathbf{p}\, {\rm Re} \int_{i\Omega\to\epsilon+i0}^{\infty} d\omega\, [\mathcal{D}(\mathbf{p},\omega)]^2\ \mathcal{M}(\mathbf{p},\omega). \tag{6d}$$

The leading correction to G has the same qualitative behavior as ΔN, Eq. (6d), while the one to H is

$$\Delta H(\Omega) \propto G \int d\mathbf{p}\ \frac{1}{\Omega} \int_0^\Omega d\omega\ \mathcal{D}(\mathbf{p},\omega)\, \mathcal{M}(\mathbf{p},\omega) \ . \tag{6e}$$

The interesting effects we discuss in this paper result from the nonanalytic $|\mathbf{p}|^{d-2}$ term in Eq. (6c). We therefore need to investigate the conditions under which this term dominates over the analytic $\mathbf{p}^2$ dependence. If we measure $\mathbf{p}$ and ω in units of $k_{\rm F}$ and $\epsilon_{\rm F}$, respectively, and remember that the dimensionless spin-triplet interaction is of $O(1)$ near a FM transition, then the coefficients a_2 and a_ω are of $O(1)$, while a_{d-2} is proportional to $1/k_{\rm F}\ell$. If we scale the wavenumber with the correlation length ξ, this means that the nonanalytic term will dominate over the analytic one when $\xi \gtrsim \ell$. With $\nu = 1$ in $D = 3$, this translates into $t \lesssim 1/k_{\rm F}\ell$. For typical values of the disorder, $k_{\rm F}\ell \approx 10$, this means that the nonanalytic term will dominate everywhere in the critical region. For less disordered samples, our effects will be present in a correspondingly narrower region around the critical point.

Performing the integral in Eq. (6d) yields $\Delta N \propto$ const.$+\epsilon^{(D-2)/D}$, while from Eq. (6e) we find $\Delta H \propto \ln\Omega$. Inserting the latter result back into the perturbative expressions leads to more complicated logarithms. Of course, insertions are at best indicative of the behavior at higher order, a complete treatment needs to take into account vertex corrections as well. We will report on the details of such a complete analysis elsewhere [8]. The result is that the fully renormalized H turns into $H\, g(\ln(1/\Omega\tau)$. Similarly, the coefficient a_{d-2} in the paramagnon propagator gets renormalized to $a_{d-2}/g(\ln(k_{\rm F}/k))$ [18]. Doing the integrals then yields expressions that are consistent with Eqs. (1) and (3). The remarkable claim that one-loop perturbation theory is exact apart from logarithmic corrections finds its explanation in the RG analysis of Refs. [7,8].

We gratefully acknowledge helpful conversations and correspondence with Lior Klein and Achim Rosch. This work was supported in part by the NSF under grant Nos. DMR–98–70597 and DMR–99–75259, by the DFG under grant No. Vo659/2, and by the EPSRC under grant No. GR/M 04426. Part of this work was performed at the Aspen Center for Physics.

---

[1] See, e.g., M.E. Fisher in *Advanced Course on Critical Phenomena*, F.W. Hahne (ed.), Springer (New York 1983).

[2] T.R. Kirkpatrick and D. Belitz, J. Stat. Phys. **87**, 1307 (1997).

[3] For reviews, see, B.L. Altshuler and A.G. Aronov in *Electron-electron interactions in disordered systems*, A.L. Efros and M. Pollak (eds.), North Holland (Amsterdam 1985); P.A. Lee and T.V. Ramakrishnan, Rev. Mod. Phys. **57**, 287 (1985).

[4] D. Belitz and T.R. Kirkpatrick, Phys. Rev. B **56**, 6513 (1997).

[5] See, e.g., the ITP Conference Proceedings published in J. Phys. Cond. Matt. **8** (1996).

[6] T.R. Kirkpatrick and D. Belitz, Phys. Rev. B **53**, 14364 (1996).

[7] D. Belitz, T.R. Kirkpatrick, Maria Teresa Mercaldo, and Sharon L. Sessions, cond-mat/0008061

[8] D. Belitz, T.R. Kirkpatrick, Maria Teresa Mercaldo, and Sharon L. Sessions, unpublished results.

[9] J.A. Hertz, Phys. Rev. B **14**, 1165 (1976).

[10] C. Pfleiderer, G.J. McMullan, S.R. Julian, and G.G. Lonzarich, Phys. Rev. B **55**, 8330 (1997).

[11] C. Castellani, G. Kotliar, and P.A. Lee, Phys. Rev. Lett. **59**, 323 (1987).

[12] A. Schmid, Z. Phys. B **271**, 251 (1974).

[13] Although this is reminiscient of the behavior predicted for the thermal ferromagnetic transition by P.G. de Gennes and J. Friedel, J. Chem. Phys. Solids **4**, 71 (1958) (see also M.E. Fisher and J.S. Langer, Phys. Rev. Lett. **20**, 665 (1968)), the underlying physics is very different since our effect is entirely dynamical in origin.

[14] A.A. Abrikosov, L.P. Gorkov, and I.E. Dzyaloshinski, *Methods of Quantum Field Theory in Statistical Physics*, Dover (New York 1975).

[15] C.M. Varma, P.B. Littlewood, S. Schmitt-Rink, E. Abrahams, and A.E. Ruckenstein, Phys. Rev. Lett. **63**, 1996 (1989).

[16] This is because the spin-triplet interaction amplitude that enters perturbation theory for σ in the theory of disordered Fermi liquids [3] gets replaced by the paramagnon propagator in the case of an itinerant ferromagnet.

[17] F. J. Wegner, in *Phase Transitions and Critical Phenomena*, vol.6, C. Domb and M. S. Green (eds.) (Academic, New York 1976).

[18] Certain logarithmic corrections were missed in Ref. [6]. Refs. [7,8] show how they arise, and that the exact critical behavior is identical with that for an unidentified phase transition discussed before by T.R. Kirkpatrick and D. Belitz, Phys. Rev. B **45**, 3187 (1992), identifying the transition treated in that paper as the ferromagnetic one.