# Successive Relaying for Two-hop Two-Destination Multicarrier Relay Channels

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***Abstract*—We propose a successive relaying decode-and-forward protocol for the half-duplex 2-relay 2-destination multicarrier channel. The proposed protocol performs significantly better than existing protocols and achieves sum rates close to the cutset upper bound. We also prove that the linear program that describes the cutset upper bound for sum capacity is optimized using only 3 of the 4 possible states of the half-duplex network. Of these 3 states, successive relaying uses the two states in which one of the relays is receiving from the source while the other relay is transmitting to the destinations. In simulations under a Rayleigh fading model, the gap between the cutset upper bound and the proposed protocol is observed to be bounded.**

## I. Introduction

Cooperative relaying enhances the coverage and capacity of cellular networks. Relay networks have attracted significant attention in the context of device-to-device (D2D) communication and self-backhauling in 5G systems [1]–[3]. In this letter, we consider a relay channel with the source transmitting messages to two destinations with the help of relays (Fig. 1). The relays are assumed to be half-duplex (HD), i.e., each relay may either receive or transmit at a time but not both. Therefore, the 2-relay network has 4 possible states (See Fig. 4).

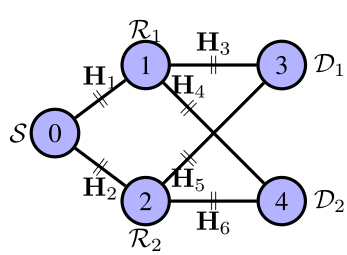


Fig. 1: Two-destination relay channel

The 2-relay channel with a single destination, called the diamond channel, was first studied in [4]. For the diamond relay channel, it was shown in [5] that Decode-and-Forward (DF) protocols can achieve rates within 0.71 bits of the capacity. These DF protocols were extended to the *multicarrier* 2-relay channel in [6]. In [7], [8], the authors studied an $M$-*relay* multicarrier channel and proposed a greedy protocol to transmit messages. This greedy protocol uses two states: (1) the multiple access (MAC) state in which the source is transmitting to all the relays and (2) the broadcast (BC) state in which all the relays are transmitting to the destination. As the first contribution in this letter, we propose a successive relaying protocol for multicarrier 2-destination 2-relay channel that uses the other 2 states of the network. In successive relaying, one of the relays is transmitting in each state while the other is receiving (See Fig. 2). Therefore, the source is always transmitting and the destination is always receiving. This helps in overcoming the multiplexing loss due to the half-duplex constraint. This protocol performs significantly better than a greedy protocol based on [8] and a protocol that time-shares between the single-destination protocols in [6]. The proposed protocol also achieves sum rates close to the cutset upper bound. In simulations, under the Rayleigh fading model, the gap between the successive relaying protocol and the cutset upper bound is bounded.

The cutset bound gives an upper bound to the capacity of the channel under any protocol. In particular, the cutset bound for a half-duplex relay network is derived in [9]. For single-destination HD relay channels, the states with non-zero time fractions required to achieve rates within a constant gap of the cutset bound have been studied in [10], [11]. In these papers, it has been shown by analyzing the cutset bound that the number of states required for an $n$-relay channel is at most $n+1$. The case of $n \leq 6$ was shown in [10], and later, this result was generalized for any $n$ in [11]. The works [10], [11] discuss the existence of $n+1$ optimal states. In [12], the algorithm to find the optimal states for a line network is described. Note that all these results are for single-destination relay channels. However, for multiple destination relay channels, to the best of our knowledge, no such results have been shown. As the second contribution in this letter, we show that for the 2-destination multicarrier relay channel, at most 3 states are required to achieve the optimal value of the linear program that describes the cutset bound for sum capacity under all channel conditions.

## II. System Model

The source ($\mathcal{S}$) transmits message $w_i$ to the destination-i ($\mathcal{D}_i$) with the help of the relays-1 and 2 ($\mathcal{R}_1, \mathcal{R}_2$), $i = 1, 2$. This relay network is shown in Fig.1. Note that there is no direct link between the source and the destinations. The received signals at $\mathcal{R}_1$, $\mathcal{R}_2$, $\mathcal{D}_1$, $\mathcal{D}_2$ are respectively given by

$$\mathbf{y}_1 = \mathbf{H}_1\mathbf{x}_0 + \mathbf{z}_1, \quad \mathbf{y}_2 = \mathbf{H}_2\mathbf{x}_0 + \mathbf{z}_2,$$
$$\mathbf{y}_3 = \mathbf{H}_3\mathbf{x}_1 + \mathbf{H}_5\mathbf{x}_2 + \mathbf{z}_3, \quad \mathbf{y}_4 = \mathbf{H}_4\mathbf{x}_1 + \mathbf{H}_6\mathbf{x}_2 + \mathbf{z}_4,$$

where $\mathbf{x}_0 \in \mathbb{R}^N$ is the signal transmitted by the source $\mathcal{S}$; $\mathbf{y}_i$ is the signal received by node $i$; $\mathbf{x}_i$ is the vector signal transmitted by the relay $\mathcal{R}_i$; $\mathbf{H}_i$ are $N \times N$ channel gain matrices; and $\mathbf{z}_i$ are Gaussian noise vectors, $i = 1, 2$. We will

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be studying *multicarrier* relay channels, so $\mathbf{H}_i$s are diagonal matrices with each diagonal entry being the channel gain of the subcarrier. We also have $\mathbb{E}||\mathbf{x}_0||^2 \leq P_S$ and $\mathbb{E}||\mathbf{x}_i||^2 \leq P_R$ $(i = 1, 2)$ where $P_S$ and $P_R$ are the power available at source and relays, respectively.

## III. Successive Relaying

In this section, we propose a successive relaying protocol to transmit messages from the source to the destinations. The states of the channel used for this protocol are shown in Fig. 2. In state-1, the source transmits signals to $\mathcal{R}_1$ and $\mathcal{R}_2$

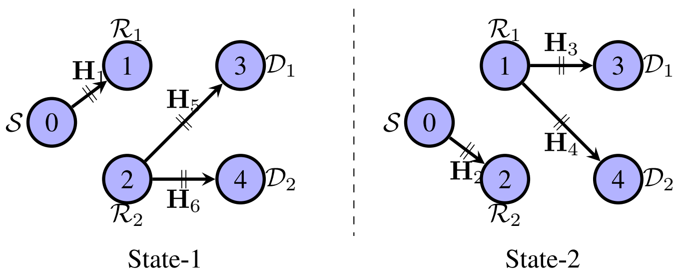


Fig. 2: Successive Relaying states

transmits signals to both the destinations; while in state-2 the roles of these relays are reversed. The subchannel selection, power allocation, and sum-rate computation of the successive relaying protocol are detailed below.

**Subchannel Selection:** In state-1, for each subcarrier, $\mathcal{R}_2$ selects best destination for transmission. Let $\bar{\mathcal{N}}_3^{(1)}$ and $\bar{\mathcal{N}}_4^{(1)}$ be the subcarriers used in state-1 by $\mathcal{R}_2$ to transmit to $\mathcal{D}_1$ (node-3) and $\mathcal{D}_2$ (node-4), respectively. If $H_5(n) \geq H_6(n)$ (where $H_i(n)$ denotes the channel coefficient corresponding to the $n^{th}$ subcarrier of $\mathbf{H}_i$) then $n$ is assigned to $\bar{\mathcal{N}}_3^{(1)}$ else $n$ is assigned to $\bar{\mathcal{N}}_4^{(1)}$. Similarly, in state-2, $\mathcal{R}_1$ selects the best destination for transmission.

**Power Allocation:** Power allocation at source in state-$i$, $P_S^{(i)}$ is done using the waterfilling for channel $\mathbf{H}_i$, $i = 1, 2$ . In state-1, $\mathcal{R}_2$ selects the best destination for each subcarrier and then performs waterfilling across channel gains $\bar{H}^{(1)}(n) = \max\{H_5(n), H_6(n)\}$ $\forall n$. This gives the power allocation at $\mathcal{R}_2$, $\bar{P}^{(1)}(n)$. Similarly, in state-2, $\mathcal{R}_1$ performs waterfilling across channel gains $\bar{H}^{(2)}(n) = \max\{H_3(n), H_4 n)\}$ $\forall n$, after selecting the best destination for each channel, to get power allocation $\bar{P}^{(2)}(n)$ at $\mathcal{R}_1$.

**Sum-rate Computation:** The sum-rate achieved by the successive relaying protocol is given by:

$$R_S = \max_{t \in [0,1]} (\min\{tR_S^{(1)}, (1-t)\bar{R}^{(2)}\} + \min\{(1-t)R_S^{(2)}, t\bar{R}^{(1)}\})$$

where $t$ is the fraction of time the protocol operates in state-1, $R_S^{(1)} = \sum_{n=1}^{N} \frac{1}{2}\log(1 + |H_1(n)|^2 P_S^{(1)}(n))$, $\bar{R}^{(2)} = \sum_{n=1}^{N} \frac{1}{2}\log(1 + |\bar{H}^{(2)}(n)|^2 \bar{P}^{(2)}(n))$, $R_S^{(2)} = \sum_{n=1}^{N} \frac{1}{2}\log(1 + |H_2(n)|^2 P_S^{(2)}(n))$, $\bar{R}^{(1)} = \sum_{n=1}^{N} \frac{1}{2}\log(1 + |\bar{H}^{(1)}(n)|^2 \bar{P}^{(1)}(n))$. Note that, the first and second term in the above expression of $R_S$ correspond to the information flowing through $\mathcal{R}_1$ and $\mathcal{R}_2$, respectively.

## IV. Cutset Bound

The cutset upper bound to the sum-rate capacity of the channel can be obtained by finding the optimum value of the linear program (LP) [9] in eqn. (1). The states and cuts of the channel considered to obtain the linear program are shown in Fig. 3 and Fig. 4, respectively. The cuts (a)-(d) in Fig. 3 are represented in cutset LP by the constraints (1a)-(1d), respectively. For each cut, the constraint is obtained by bounding the information flow across the cut. Let $t_1, t_2, t_3$

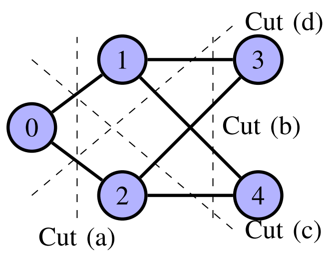


Fig. 3: Cuts of the two-destination relay channel

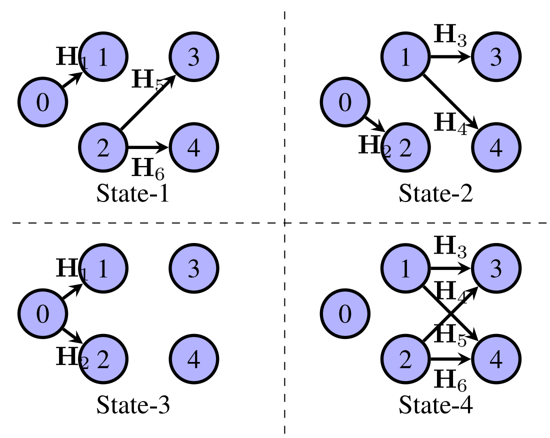


Fig. 4: States of the two-destination relay channel

and $t_4$ be the fraction of time the channel operates in states-1, 2, 3 and 4. Let $P$ be the power available at source and the relays. The LP used to find cutset bound is given by:

$$\begin{aligned}
&\text{Maximize} && R_c && (1)\\
&\text{Subject to} && R_c \leq t_1 C_1 + t_2 C_2 + t_3 C_{13} + t_4 .0 && (1a)\\
& && R_c \leq t_1 C_3 + t_2 C_4 + t_3 .0 + t_4 (C_3 + C_4) && (1b)\\
& && R_c \leq t_1 (C_1 + C_3) + t_2 .0 + t_3 C_1 + t_4 C_3 && (1c)\\
& && R_c \leq t_1 .0 + t_2 (C_2 + C_4) + t_3 C_2 + t_4 C_4 && (1d)\\
& && \sum_{i=1}^{4} t_i = 1, \qquad t_i \geq 0 && (1e)
\end{aligned}$$

In this linear program, we have $C_1 \triangleq \max \sum_{i=1}^{N} \log(1 + |H_1(n)|^2 P_i)$ where the maximization is over the set $\{(P_1, P_2, \dots P_N) \in \mathbb{R}_+^N | \sum P_i = P\}$. Similarly, $C_2 \triangleq \max \sum \log(1 + |H_2(n)|^2 P_i)$. Let function $C(.,.)$ be defined as $C(\mathbf{H}^H, P) \triangleq \max \log |I + \mathbf{H}\mathbf{Q}\mathbf{H}^H|$ where the maximization is over the set of positive semi-definite matrices

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$\{\mathbf{Q} \in \mathbf{S}^N | tr(\mathbf{Q}) = P\}$. Also, $C_{13} \triangleq C([\mathbf{H}_1^H \mathbf{H}_2^H], P)$; $C_4 \triangleq C([\mathbf{H}_3^H \mathbf{H}_4^H], P)$ and $C_3 \triangleq C([\mathbf{H}_5^H \mathbf{H}_6^H], P) + \delta$ where $\delta = [C_{PAPC} - C_4 - C([\mathbf{H}_5^H \mathbf{H}_6^H], P)]^+$, $C_{PAPC}$ is the per-antenna power constrained MIMO capacity of the channel formed by the relays $(\mathcal{R}_1, \mathcal{R}_2)$ and destinations $(\mathcal{D}_1, \mathcal{D}_2)$ shown in State-4 of Fig. 4 .

The most important states of the channel are states-1 and 2. This is because (1) the source and the destination are always 'ON' in states-1, 2 unlike states-3, 4 and (2) $t_3^* = 0$ or $t_4^* = 0$ where $t_3^*$ and $t_4^*$ are the optimal values of $t_3$ and $t_4$ for LP-(1) as shown later in Theorem-1. Before we state the theorem, we show the inequality $C_{13} < C_1 + C_2$, which is used in the proof of Theorem-1.

**Lemma 1.** $C_{13} < C_1 + C_2$

*Proof.* Let $\mathbf{H}^H = [\mathbf{H}_1^H \mathbf{H}_2^H]$ then

$$
\begin{aligned}
C_{13} &= \max \log |I + \mathbf{H}\mathbf{Q}\mathbf{H}^H| \\
&\overset{(a)}{=} \max \log |I + \mathbf{H}^H \mathbf{H}\mathbf{Q}| \overset{(b)}{=} \max \log |I + \mathbf{D}\mathbf{Q}| \\
&\overset{(c)}{=} \sum_{i=1}^{N} \log(1 + (|H_1(n)|^2 + |H_2(n)|^2)P_i) \\
&\overset{(d)}{<} \sum \log(1 + |H_1(n)|^2 P_i) + \sum \log(1 + |H_2(n)|^2 P_i) \\
&\leq C_1 + C_2,
\end{aligned}
$$

where (a) uses the identity $|I+AB| = |I+BA|$; (b) is because $\mathbf{H}^H\mathbf{H}$ is a diagonal matrix $\mathbf{D}$ (remember $\mathbf{H}$ is formed from the multicarrier diagonal matrices $\mathbf{H}_1$ and $\mathbf{H}_2$), whose $n^{th}$ diagonal entry is given by $D(n)^2 = |H_1(n)|^2 + |H_2(n)|^2$; in (c), $P_i$ are powers obtained by waterfilling across multicarrier subchannels $\mathbf{D}^{\frac{1}{2}}$; (d) follows from the monotonicity of the log function; and the last step follows because $C_1$ and $C_2$ are the capacities of the channels $\mathbf{H}_1$ and $\mathbf{H}_2$, respectively. ☐

**Theorem 1.** *For the linear program-*(1)*,* $t_3^* = 0$ *or* $t_4^* = 0$*.*

*Proof.* Consider the dual program of LP-(1):

$$
\begin{aligned}
&\text{Minimize } R_D && (2)\\
&\text{Subject to } R_D \geq \tau_1 C_1 + \tau_2 C_3 + \tau_3 (C_1 + C_3) + \tau_4 .0 && (2a)\\
&\qquad R_D \geq \tau_1 C_2 + \tau_2 C_4 + \tau_3 .0 + \tau_4 (C_2 + C_4) && (2b)\\
&\qquad R_D \geq \tau_1 C_{13} + \tau_2 .0 + \tau_3 C_1 + \tau_4 C_2 && (2c)\\
&\qquad R_D \geq \tau_1 .0 + \tau_2 (C_3 + C_4) + \tau_3 C_3 + \tau_4 C_4 && (2d)\\
&\qquad \sum_{i=1}^{4} \tau_i = 1, \qquad \tau_i \geq 0 && (2e)
\end{aligned}
$$

We prove the theorem in two parts: first for Case-1, then for Case-2. In case-1, i.e., for channels that satisfy the condition $\frac{C_2}{C_2+C_3} - \frac{C_4}{C_1+C_4} < 0$, we show that $t_3^*$ or $t_4^*$ is zero by the method of contradiction. If we assume that $t_3^*$ and $t_4^*$ are strictly greater than zero, then optimal $\tau_2^*$ will be strictly negative, which is impossible. Case-2 considers all channels that do not come under case-1. In case-2, we show that $t_3^* = 0$ by finding the optimal solutions of LP-(1) and its dual LP-(2). Finally, it is worth noting that the condition $\frac{C_2}{C_2+C_3} - \frac{C_4}{C_1+C_4} < 0$ is equivalent to the condition $C_1 C_2 - C_3 C_4 < 0$ used in [5] for the single-carrier single-destination diamond channel.

**Case-1** (channels with $\frac{C_2}{C_2+C_3} - \frac{C_4}{C_1+C_4} < 0$)
For this case, we show $t_3^* = 0$ or $t_4^* = 0$ by the method of contradiction. Assume $t_3^* > 0$ and $t_4^* > 0$. This implies that by the complementary slackness theorem, (2c) and (2d) should be satisfied by equality. Therefore

$$R_D^* = \tau_1^* C_{13} + \tau_2^* .0 + \tau_3^* C_1 + \tau_4^* C_2, \tag{3}$$
$$R_D^* = \tau_1^* .0 + \tau_2^* (C_3 + C_4) + \tau_3^* C_3 + \tau_4^* C_4. \tag{4}$$

Using (3) and (4) we have

$$\tau_1^* C_{13} + \tau_3^* C_1 + \tau_4^* C_2 = \tau_2^* (C_3 + C_4) + \tau_3^* C_3 + \tau_4^* C_4$$

The above equation can be simplified using $\sum_i \tau_i = 1$ to get:

$$\tau_1^* (C_{13} - C_2 + C_4) - \tau_2^* (C_2 + C_3) + \tau_3^* (C_1 + C_4 - C_2 - C_3) = C_4 - C_2 \tag{5}$$

Now we need to ensure that (2a) and (2b) are satisfied. Therefore

$$R_D^* \geq \tau_1^* C_1 + \tau_2^* C_3 + \tau_3^* (C_1 + C_3) + \tau_4^* .0, \tag{6}$$
$$R_D^* \geq \tau_1^* C_2 + \tau_2^* C_4 + \tau_3^* .0 + \tau_4^* (C_2 + C_4). \tag{7}$$

Using (3), (6) and $\sum_i \tau_i = 1$ we have

$$\tau_1^* (C_1 + C_2 - C_{13}) + \tau_2^* (C_2 + C_3) + \tau_3^* (C_2 + C_3) \leq C_2 \tag{8}$$

Similarly, using (4), (7) and $\sum_i \tau_i = 1$ we have

$$\tau_2^* (C_2 + C_3) + \tau_3^* (C_2 + C_3) \geq C_2 \tag{9}$$

From Lemma-1, note that $C_1 + C_2 - C_{13} > 0$ in (8). Therefore, the $(\tau_1^*, \tau_2^*, \tau_3^*, \tau_4^*)$ that satisfies (8), (9) has to satisfy the following conditions:

$$\tau_2^* (C_2 + C_3) + \tau_3^* (C_2 + C_3) = C_2, \tag{10}$$
$$\tau_1^* = 0. \tag{11}$$

Using (11) in (5) we have

$$-\tau_2^* (C_2 + C_3) + \tau_3^* (C_1 + C_4 - C_2 - C_3) = C_4 - C_2 \tag{12}$$

Solving (10) and (12) simultaneously, we have $\tau_3^* = \frac{C_4}{C_1+C_4}$ and $\tau_2^* = \frac{C_2}{C_2+C_3} - \frac{C_4}{C_1+C_4}$. $\tau_2^* < 0$ because we have considered channels for which $\frac{C_2}{C_2+C_3} - \frac{C_4}{C_1+C_4} < 0$. This is a contradiction. Therefore, our assumption that $t_3^* > 0$ and $t_4^* > 0$ is incorrect, and either $t_3^* = 0$ or $t_4^* = 0$.

**Case-2** (channels with $\frac{C_2}{C_2+C_3} - \frac{C_4}{C_1+C_4} \geq 0$)
Consider a point FS-1 $= (t_1, t_2, t_3, t_4, R_C) = (\frac{C_4}{C_1+C_4}, \frac{C_3}{C_2+C_3}, 0, 1 - \frac{C_4}{C_1+C_4} - \frac{C_3}{C_2+C_3}, \frac{C_1 C_4}{C_1+C_4} + \frac{C_2 C_3}{C_2+C_3})$. It can be easily shown that this point is a feasible point of LP-(1) as it satisfies all constraints (1a)-(1e); in fact, the conditions (1a)-(1d) are satisfied by equality.

Similarly, consider another point FS-2 $= (\tau_1, \tau_2, \tau_3, \tau_4, R_D) = (0, \frac{C_2}{C_2+C_3} - \frac{C_4}{C_1+C_4}, \frac{C_4}{C_1+C_4}, 1 - \frac{C_2}{C_2+C_3}, \frac{C_1 C_4}{C_1+C_4} + \frac{C_2 C_3}{C_2+C_3})$. It can be shown easily that this point is a feasible point of LP-(2) as it satisfies all constraints (2a)-(2e); in fact, the conditions (2a)-(2d) are satisfied by equality.

Note that $R_C$ of FS-1 is equal to $R_D$ of FS-2. Therefore, by the strong duality theorem, FS-1 and FS-2 are the solutions of LP-(1) and LP-(2), respectively. Hence, $t_3^* = 0$. ☐

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**Remark 1.** *From the proof of Theorem-1, we see that for case-1 states 1, 2, 4 optimize the cutset bound LP. For case-2, states 1, 2, 3 or states 1, 2, 4 optimize the cutset bound LP. Therefore, for all channel conditions, states 1 and 2 are always part of the optimal states. These are exactly the states used in successive relaying.*

## V. Simulation Results

We compare the performance of successive relaying protocol with cutset upper bound [9], a greedy protocol based on [7], [8] and a time-sharing protocol based on the single-destination MDF protocols in [6]. The time-sharing protocol used for comparison is as follows. The messages, in the time-sharing (TS) protocol, are transmitted to the destinations in two phases. In phase-i ($i \in \{1, 2\}$) the source transmits messages exclusively to $\mathcal{D}_i$ at the rate $R_{\mathcal{S}\mathcal{D}_i}$ for $t_i$ fraction of time, using the MDF (multi-hop DF) protocol for multicarrier diamond channel [6]. Therefore, the sum-rate of time-sharing protocol is $t_1 R_{\mathcal{S}\mathcal{D}_1} + t_2 R_{\mathcal{S}\mathcal{D}_2}$. We consider two modes of the time-sharing protocol: *TS half* and *TS best*. *TS half* sum-rate is obtained by assigning $t_1 = t_2 = 1/2$ while *TS best* sum-rate is obtained by computing $\max_{t_1+t_2=1}\{t_1 R_{\mathcal{S}\mathcal{D}_1} + t_2 R_{\mathcal{S}\mathcal{D}_2}\} = \max\{R_{\mathcal{S}\mathcal{D}_1}, R_{\mathcal{S}\mathcal{D}_2}\}$. Since we are considering sum rate, the greedy protocol for the 2-destination case is obtained in the following manner: first, the best destination is chosen for each subcarrier from each relay, and then the greedy algorithm in [7], [8] is used.

We compare the performance of successive relaying protocol for three different topologies. We consider 256 subcarriers for simulation. For the cases where i.i.d. subchannels are simulated, the subchannels are drawn from the normal distribution $\mathcal{N}(0, \frac{1}{d^2})$ where $d$ is the distance between the two nodes. For the correlated subcarriers simulations, a multipath channel with $L$ taps is used with each tap drawn from $\mathcal{N}(0, \frac{256}{d^2 L})$.

*1) Topology-1:* In topology-1, the relays are positioned on a plane between the source and the destination plane as shown in Fig. 5. Let $d_{\mathcal{S}}$ be the source to relay plane distance. Similarly, $d_{\mathcal{D}}$ is the distance between the relay plane and the destination plane. We consider three scenarios: (1) scenario-1 with $d_{\mathcal{S}} : d_{\mathcal{D}} = 1 : 1$, (2) scenario-2 with $d_{\mathcal{S}} : d_{\mathcal{D}} = 1 : 2$ and i.i.d subcarriers, (3) scenario-3 with $d_{\mathcal{S}} : d_{\mathcal{D}} = 1 : 2$ and correlated subcarriers with $L = 6$. The average rate plots for these scenarios are shown in Figs. 6, 7, 8. From these plots we see that Successive Relaying protocol achieves rates very close to the cutset upper bound. The Successive Relaying protocol also performs significantly better than the greedy protocol. The time-sharing protocol rates are almost as good as the successive relaying in scenario-1. However, for scenario-2 when the destinations are away from the relays the successive relaying rate is significantly better than the time-sharing protocol rates. The use of successive relaying reduces the multiplexing loss from half-duplex relaying [13]. This is captured in Fig. 9 by plotting the *Throughput ratio*, defined as the ratio of achievable rate of a scheme to the cutset bound, of various schemes for scenario-2 in Fig. 9. From Fig. 9, we see that at high power, throughput ratio of successive relaying is significantly better than that of other schemes and approaches 1, i.e., it approaches the maximum possible multiplexing gain.

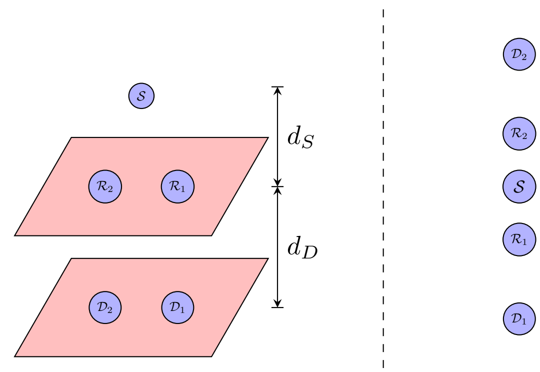


Fig. 5: Topologies-1 and 2

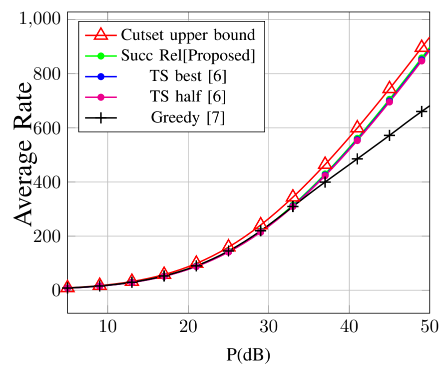


Fig. 6: Average rate plots of scenario-1. Here, the average rates due to Succ. Rel., TS half, TS best are almost the same.

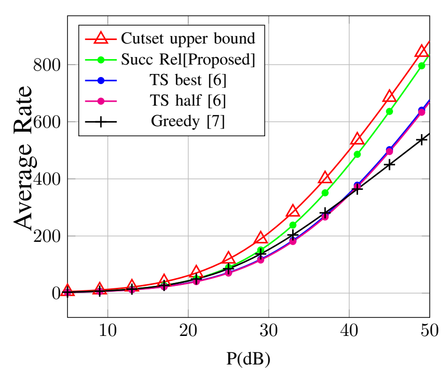


Fig. 7: Average rate plots of scenario-2. Note that succ. rel. performs much better than greedy and TS protocols.

*2) Topology-2:* In topology-2 shown in Fig. 5, $\mathcal{R}_1$ and $\mathcal{D}_1$ are arranged on one side of the source while $\mathcal{R}_2$ and $\mathcal{D}_2$ are on the other side of the source. Let $d_{\mathcal{S}\mathcal{R}_i}$ be the $\mathcal{S}$-$\mathcal{R}_i$ distance and $d_{\mathcal{R}_i\mathcal{D}_i}$ be the $\mathcal{R}_i$-$\mathcal{D}_i$ distance for $i = 1, 2$. We generate average rate plots for the case when $d_{\mathcal{S}\mathcal{R}_1}{:}d_{\mathcal{R}_1\mathcal{D}_1} = d_{\mathcal{S}\mathcal{R}_2}{:}d_{\mathcal{R}_2\mathcal{D}_2} = 1{:}2$. The average rates for successive relaying and other protocols are shown in Fig. 10. Once again, we see that the successive relaying protocol gives rates significantly better than the greedy and time-sharing protocols.

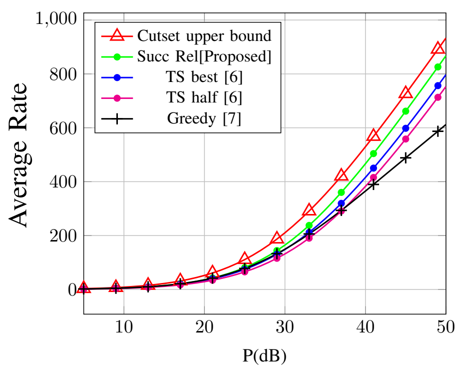


Fig. 8: Average rate plots of scenario-3

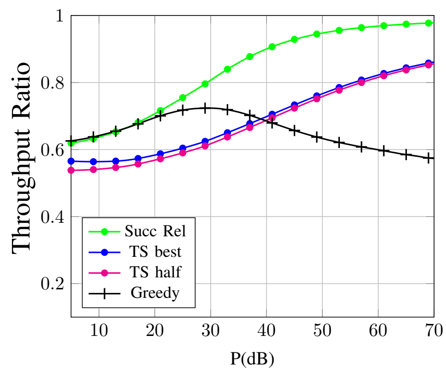


Fig. 9: Throughput ratio plots of scenario-2.

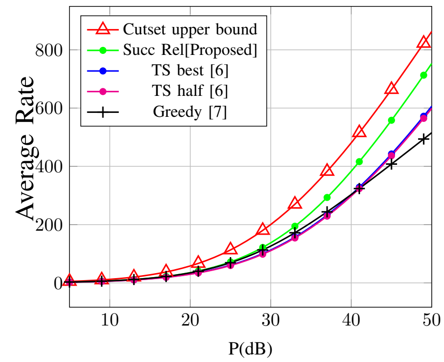


Fig. 10: Average rate plots of topology-2

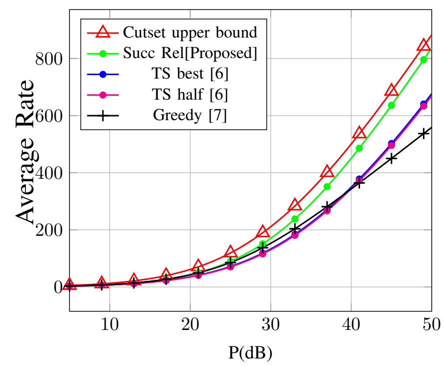


Fig. 11: Average rate plots of topology-3

*3) Topology-3:* In topology-3, the relays are placed on either side of $\mathcal{S}$ on a plane. The destinations are positioned on either side of this plane. Let $d_{\mathcal{D}_1}$ and $d_{\mathcal{D}_2}$ be the $\mathcal{D}_1 - \mathcal{S}$ and $\mathcal{S} - \mathcal{D}_2$ distances, respectively. We generate average plots for $d_{\mathcal{D}_1} : d_{\mathcal{D}_2} = 2 : 2$ in Fig. 11, where $d_{\mathcal{D}_i}$ is the distance of $\mathcal{D}_i$ from relay plane. The successive relaying protocol performs significantly better than the greedy and time-sharing protocols.

## VI. Conclusions

We proposed the Successive Relaying protocol for two-destination relay channels. From the analysis of the cutset upper bound, we showed that the successive relaying states are more important than the other two states. Our simulations showed that, under the Rayleigh fading model, the ratio of the rate achieved by the Successive Relaying protocol and the cutset bound approaches 1 at high SNR. The proposed protocol performs better than the modified versions of protocols in the literature, especially when the destinations are away from the source and the relays.

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