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SEREP Integrated Control of Flexible Structures

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Abstract: Flexible structures are continuum modeled as infinite degrees of freedom. Most of the time finite element models are made with large number of degrees-of-freedom to analyse flexible structures. Continuous monitoring, analysis and control of dynamics of flexible structures need simulation of large degrees of freedom in real time. This is computationally expensive and realtime control fails. This study focuses on the development of a reduced order framework for dynamics of flexible structures and use the reduced order model for controlling the structure. Essentially, in the proposed framework, full order state space obtained from finite element modeling of the flexible structure has been reduced to lower subspace using a reduced order algorithm keeping the dynamical characteristic intact. The transformation matrix for reduction has been calculated using system equivalent reduction expansion process (SEREP). Traditionally, full order dynamical system is being used to find the gain matrix to suppress the uncontrolled dynamics associated with the dynamical system. Here in this methodology, the reduced dimension of state space of the system is used for estimating the gain matrix using optimal linear quadratic regulator (LQR). The gain obtained through the reduced system is subsequently used as a feedback to the attached actuators which produce the required force to control the system. A numerical example of a flexible cantilever beam has been shown to investigate the effectiveness of the algorithm.

Keywords: reduced order model, vibration control, finite element model, Euler Bernoulli beam, SEREP, Transformation matrix, LQR.

1. INTRODUCTION

Flexible structures are widely used in many applications ranging from robotics, bio mechanics, aerial vehicle to space structure applications. These structures have less inherent/material damping and hence vibrations caused by external sources take long time to decay. This may hamper proper functioning and/or efficiency of the complete system (Ali and Ramaswamy, 2009). To overcome this problem, active, passive or semi-active mechanisms are deployed for structural vibration control (Refer papers listed). Various control algorithms have been developed and deployed in the past (Saaed et al., 2013).

There is a growth in research integrated with industrial demand for efficient MEMs devices, soft robotics applications, Micro-air-vehicles and underwater flexible robotics. The design space demanded advance features and accurate performance that can only be catered real time control of their flexible parts (Sodano et al., 2004). Traditionally structural control is achieved using passive mechanisms (Smith, 2002), where a particular frequency is targeted for control. The controller becomes sub optimal or inefficient at other operating frequencies. Furthermore, passive mechanism is bulky and heavy in nature (Krenk and Hogsberg, 2014). Increase in application of light weight flexible structures (Obradovic and Subbarao, 2011) has opened the scope of controller design of flexible system using active means (Ali and Padhi, 2009; Ali and Ramaswamy, 2009). Distributed parameter modeling and control is considered necessary in such classes of problems. Distributed control is achieved using advanced smart materials like, piezocomposites (Sodano et al., 2004), shape memory actuators (Baz et al., 1990), shape memory polymers (Saadat et al., 2002). For example Structures embedded with piezoelectric sensors and actuators is used for active vibration control of flexible structure (Hu and Ng, 2005).

In addition to that there are many different active control techniques, to control the shape and dynamics of structure, which have been applied to piezo-actuated flexible structures. For example, passivity based control methods (Kugi and Schlacher, 2002), Lyapunov based control (Hu and Ng, 2005), Fractional-order positive position feedback PPF control (Marinangeli et al., 2018), back propagation neural network controller (Qiu et al., 2012) and many more.

As mentioned, these flexible structures are modeled as a continuous system using the coupled partial differential equations for which obtaining an analytical solution is a tedious phenomenon. Therefore, numerical techniques

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are used to get the approximate solution. To estimate the solution numerically using finite element (FE) approach, the strong form of the equations are written into weak form. This full order FE model contains a large number of degrees of freedom (dofs) and its solution becomes computationally exhaustive and time consuming for real time application. To bypass the complexity associated with computations without sacrificing accuracy, reduced order models (ROMs) are used for analysis. A wide range of methods have been reported in literature for linear as well as non linear dynamical systems. For example, modal truncation (Hughes and Skelton, 1981), balanced truncation (Willcox and Peraire, 2002) are used for linear systems and proper orthogonal decomposition (POD) which is also known as principle component analysis (PCA) (Hsieh, 2001) is used for non linear systems. For implementing Balanced truncation, POD/PCA, snapshot matrix is needed for singular value decomposition (SVD) (Hsieh, 2001; Willcox and Peraire, 2002) which is cumbersome for large order matrices. Another, very popular and frequently used eigenvalue analysis based ROM technique in linear structural dynamics is system equivalent reduction expansion process (SEREP) (Callahan et al., 1989) which also preserves the modes along with the dynamics of system under consideration.

In this study the application of SEREP has been explored in controlling the dynamics of piezo-actuated flexible cantilever beam. The eigenvalue analysis of the full order system matrices is carried out to reduce the system dimension using SEREP. Subsequently, optimal linear quadratic regulator (LQR) is employed on the reduced system matrices which have been obtained through SEREP. Traditionally, the gain matrix, to control the system dynamics, is obtained by solving full order dynamical systems which is not computationally efficient. On the other hand, in present analysis, this control gain matrix is obtained in reduced subspace which reduces the computational cost. The gain matrix obtained through SEREP is further used in estimating the required control forces for controlling the dynamics of full order system.

The outline of this paper is as follows: Section 2 describes the full order FE modelling of flexible cantilever beam with piezoelectric actuators. The philosophy and procedure to obtain the transformation matrix which maps the full system to reduced dimension using SEREP has been explained in Section 3. Section 4 discusses the procedure to apply the control algorithm to calculate the gain matrix and subsequently estimating the control force in reduced subspace. The results obtained through the solution of a benchmark problem of cantilever beam have been discussed in Section 5. The findings of this study have been concluded in Section 6.

2. MATHEMATICAL FORMULATION

In this study, an Euler Bernoulli cantilever beam with multiple piezoelectric actuators has been considered as shown in Fig. 1. Here, the piezoelectric layers are perfectly bonded using adhesive to the surface of the host beam structure. The layer of adhesive is very thin compared to other dimensions and is neglected in developing the mathematical formulation. However, contribution of piezoelectric actuators on the mass and stiffness matrices of the beam has been considered in the formulation.

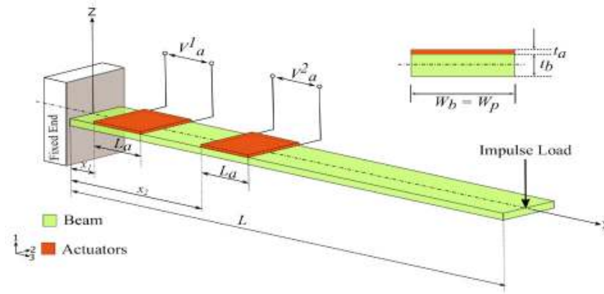

Fig. 1. Schematic of the cantilever beam attached with two actuators.

A two noded beam element with two mechanical(w,θ) and one electric(ϕ) dofs at each node is considered. The transverse displacement w and the rotation θ are interpolated using Hermite cubic interpolation shape functions $N = [N_1, N_2, N_3, N_4]$ which are given in (Petyt, 2010) Linear constitutive relation of piezoelectric material in terms of strain and electric field is given as

$$\sigma_{ij} = C_{ijkl}\epsilon_{kl} - e_{ijk}E_k \tag{1}$$

$$D_i = e_{ikl}\epsilon_{kl} + \chi_{ik}E_k \tag{2}$$

where, σ, D, ϵ, E, C_{ijkl}, e_{kij} and χ_{ij} denote the mechanical stress tensor, electric displacement vector, strain tensor, electric field vector elastic, piezoelectric and permittivity constants respectively. In this study, Strain ϵ and electric field E are expressed in terms of displacement and electric voltage respectively which will be the obvious choices as degrees of freedom in the finite element analysis. This necessitates the use of strain formulation of the piezoelectric constitutive equations (Chee et al., 1999). The electric field components are related to the electrostatic potential ϕ, which is given as

$$E_k = -\frac{\partial \phi}{\partial z} \tag{3}$$

In finite element discretization of the piezoelectric parts of the system and for getting the elemental matrix, the same shape functions as the structural elements are used and electric dof is assumed at each node. The electric potential is considered as a function of the thickness (h) and the length (L) of the beam, thus the shape function for electric field will be of the form (Bendary et al., 2010):

$$E(x,z) = E_1\zeta_1 + E_2\zeta_2 = [E]^T \zeta \tag{4}$$

where, ζ_1 and ζ_2 are the shape function

$$\zeta_1 = \left(\frac{1}{2} + \frac{z}{h}\right)\left(1 - \frac{x}{L}\right) \qquad \zeta_2 = \left(\frac{1}{2} - \frac{z}{h}\right)\left(\frac{x}{L}\right) \tag{5}$$

The equations of motion for the system shown in Fig. 1 is derived using Hamilton's principle given as follows,

$$\delta \int_{t1}^{t2} (T - U + W_{nc})dt = 0 \tag{6}$$

where, T denotes kinetic energy of the system and U is the potential energy which is sum of internal strain energy and the electrical energy and W_{nc} represents work done by non-conservative forces (Bendary et al., 2010). The weak form of the governing equation for this system is derived by substituting back the variation of the total energy and variation of work done by non-conservative forces into Hamilton's Eq. (6). Now, substituting Hermite cubic interpolation shape functions N and Eq. (4) into weak form, yields

$$\begin{aligned}\{\delta u\}_e^T[M_{uu}]\{\ddot{u}\}_e + \{\delta u\}_e^T[K_{uu}]\{u\}_e \\ - \{\delta u\}_e^T[K_{u\phi}]\{E_z\}_e - \{\delta E_z\}_e^T[K_{\phi u}]\{u\}_e \\ - \{\delta E_z\}_e^T[K_{\phi\phi}]\{E_z\}_e - \{\delta u\}_e^T\{\bar{F}\} = 0\end{aligned} \tag{7}$$

where,

$$\begin{aligned}
[M_{uu}] &= \int_v [N]^T \rho [N]\, dv & [K_{uu}] &= \int_v [B_u]^T [C][B_u]\, dv \\
[K_{u\phi}] &= \int_v [B_u]^T [e][B_E]\, dv & [K_{\phi u}] &= \int_v [B_E]^T [e][B_u]\, dv \\
[K_{\phi\phi}] &= \int_v [B_E]^T [\chi][B_E]\, dv & \{\bar{F}\} &= \int_s [N]^T \{F_s\} ds \\
\{Q_a\} &= \int_s \{\sigma\} ds
\end{aligned} \tag{8}$$

The equation of motion for undamped system in state space form is written as,

$$M_{uu}\ddot{U} + K_{uu}U + K_{u\phi}\Phi = \bar{F} \tag{9a}$$

$$K_{\phi u}U + K_{\phi\phi}\Phi = Q_a \tag{9b}$$

where, M_{uu} is the global mass matrix, K_{uu} is the global stiffness matrix corresponding to mechanical degrees of freedom, $K_{u\phi}$ is the global stiffness matrix due to electromechanical coupling and $K_{\phi\phi}$ the global stiffness matrix due to electrical dofs alone. Also, U is the vector consists of transverse displacement and rotation of the structural part of the system and Φ is vector of electrical dofs corresponding to piezoelectric actuators. Here, $\bar{F}$ is the externally applied load on the system and Q_a is the vector of electrical charge. Now using back substitution for Φ from Eq. (9b) into Eq. (9a) and considering the damping effect of structure, the governing second order differential equation for the damped system in compact form is given by

$$M\ddot{X}(t) + C\dot{X}(t) + KX(t) = F(t) \tag{10}$$

where M, C and K denote mass, Rayleigh damping and stiffness matrices of full order system of size $n \times n$. Also, $X(t)$ (same as $(U(t))$, $\dot{X}(t)$ and $\ddot{X}(t)$ represent the displacement, velocity and acceleration vector of size $n \times 1$, respectively. For all the numerical simulations in this manuscript, Eq. (10) is used.

3. SYSTEM EQUIVALENT REDUCTION EXPANSION PROCESS

System equivalent reduction expansion process (SEREP) is a global reduction algorithm which is based on the eigenvalue analysis of high fidelity finite element model. It is a two stage reduction process; reduction in number of modes as well as the reduction in dofs. The transformation matrix obtained using this algorithm maps the full model to a lower subspace using selected eigenmodes with fewer arbitrary dofs. Eq. (10) describes the dynamics of the full order system and eigenvalue analysis using full order mass (M) and stiffness (K) matrices will give the modal matrix $(\psi \in \Re^{n\times n})$. Using this modal matrix, system can be written in terms of modal coordinates $(q(t) \in \Re^{n\times 1})$ as

$$X(t) = \psi_{n\times n} q_{n\times 1}(t) \tag{11}$$

Next, if only a eigenmodes are retained, the solution can be written as

$$X(t) = \psi_{n\times a} q_{a\times 1}(t) \tag{12}$$

Now, splitting the full order system into master and slave dofs and retaining only a eigenmodes in Eq. (12), which can be written in terms of active dofs as

$$X(t) = \begin{Bmatrix} X^m(t) \\ X^s(t) \end{Bmatrix} \approx \begin{Bmatrix} \psi^m \\ \psi^s \end{Bmatrix} q_a(t) \tag{13}$$

where q_a is the modal matrix having a modes, the superscript m and s corresponds to master and slave, respectively. Also here, $\psi^m \in \Re^{m\times a}$ and $\psi^s \in \Re^{(n-m)\times a}$. Considering only the master dofs, the modal response in terms of master dofs can be given as

$$q_a(t) = \psi_p^m X^m(t) \tag{14}$$

where, ψ_p^m is the generalized inverse of ψ^m. The number of modes and number of dofs in the analysis may be same, it depends on the problem and hence the matrix ψ^m always may not be a square. The different possible cases of the selection of modes and dofs has been discussed in (Lal et al., 2017). Now substituting back Eq. (14) in Eq. (13), the full order response can be written as

$$X(t) = \psi\, \psi_p^m X^m(t) = T_r X^m(t) \tag{15}$$

Here, $\psi^m \psi_p^m$ is the transformation matrix (T_r) which maps the full system into reduced subspace. Using this transformation matrix, the full order system matrices in reduced dimension can be given as

$$M_r = T_r^T M T_r \qquad K_r = T_r^T K T_r \qquad C_r = T_r^T C T_r \tag{16}$$

where, M_r, K_r and C_r represent the reduced mass, stiffness and damping matrices in $\Re^{m\times m}$. The superscript T in Eq. (16) denotes the transpose operator. The same transformation matrix is used for reducing the force vector in reduced domain by $T_r^T F(t)$. Using SEREP, the modes of full order dynamical systems are also retained in the reduced subspace. To retrieve the full order system dynamics, the same transformation matrix (T_r) can be used. In fact, selection of modes and dofs are governed by the type of system as well as the external loading conditions. However, selection of active dofs and modes in the formulation from entire domain is preferable.

4. CONTROL ALGORITHM

The dynamic equations for a cantilever beam with PZT in reduced dimension using SEREP can be expressed as

$$M_r\ddot{X}^m(t) + C_r\dot{X}^m(t) + K_rX^m(t) = F_r(t) \tag{17}$$

For efficient handling of MIMO systems, Eq. (17) in the state space form can be written as

$$\dot{X}^m = A^m X^m + B^m u^m \tag{18}$$

where

$$A^m = \begin{bmatrix} 0 & I \\ -M_r^{-1}K_r & -M_r^{-1}C_r \end{bmatrix}, \qquad B^m = \begin{bmatrix} 0 \\ -M_r^{-1}F_r \end{bmatrix} \tag{19}$$

where $\dot{X}^m$ is $(m \times 1)$ state vector.

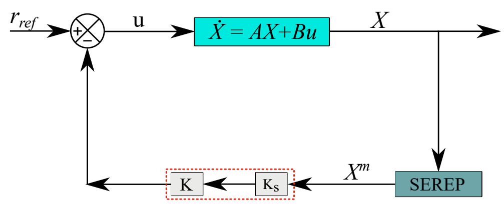

Fig. 2. Block diagram to show the flow of proposed control algorithm.

In this study, Linear Quadratic Regulator (LQR) controller is used which has been implemented on the proposed reduced order model algorithm. The optimal control input to actuator for vibration suppression is obtained through LQR which minimizes the cost function. This cost function (J) is defined as,

$$J = \lim_{\tau \to \infty} \frac{1}{\tau} \left[\int_0^\infty \{(X^m)^T Q(X^m) + (u^m)^T R(u^m)\} dt \right] \tag{20}$$

where Q and R are weighting matrices which penalize the system states X^m and control input u^m. Minimization of the quadratic cost function defined in Eq. (20) subjected to system dynamics, Eq. (18) gives the state feedback in form of required control input u^m which is given by,

$$u^m = -K_s X^m \tag{21}$$

where K_s is the $(m \times r)$ gain matrix obtained through the

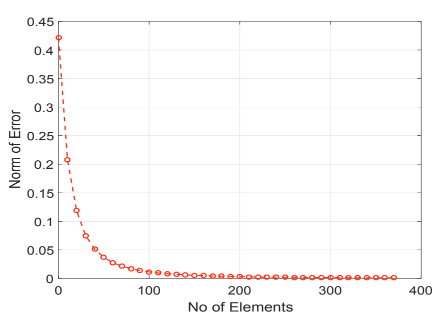

Fig. 3. Convergence of number of elements for finite element analysis.

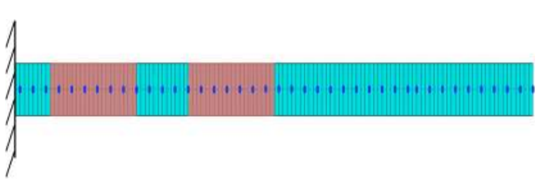

Fig. 4. Location of selected nodes for analysis of system using SEREP in Case 1.

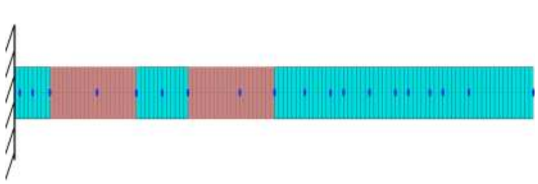

Fig. 5. Location of selected nodes for analysis of system using SEREP in Case 2.

proposed reduced order model. The closed loop optimal gain is calculated by solving the reduced-matrix Riccati Equation. Using this gain matrix, the control input needed to control the dynamics of full order system is estimated. The block diagram of the proposed framework is shown in Fig. 2. Here, in this analysis the value of r_{ref} has been assumed to be zero.

5. NUMERICAL EXAMPLE: CANTILEVER BEAM

An example of flexible cantilever beam length with piezoelectric patches as shown in Fig. 1 has been solved numerically to verify the efficiency of proposed reduce order formulation. The geometric dimension of the beam has been taken to be $400 \times 10 \times 0.3$ (in mm) and modulus of elasticity is assumed to be 200 Gpa. Figure 3 shows the convergence study of the number of elements in the finite discretization of the system. It can be seen that the after 200 elements the error norm of natural frequencies has become constant. Hence, based on the convergence study, the system has been discretized using 200 elements in the current analysis. Each element has two nodes and each node has two dofs.

To discuss the importance of selection of nodes in the formulation of SEREP, two different cases have been considered.

- Case 1: Nodes are selected at equal interval in the mesh as shown in Fig. 4. Here, in this way of selection of nodes some of the critical points have been missed. In this case, around 48 nodes has been chosen and hence a total of 96 dofs will participate in analysis.
- Case 2: Nodes are selected randomly from the entire domain as reported in Fig. 5. Here, all the critical nodes has been taken into consideration and only 19 nodes has been chosen and hence a total of 38 dofs.

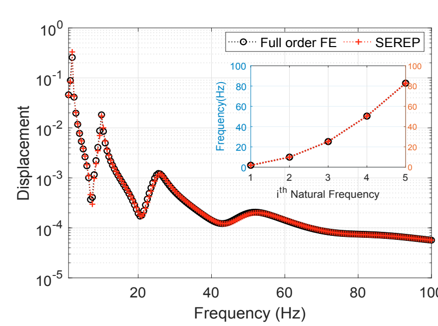

Fig. 6. Comparison of frequency response obtained through full order FE analysis and SEREP and comparison of first five natural frequencies of the system is shown in inset (for Case 2).

Figure 6 shows the comparison of full order system response to that with obtained through SEREP. In inset of Fig. 6, the comparison of first five natural frequencies have also been depicted. It can be noticed here that the results obtained through SEREP are in very good agreement with that obtained through the full order system. To show the comparison of mode shapes obtained through SEREP with that of full order system, modal assurance criteria (MAC) has been shown in Fig. 7. The definition of MAC is given as (Ewins, 1984)

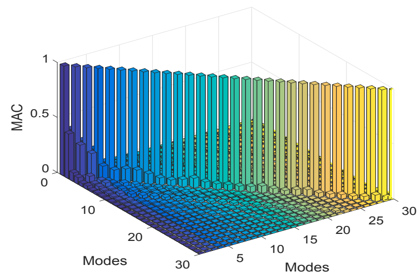

Fig. 7. Modal assurance criteria of modes obtained through SEREP with the modes of full order system.

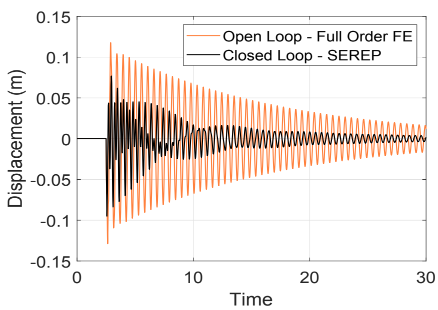

Fig. 8. Comparison of controlled response with that of the uncontrolled dynamics obtained for Case 1.

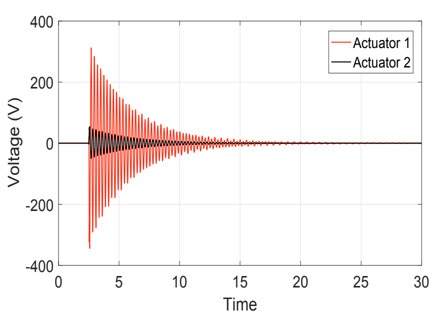

Fig. 9. Required voltage needed given to the actuators to control the dynamics for Case 1.

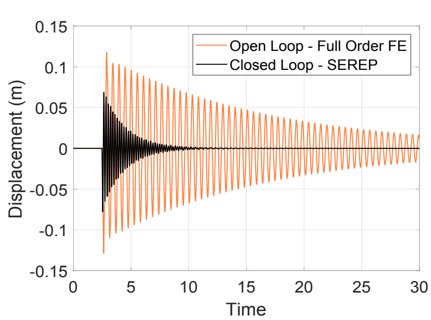

Fig. 10. Comparison of controlled response with that of the uncontrolled dynamics obtained for Case 2.

$$MAC_i = \frac{|v_{ir}^T v_{if}|^2}{(v_{ir}^T v_{ir})(v_{if}^T v_{if})} \tag{22}$$

where, v_{ir} and v_{if} are the i^{th} normal modes computed using the reduced order modal and the full order system, respectively. MAC value of one represents the best match

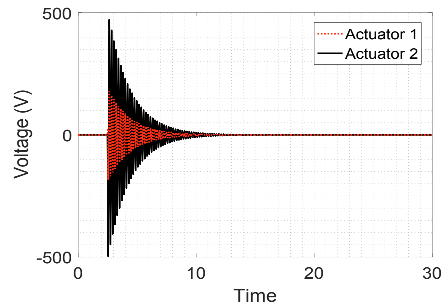

Fig. 11. Required voltage needed given to the actuators to control the dynamics for Case 2.

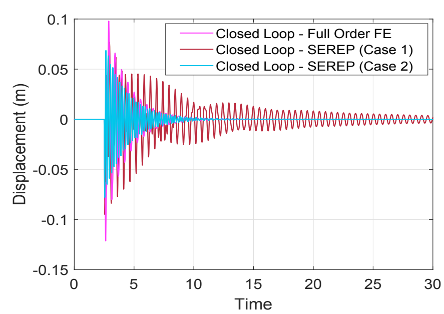

Fig. 12. Comparison of controlled response obtained through full order FE model and proposed algorithm.

of modes obtained through ROM to that with the full order modes. In Fig. 7, MAC value for first 30 modes has been shown which are very close to one and hence the modes of full order system has been retained in the reduced space.

The controlled response obtained through ROM using the set of selected dofs as mentioned in Case 1 has been shown in Fig. 8. The dynamic response of the system is getting reduced but not going to zero. The amount of required voltage given to both the actuators have been reported in Fig. 9. On the other hand, with less number of dofs in SEREP formulation (Case 2), the controlled dynamic response is almost going to zero after 10 seconds as shown in Fig. 10. The corresponding voltage required for controlling the uncontrolled response has been shown in Fig.11.

The controlled responses obtained through SEREP (Case 1 and Case 2) have been compared with the controlled response obtained with the help of gain matrix using the full order system matrices as shown in Fig. 12. In case 1, the way nodes are selected (at equal interval), the nodes corresponding to junctions of actuators and beam are missing in the formulation. Whereas in Case 2, there is no such order in selection of nodes, but those critical nodes have been taken into account in the formulation. Therefore, even though the number of dofs in Case 1 is more than that in Case 2, the response is getting better controlled in the later case as can be seen in Fig. 12.

5.1 COMPUTATIONAL COST

The full order finite element of the system has a total of 200 elements which results in a sum of 402 dofs and consequently the state dimension will be 804. Here, in

SEREP framework, two cases have been considered: one with 96 dofs and another case with 38 dofs. The reduction in computational cost for first case has been achieved by 95% whereas for the other case this value has been increase to 97.5%. Even though the reduction in number of dofs in Case 2 is approximately 60% relative to Case 1, the global improvement in computational cost is around 2.5%. Essentially, SEREP is based on the eigenvalue analysis of full system matrices to get the transformation matrix. This eigenvalue analysis is a costly process and an inevitable step in this framework, which has been reflected in the difference among the computational cost of the two aforementioned cases.

6. CONCLUSIONS

In this study, a reduced order model has been proposed to control the dynamics of a flexible structures. To map the full order FE system dimension to reduced subspace, the transformation matrix is obtained through SEREP. Here, the system is solved in reduced dimension to get the gain matrix. The gain matrix obtained in reduced dimension is further used in estimating the required force to control the dynamics of full order FE system. To describe the efficiency of the developed framework, an example of flexible cantilever beam with multiple piezoelectric patches has been solved. Two different cases using SEREP have been explored to show the importance of the selection of dofs to get the desired dynamics. In first case, a total of 96 dofs are selected at equal interval from the entire system. Whereas in the other case, only 38 dofs has been chosen randomly from the entire domain. The analysis reveals that the controlled response obtained in the later case is much better than the earlier case with less computational effort.

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