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Role of thickness variation on the tailored bistability of unsymmetric composite laminates

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Abstract. Unsymmetric composite laminates exhibit two stable configurations at room temperature after being cured at elevated temperature. Bistability of cross-ply laminates is due to the presence of residual strains imparted during the curing process. A simple snap-through process can be used to switch from one stable shape to another. The amount of snap-through force required for attaining the other stable shape depends on the total strain energy of the laminate in the first cured shape. Bistable shapes of the conventional square laminates are in the same energy level, which means the stable configurations are equally stable and results in equal snap-through and snap-back forces. By tailoring the energy levels of stable shapes, snap-through and snap-back forces can be varied. From various studies in the past, it is observed that bistable laminates have enormous potential for morphing applications. In some of the practical applications, one may require stable shapes at different energy levels, with unequal snap-through and snap-back energies for the shape transition. This study aims to tailor the energy levels of bistable shapes of the square laminate by varying the thickness of individual layers in the considered [0/90] square laminates. A detailed parametric study is performed in commercially available finite element software. It is concluded from the study that individual lamina thickness has a noticeable influence on the bistability of cross-ply laminates.

1. Introduction

Unsymmetrical composite laminates exhibit multiple equilibrium states when they are cooled from the curing temperature to the room temperature. The formation of multiple equilibrium states, rather than just a single equilibrium state, depends on the laminate dimensions, as well as the lamina fiber alignments and the material properties, specifically the mismatch of thermal expansion coefficients in the fiber direction between the layers. Hyer [1] in his pioneering works conducted a series of experiments [2] and formulated an analytical model [3, 4] to demonstrate the bistable behavior of thin unsymmetric composite laminates (two stable cylindrical equilibrium shapes and one unstable saddle shape) upon curing as shown in figure 1.

Since multiple equilibrium states are possible at the end of the curing process, the cured shape of unsymmetrical laminates can be changed from one stable state to another by appropriate snap-through mechanisms. Hyer modified his earlier model to precisely depict the behavior of thin cross-ply laminates considering geometric nonlinearities with Classical Laminate Theory (CLT) [5]. This anomalous behavior of unsymmetrical laminates attained much research interest, and many researchers contributed by improving Hyer's theory to take advantage of this unusual characteristic [6-11]. Thereafter, several analytical models have been reported in the literature to analyze the cured room-temperature shapes of



unsymmetric laminates [12-19]. Analytical models developed by various researchers explained the difficulty predicting the exact cured shapes of the unsymmetric laminates, especially at its boundaries. To predict the shapes and snap through loads accurately, one requires a higher order of polynomial functions in the analytical model, which is computationally expensive.

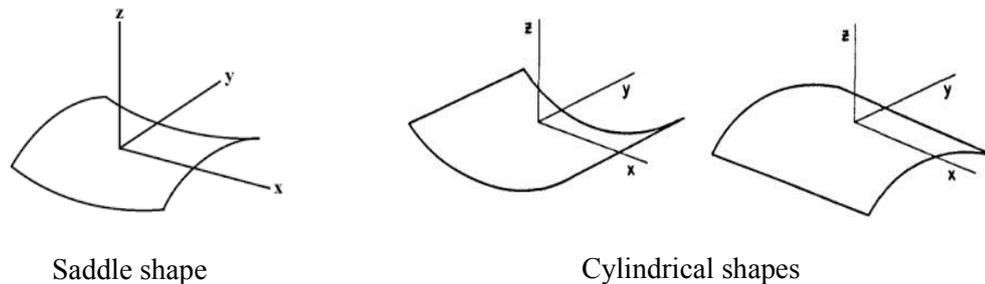


Figure 1: Equilibrium shapes of bistable laminates

Besides the analytical approaches, some of the researchers have also employed the finite element method to study the behavior of bistable plates. With refined FE models [20, 21], it is possible to predict the bistable shapes with higher accuracy, and they are closer to the experimentally observed shapes than the predictions from analytical models developed so far. Unlike analytical modelling, by using FE, one can model even the complex shapes using these bistable laminates. One of the main advantages of FE modelling is that it is possible to predict complex shapes generated by the bistable laminates numerically with at high accuracy [22-28].

The bistable nature of a structure can be explained based on its two distinct potential energy minima's. Each of the minima corresponds to one of the stable geometric shapes of the structure, and the graph for a bistable structure with equally stable shapes is shown in figure 2. From figure 2, one can observe that the structure has two stable states SS_1 and SS_2 . External energy needs to be supplied to the structure to actuate between one stable shape and the other. When sufficient external energy is given in the form of actuation force, the potential energy of the structure increases until it reaches the curve's peak, and the structure will snap-through to another stable shape. If the external energy is not sufficient, snap-through will not occur, and the structure remains in the same stable shape. If the values SS_1 and SS_2 are equal, then the structure can have equilibrium shapes with equal stability, and if the values of SS_1 and SS_2 are not equal, then the structure can have equilibrium shapes with tailored stability. In such a case, the state with less energy is more stable compared to the other shape.

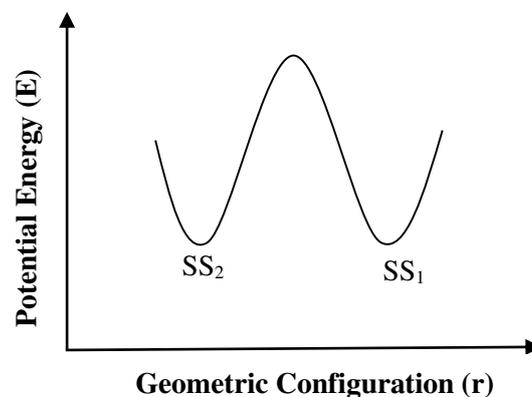


Figure 2: Potential energy curve (E: Potential Energy and r: Geometric Configuration)

In many practical applications like aerospace applications, there is a huge demand for structures that can undergo shape change based on the environmental conditions called adaptive structures. Bistable laminates have attained significant attention in the design of adaptive structures due to the low energy requirements for the shape transition. In some of the applications, the designer may have to use bistable shapes, where one stable shape is required to be more stable than the other. This helps in further reduction of energy consumption compared to the stable shapes, which are in equal energy levels (Figure 2). In 2002, Hufenbach et al. [29] incorporated multistable composites in adaptive structures by investigating various aspects like laminate edge length L , different angle-ply laminates, and variable layer ratio. The study showed the possibility of utilizing the residual stress field to generate monostable and multistable equilibrium states. Taking the studies by Hufenbach et al. [29] as a reference, a detailed parametric study is proposed in this study to understand the role of lamina thickness variations to tailor energy levels of bistable shapes of unsymmetric composite laminates. The proposed parametric investigation is performed in commercially available finite element software, AbaqusTM. Details of the steps followed in the study are given in the subsequent sections.

2. Problem definition

Composite bistable shells obtained from the cool down process of $[0/90]$ square laminate is used for the present analysis. The material properties of the laminate are given in table 1.

Table 1: Properties of laminate

| Engineering constants | | Value |
|----------------------------------|---------------|---------------------------------------|
| Elastic Modulus | E_1 | 294 GPa |
| | E_2 | 9.5 GPa |
| Poisson Ratio | ν_{12} | 0.3 |
| Shear Modulus | G_{12} | 5.0 GPa |
| Coefficient of thermal expansion | α_{11} | $-2 \times 10^{-8}/^\circ\text{C}$ |
| | α_{22} | $-2.25 \times 10^{-5}/^\circ\text{C}$ |

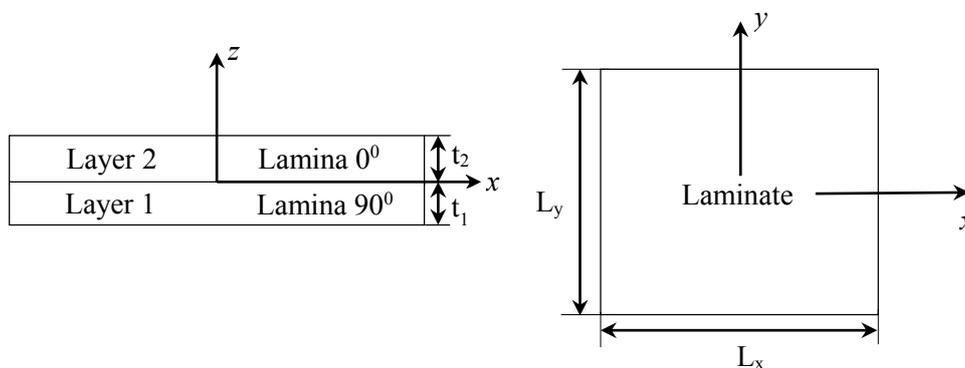


Figure 3: Laminate geometry

Figure 3 shows the schematic representation of the geometry used for the present analysis. Where t_1

is the thickness of lamina, which is fixed, t_2 is the thickness of lamina, which is varying for the study, and “p” is the ratio of the thickness (t_2/t_1). To investigate the effect of individual lamina thickness on the cured shape of laminate, one of the lamina thickness is increased from 0.100 mm to 0.150 mm while keeping the thickness of another lamina constant (0.100 mm). Five square geometries are taken for the numerical parametric study ($L=L_x=L_y$), $L = 100$ mm, 125 mm, 150 mm, 175 mm, 200 mm. Details of the proposed study are given in the following sections.

3. Numerical investigation

Thermally induced shapes of bistable unsymmetric composite laminates are modelled in a commercial finite element package, Abaqus™ for the analysis. Static non-linear analysis is performed by considering geometric non-linearities in the numerical model. Imperfections are introduced to the model of proposed laminate to avoid the convergence to unstable saddle shape during the analysis. Four-node quadrilateral general-purpose composite shell elements, type S4R in the Abaqus™ is taken for the finite element analysis. Mesh convergence studies are performed to find a suitable mesh characteristics for the described problem. Steps followed in the numerical investigation are explained in the subsequent subsections.

3.1 Cool-down step

The modelled plate is subjected to a curing temperature of 180° C during the initial stage. The temperature field of the laminate assembly is then brought down to room temperature at 20° C. The assigned temperature difference produces residual stresses, which makes the modelled plate deviate to any one of its stable shapes.

3.2 Snap-through and stability check

The static transverse load is applied on the corners of the modelled cured shape of laminate. Load higher than the required snap-through are applied so that the laminate will deform to another equilibrium shape. An additional step (stability check) is imposed to ensure the stability of obtained stable state even after the removal of the applied load.

4. Results and discussion

Figure 4 shows the cured bistable shapes of an unsymmetric cross-ply laminate. Figure 5 shows the variation of out of plane displacement with curing temperature for lamina with 150 mm side length. A noticeable increase in maximum out-of-plane displacement of the other stable shape, even after the removal of snap-through load, is observed with the increase in thickness ratio. Strain energy levels of both stable shapes are extracted from the developed numerical model.

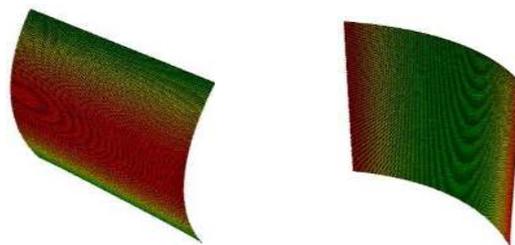


Figure 4: Modelled bistable shapes

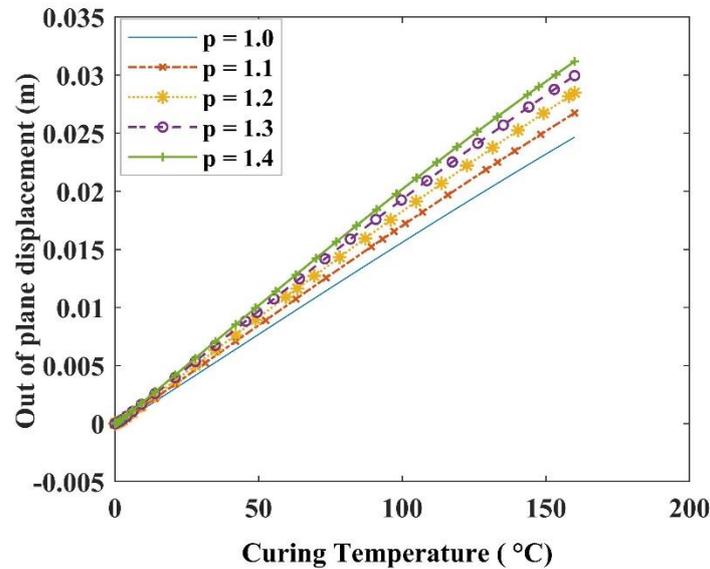


Figure 5: Out of plane displacement with curing temperature for L=150 mm

Table 2: Difference in SE with thickness variation

| P | Difference in SE | | | | |
|-----|------------------|----------|---------|---------|---------|
| | L=100 mm | L=125 mm | L=150mm | L=175mm | L=200mm |
| 1.0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1.1 | 4.85 | 5.59 | 5.15 | 5.36 | 5.30 |
| 1.2 | 9.52 | 9.76 | 10.59 | 10.56 | 10.45 |
| 1.3 | 14.15 | 15.06 | 15.00 | 15.34 | 15.46 |
| 1.4 | 18.69 | 19.05 | 19.83 | 20.36 | 20.47 |
| 1.5 | - | - | - | - | 25.12 |

Table 3: Ratio of snap forces with thickness variation

| P | Ratio of snap force | | | | |
|-----|---------------------|----------|---------|---------|---------|
| | L=100 mm | L=125 mm | L=150mm | L=175mm | L=200mm |
| 1.0 | 1.01 | 1.01 | 1.01 | 1.01 | 1.01 |
| 1.1 | 1.43 | 1.33 | 1.37 | 1.40 | 1.45 |
| 1.2 | 2.45 | 2.30 | 2.47 | 2.16 | 2.20 |
| 1.3 | 5.20 | 4.53 | 4.98 | 4.82 | 4.07 |
| 1.4 | 25.26 | 15.19 | 13.82 | 10.28 | 9.31 |
| 1.5 | - | - | - | - | 12.50 |

Table 2 and Table 3 show the difference in strain energy (SE) and the ratio of snap forces (which is defined as the ratio between snap-through and snap-back forces), respectively with the thickness variation. There is a noticeable difference in SE with the increase of thickness of one lamina. It shows

the presence of tailoring in bistability. The two stable shapes obtained as a result of the cool-down process seem to be in different energy levels. The above observation shows a higher degree of stability for one of the stable shapes. Calculated snap-through and snap-back forces are given in table 3.

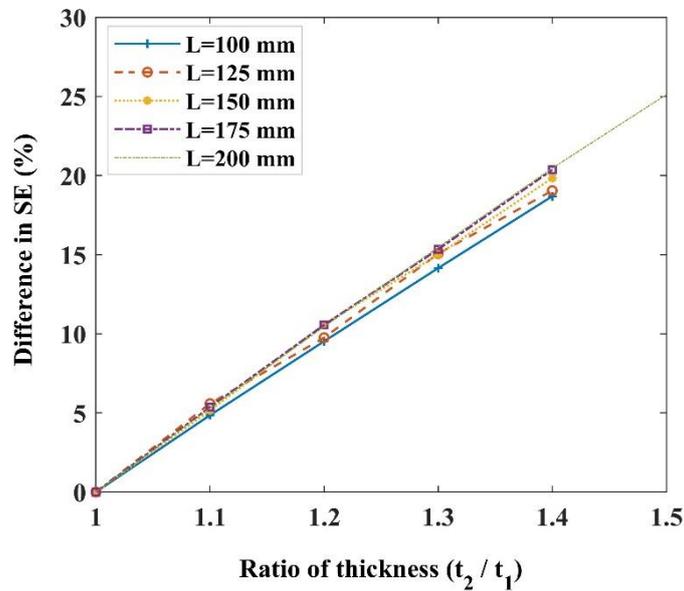


Figure 6: Difference in SE with thickness variation

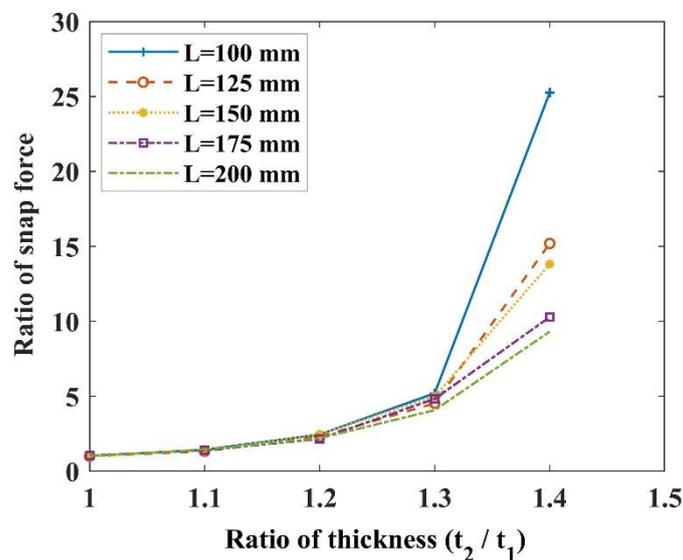


Figure 7: Ratio of snap forces with thickness variation

There is a noticeable difference in SE with the increase of thickness of one lamina. It shows the presence of tailoring in bistability. The two stable shapes obtained as a result of the cool-down process seem to be in different energy levels. The above observation shows a higher degree of stability for one of the stable shapes. The effect of change in “p” on the ratio between the snap-through/snap-back is

shown in table 3. Bistability of the laminate is lost when “p” is 1.5 for $L = 100$ mm, 125 mm, 150 mm and 175 mm. Graphical representation of table 2 and table 3 are given in figure 6 and figure 7, respectively. After a specific point ($t_2 = 0.150$ mm), loss in bistability is observed. Figure 8 shows the schematic representation of energy levels for equally stable shapes and tailored stable shapes of the square laminate.

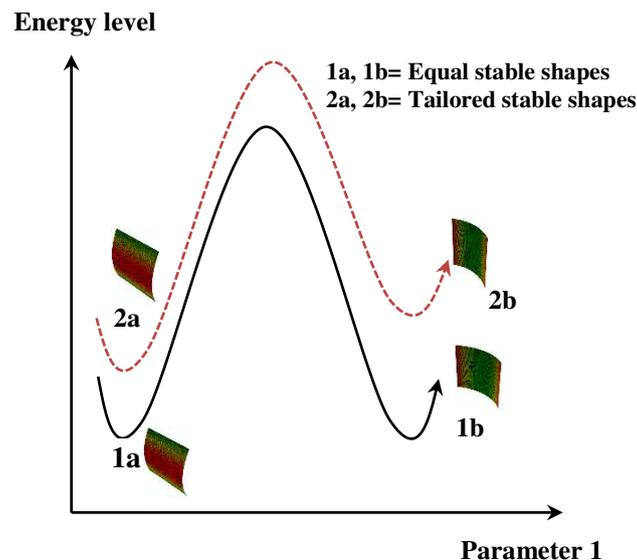


Figure 8: Schematic representation of energy levels

5. Conclusion

With the increase in the thickness of individual lamina, the stiffness of the laminate gets increases. It is observed that after a specific point (limit of $t_2/t_1 = 1.5$), laminate loses its bistability completely. From the ratio of snap forces, it is observed that the laminate with $L = 100$ mm is having a more stable second shape compared to the first shape. This is due to the higher snap-back force compared to the snap-through force in $L = 100$ mm compared to other different length cases considered. It can be concluded that in order to have laminate with tailored bistability, a bistable laminate with shorter length is more preferred. The focus needs to be laid on the thickness ratio of the laminates being used for this application as we may lose the bistability completely.

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