

Punctuated inflation and the low CMB multipoles

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Abstract. We investigate inflationary scenarios driven by a class of potentials which are similar in form to those that arise in certain minimal supersymmetric extensions of the standard model. We find that these potentials allow a brief period of departure from inflation sandwiched between two stages of slow roll inflation. We show that such a background behavior leads to a step like feature in the scalar power spectrum. We set the scales such that the drop in the power spectrum occurs at a length scale that corresponds to the Hubble radius today—a feature that seems necessary to explain the lower power observed in the quadrupole moment of the Cosmic Microwave Background (CMB) anisotropies. We perform a Markov Chain Monte Carlo analysis to determine the values of the model parameters that provide the best fit to the recent WMAP 5-year data for the CMB angular power spectrum. We find that an inflationary spectrum with a suppression of power at large scales that we obtain leads to a much better fit (with just one extra parameter, χ^2_{eff} improves by 6.62) of the observed data when compared to the best fit reference Λ CDM model with a featureless, power law, primordial spectrum.

PACS numbers: 98.80.Cq, 98.70.Vc, 04.30.-w

1. Introduction

Measurements of the Cosmic Microwave Background (CMB) anisotropies—from the early days of the COsmic Background Explorer (COBE) satellite until the most recent observations of the Wilkinson Microwave Anisotropy Probe (WMAP)—have consistently indicated a low value of the quadrupole, below the cosmic variance of the concordant Λ CDM cosmological model with a nearly scale invariant, primordial spectrum [1, 2, 3, 4, 5]. While there has been a recurring debate about the statistical significance of the quadrupole and the other outliers (notably, near the multipole moments 22 and 40) in the CMB angular power spectrum (see Refs. [6, 7, 8, 9] and references therein), there has also been constant activity to understand possible underlying physical reasons for the outliers (see, for an inexhaustive list, Refs. [10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32]).

Given the CMB observations, different model independent approaches have been used to recover the primordial spectrum (see, for example, Refs. [4, 33, 34, 35, 36, 37]). While all these approaches arrive at a spectrum that is nearly scale invariant at the smaller scales, most of them inevitably seem to point to a sharp drop in power at the scales corresponding to the Hubble scale today. Within the inflationary scenario, a variety of single and two field models have been constructed to produce such a drop in power at the large scales [10, 12, 18, 19, 20, 27, 29, 30, 32]. However, in single field inflationary models, in order to produce such a spectrum, we find that many of the scenarios either assume a specific pre-inflationary regime, say, a radiation dominated epoch, or special initial conditions for the background scalar field, such as an initial period of fast roll [18, 19, 29, 30]. Moreover, some of them impose the initial conditions on the perturbations when the largest scales are outside the Hubble radius during the pre-inflationary or the fast roll regime [18, 29, 30]. Such requirements are rather artificial and, ideally, it would be preferable to produce the desired power spectrum during an inflationary epoch without invoking any specific pre-inflationary phase or special initial conditions for the inflaton. Furthermore, though a very specific pre-inflationary phase such as the radiation dominated epoch may allow what can be considered as natural (i.e. Minkowski-like) initial conditions for the perturbations even at super-Hubble scales, we believe that choosing to impose initial conditions for a small subset of modes when they are outside the Hubble radius, while demanding that such conditions be imposed on the rest of the modes at sub-Hubble scales, can be considered unsatisfactory.

It has long been known that power spectra with large deviations from scale invariance can be generated in inflationary models that admit one or more periods of departure from the slow roll phase (see, for instance, Refs. [10, 38, 39, 40, 41, 42, 43, 44, 45]). The degree of the deviation from a nearly scale invariant spectrum would be determined by the extent and duration of the departure, which are, in turn, controlled by the parameters of the model. A departure from slow roll affects the evolution of modes that leave the Hubble radius just before the departure. Rather than remaining constant, the curvature perturbations, say, \mathcal{R}_k , corresponding to these modes evolve at super-

Hubble scales, sourced by the intrinsic entropy perturbations of the inflaton field which, typically, exhibit a rapid growth during the fast roll regime [46, 47]. Such an evolution on super-Hubble scales results in dips or bursts of oscillations in the scalar power spectrum. Usually, such a departure is induced by introducing a sharp feature in the potential of the inflaton field, such as a step or a sudden change in the slope [10, 26, 31, 45]. However, this is not necessary, and transitions to fast roll for brief periods can be generated even with smooth and better motivated effective potentials [38, 40, 46, 47].

Our purpose in this paper is to present a simple model of inflation that suppresses the power spectrum on large scales, a feature—as we discussed above—that seems to be necessary to fit the lower power in the quadrupole (and, to some extent, in the deviant power at other lower multipoles such as the octopole and the multipole $\ell = 22$) of the CMB angular power spectrum, using an effective potential of the canonical scalar field *without* introducing any ad hoc sharp feature. We find that the form of the potentials motivated by a class of certain minimal supersymmetric extensions of the standard model provide us with the desired behavior [48, 49, 50, 51]. These large field models allow a period of fast roll sandwiched between two stages of slow roll inflation[‡]. The first phase of slow roll inflation allows us to impose the standard Bunch-Davies initial conditions on the modes which exit the Hubble radius during the subsequent fast roll regime, an epoch due to which the curvature perturbations on the super-Hubble scales are suppressed. The second slow roll phase lasts for about 50-60 e -folds, thereby allowing us to overcome the standard horizon problem associated with the hot big bang model. The advantages of our approach over other single field models mentioned earlier are twofold. Firstly, we do not need to assume any specific pre-inflationary phase. The entire evolution of the inflationary era is described by a single inflaton potential and, therefore, is much simpler. Secondly, the modes which exit the Hubble radius during the fast roll regime are inside the Hubble radius during the first stage of slow roll inflation and, hence, we do not have to impose any special initial conditions on the large scale modes.

This paper is organized as follows. In Sec. 2, we shall review the essential features of the effective inflaton potential that we shall consider, and describe the background dynamics in situations of our interest. In Sec. 3, after an outline of the slow roll ‘expectations’ of the scalar spectrum that can arise in such a background, we shall discuss the spectra that we obtain through numerical integration. In Sec. 4, using the cosmological Boltzmann code CAMB and the Monte Carlo code COSMOMC, we shall compare the power spectra from the models we consider with the recent WMAP 5-year data. Finally, we shall close with Sec. 5, wherein after a brief summary of our results, we shall discuss as to how the results from our model compare with those that have been obtained in another closely related single field model.

In the discussions below, we shall set \hbar and c as well as $M_{\text{pl}} = (8\pi G)^{-1/2}$ to unity.

[‡] Earlier, in the literature, two successive stages of slow roll inflation have often been driven by two scalar fields [52, 53, 54, 55, 56]. Instead, in this paper, we achieve the two stages of slow roll inflation including a brief period of departure from inflation, all with just a single scalar field.

Also, throughout, an overdot and an overprime shall denote differentiation with respect to the cosmic and the conformal times, respectively.

2. The inflaton potential and the background dynamics

The effective potential for the inflation field that we shall consider is described by two parameters m and λ , and is given by

$$V(\phi) = \left(\frac{m^2}{2}\right) \phi^2 - \left(\frac{\sqrt{2\lambda(n-1)}m}{n}\right) \phi^n + \left(\frac{\lambda}{4}\right) \phi^{2(n-1)}, \quad (1)$$

where $n > 2$ is an integer. Such potentials are known to arise in certain minimal supersymmetric extensions of the standard model [48], and their role as an inflaton and its related effects have been studied recently [49, 50, 51]. (We should also hasten to add that the specific case of $n = 3$ has been considered much earlier for reasons similar to ours, viz. producing certain features in the primordial spectrum [38].) In the above potential, the coefficient of the ϕ^n term has been chosen in such a way that the potential has a point of inflection at $\phi = \phi_0$ (i.e. the location where both $V_\phi \equiv (dV/d\phi)$ and $V_{\phi\phi} \equiv (d^2V/d\phi^2)$ vanish), with ϕ_0 given by

$$\phi_0 = \left[\frac{2m^2}{(n-1)\lambda} \right]^{\frac{1}{2(n-2)}}. \quad (2)$$

Near this point of inflection, the potential exhibits a plateau with an extremely small curvature, which, as we shall discuss below, proves to be crucial for the desired evolution of the inflaton field. The potential (1) for the case $n = 3$ is depicted in Fig. 1.

Note that the potential (1) roughly behaves as

$$V(\phi) \sim \begin{cases} \phi^{2(n-1)}, & \text{for } \phi > \phi_0, \\ \phi^2, & \text{for } \phi < \phi_0. \end{cases} \quad (3)$$

Recall that, the first potential slow roll parameter is given by [57, 58]

$$\epsilon_1 = \left(\frac{1}{2}\right) \left(\frac{V_\phi}{V}\right)^2, \quad (4)$$

and inflation ends as ϵ_1 crosses unity. It is then clear that, in a power law potential of the form $V \sim \phi^{2(n-1)}$, slow roll inflation will occur (i.e. $\epsilon_1 \ll 1$) when $\phi \gg 1$, and inflation will end when $\phi_{\text{end}} \simeq [\sqrt{2}(n-1)] \sim \mathcal{O}(1)$. Thus, for a transition from slow roll to fast roll to occur, we need to choose the two parameters in the potential (1) so that $\phi_0 \sim \mathcal{O}(1)$, i.e. of the order of the (reduced) Planck scale. Restarting inflation after the fast roll phase and the number of e -folds that can be achieved during the second phase of slow roll crucially depends on the value of ϕ_0 . We rely on the numerics to choose this parameter carefully since the above slow roll estimate only provides a rough order of magnitude. Choosing ϕ_0 in such a way is actually fine tuning, but it seems to be inevitable if we are to achieve the desired slow-fast-slow roll transition as well as the required number of e -folds. Once the point of inflection has been identified, we find

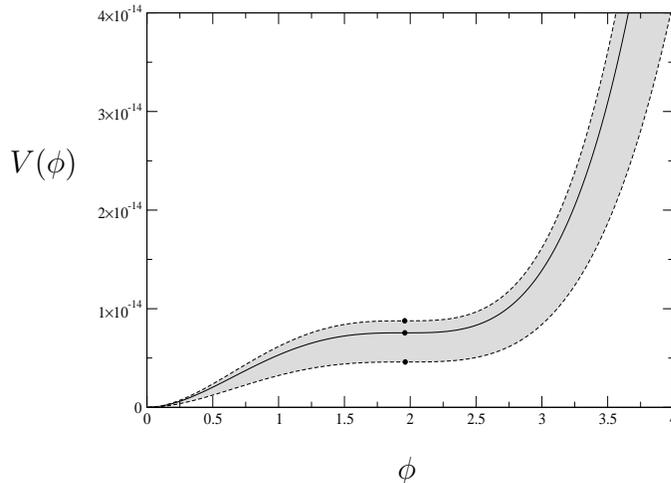


Figure 1. Illustration of the inflaton potential (1) for $n = 3$. The solid line corresponds to the following values for the potential parameters: $m = 1.5368 \times 10^{-7}$ and $\lambda = 6.1517 \times 10^{-15}$ (corresponding to $\phi_0 = 1.9594$), values which turn out to provide the best fit to the WMAP 5-year data (cf. Tab. 4.2). The dashed lines correspond to values that are $1\text{-}\sigma$ away from the best fit ones. The black dots denote the points of inflection.

that the normalization to the CMB angular power spectrum data provides the second constraint, thereby determining the value of the other free parameter m .

The equation of motion governing the scalar field described by the potential (1), when expressed as two first order equations for the coupled variables ϕ and $\dot{\phi}$, has one attractive fixed point located at the origin, i.e. at $(\phi, \dot{\phi}) = (0, 0)$. For positive values of ϕ , we find that there exists an attractor trajectory towards which all other trajectories with arbitrary initial conditions on ϕ and $\dot{\phi}$ quickly converge. For a suitably chosen ϕ_0 , we find that the attractor trajectory exhibits two regimes of slow roll inflation sandwiching a period of fast roll. Hence, if we start the evolution with $\phi \gg \phi_0$, then the initial values of ϕ and $\dot{\phi}$ prove to be irrelevant for the subsequent dynamics as they approach the attractor. This behavior is evident from Fig. 2 where we have plotted the phase portrait for the $n = 3$ case. Once the field reaches close to ϕ_0 , due to the extreme flatness of the potential (1), it relaxes and then moves very slowly, commencing the second stage of the slow roll inflation. This stage ends when the field finally rolls down towards the minima of the potential at $\phi = 0$.

We had mentioned earlier that potentials of the type (1) are encountered in the Minimal Supersymmetric Standard Model (MSSM) [48], and that their role as an inflaton has been analyzed recently [49, 50, 51]. At this point it is important that we highlight the differences between MSSM inflation and the scenario we are considering. In MSSM inflation, the point of inflection is located at sub-Planckian values (i.e. $\phi_0 \ll 1$) which can be an advantage as it avoids the problems associated with having super-Planckian values for the field. In contrast, in our case, as emphasized above, the saddle

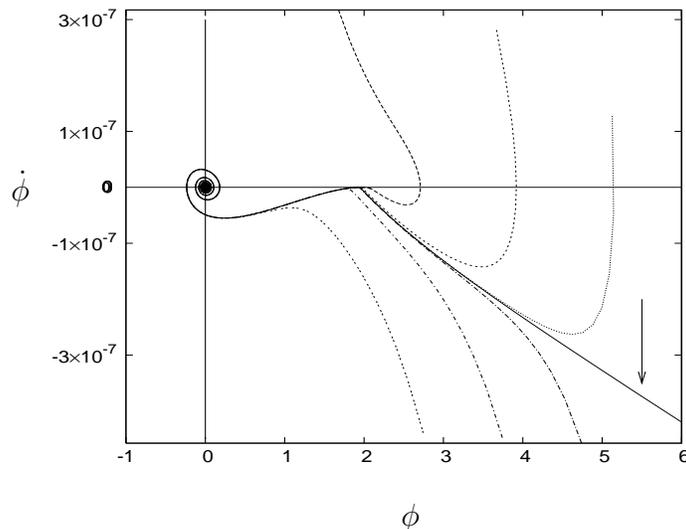


Figure 2. The phase portrait of the scalar field described by the potential (1) in the case of $n = 3$ and for the values of the parameters m and λ mentioned in the last figure. The arrow points to the attractor. Note that, as discussed in the text, all the trajectories quickly approach the attractor. We should mention that, though we have plotted the phase portrait for just the $n = 3$ case, we find that such a behavior is exhibited by higher values of n (such as, for example, $n = 4, 6$) as well.

point should be located around the Planck scale (i.e. $\phi_0 \gtrsim 1$), if we are to achieve the second period of slow roll before the end of inflation. However, in the MSSM case, to have successful inflation, the initial values of ϕ and $\dot{\phi}$ have to be finely tuned so that $\phi_{\text{ini}} \simeq \phi_0$ and $\dot{\phi}_{\text{ini}} \simeq 0$. But, in our scenario, we do not require such fine tuning of the initial conditions on ϕ and $\dot{\phi}$. Instead, we require for the location of ϕ_0 .

Though the parameters of the potential that we work with are different from the MSSM case, we nevertheless believe that it may be possible to realize the potential (1) in theories beyond the standard model, such as, for instance, string theory (in this context, see, for example, Refs. [59, 60]). For example, it is known that the existence of a number of string axion fields can give rise to the following potential describing multi-field chaotic inflation [61]:

$$V(\phi_i) = \sum_i \left(\frac{1}{2} \right) m_i^2 \phi_i^2, \quad (5)$$

with the initial field displacements smaller than unity. The dynamics and the inflationary predictions in such examples are surprisingly similar to the corresponding single field chaotic inflation models [62, 63, 64], due to the assisted inflation mechanism [65]. Similarly, with enough number of fields and with the non-renormalizable superpotential

$$W = \left(\frac{\lambda}{n} \right) \left(\frac{\phi^n}{M_{\text{Pl}}^{n-3}} \right), \quad (6)$$

and the corresponding A term and the soft mass term, one might be able to build an inflation model that is effectively equivalent to the single field one described by the potential (1). (Note that, for clarity, we have temporarily restored M_{pl} in the expression (6) above.)

3. The scalar power spectrum

In this section, after providing general arguments for the form of the scalar spectra that we can expect from our model, we present the spectra evaluated numerically.

3.1. Key equations and essential quantities

Let us begin by quickly summarizing the essential equations and the quantities that we are interested in [57, 58]. The curvature perturbation \mathcal{R}_k satisfies the differential equation

$$\mathcal{R}_k'' + 2 \left(\frac{z'}{z} \right) \mathcal{R}_k' + k^2 \mathcal{R}_k = 0, \quad (7)$$

where the quantity z is given by

$$z = (a \phi' / \mathcal{H}). \quad (8)$$

The quantity a denotes the scale factor, ϕ the background inflaton, and \mathcal{H} is the conformal Hubble factor given by (a'/a) . The scalar power spectrum $\mathcal{P}_s(k)$ is then defined as

$$\mathcal{P}_s(k) = \left(\frac{k^3}{2\pi^2} \right) |\mathcal{R}_k|^2, \quad (9)$$

with the amplitude of the curvature perturbation \mathcal{R}_k evaluated, in general, in the super-Hubble limit. The tensor perturbation \mathcal{U}_k satisfies the equation

$$\mathcal{U}_k'' + 2 \left(\frac{a'}{a} \right) \mathcal{U}_k' + k^2 \mathcal{U}_k = 0, \quad (10)$$

with the tensor power spectrum $\mathcal{P}_T(k)$ being given by

$$\mathcal{P}_T(k) = \left(\frac{k^3}{2\pi^2} \right) |\mathcal{U}_k|^2, \quad (11)$$

where, as in the scalar case, the tensor amplitude \mathcal{U}_k is evaluated at super-Hubble scales. Finally, the tensor-to-scalar ratio r is defined as follows:

$$r \equiv \left(\frac{\mathcal{P}_T}{\mathcal{P}_S} \right). \quad (12)$$

3.2. Physical ‘expectations’

Before we evaluate the scalar spectra numerically, let us broadly try and understand the spectra that we can expect to arise in the slow-fast-slow roll scenario that we are interested in.

3.2.1. The evolution of the scalar modes and the scalar spectrum Consider modes that exit the Hubble scale during an epoch of slow roll inflation. Provided there is no deviation from slow roll soon after the modes leave the Hubble radius, the amplitude of these modes will remain constant at super-Hubble scales. Therefore, their amplitude is determined by their value at Hubble exit, and the scalar power spectrum corresponding to these modes can be expressed in terms of the potential as follows [57, 58]:

$$\mathcal{P}_s(k) \simeq \left(\frac{1}{12\pi^2} \right) \left(\frac{V^3}{V_\phi^2} \right). \quad (13)$$

However, if there is a period of deviation from slow roll inflation, then the asymptotic (i.e. the extreme super-Hubble) amplitude of the modes that leave the Hubble radius *just before* the deviation are enhanced when compared to their value at Hubble exit [46]. While modes that leave well before the deviation remain unaffected, it is found that there exists an intermediate range of modes whose amplitudes are actually *suppressed* at super-Hubble scales [47]. As a result, in the slow-fast-slow roll scenario of our interest, the scalar power spectrum is initially characterized by a sharp dip and a rise corresponding to modes that leave the Hubble radius just before the transition to fast roll. Then arises a regime of nearly scale invariant spectrum corresponding to modes that leave during the second stage of slow roll inflation.

3.2.2. The effects on the tensor modes and the tensor spectrum Let us now understand the behavior of the tensor modes. In the case of the scalar modes, the quantity (z'/z) that appears in the differential equation (7) turns out to be negative during a period of fast roll, and it is this feature that proves to be responsible for the amplification or the suppression of the modes at super-Hubble scales [46, 47]. In contrast, the coefficient of the friction term in the equation (10) that describes the tensor modes, viz. $(2\mathcal{H})$, is a positive definite quantity. Hence, we do not expect any non-trivial super-Hubble evolution of \mathcal{U}_k . We find that, in the models that we consider, the tensor-to-scalar ratio r remains smaller than 10^{-4} over scales of cosmological interest, which is below the levels of possible detection by forthcoming missions such as PLANCK [66].

3.3. Numerical results

It is the background quantity (z'/z) that appears in the differential equation (7) for the curvature perturbation which essentially determines the form of the scalar power spectrum. The quantity $(z'/z\mathcal{H})$ can be expressed in terms of the first two Hubble slow roll parameters, viz. $\epsilon = -(\dot{H}/H^2)$ and $\delta = (\ddot{\phi}/H\dot{\phi})$, with $H = (\dot{a}/a)$ being the standard Hubble parameter. It is given by [46]

$$\left(\frac{z'}{z\mathcal{H}} \right) = (1 + \epsilon + \delta), \quad (14)$$

and it is clear from this expression that, during slow roll inflation (i.e. when $\epsilon \ll 1$ and $\delta \ll 1$), the quantity $(z'/z\mathcal{H})$ will remain close to unity [46, 47]. In Fig. 3, we have

plotted the evolution of $(z'/z\mathcal{H})$ as a function of the number of e -folds N for the cases of $n = 3$ and $n = 4$ in the potential (1). And, in Fig. 4, we have plotted the evolution of the field in the plane of the Hubble slow roll parameters ϵ and δ for the $n = 3$ case. It

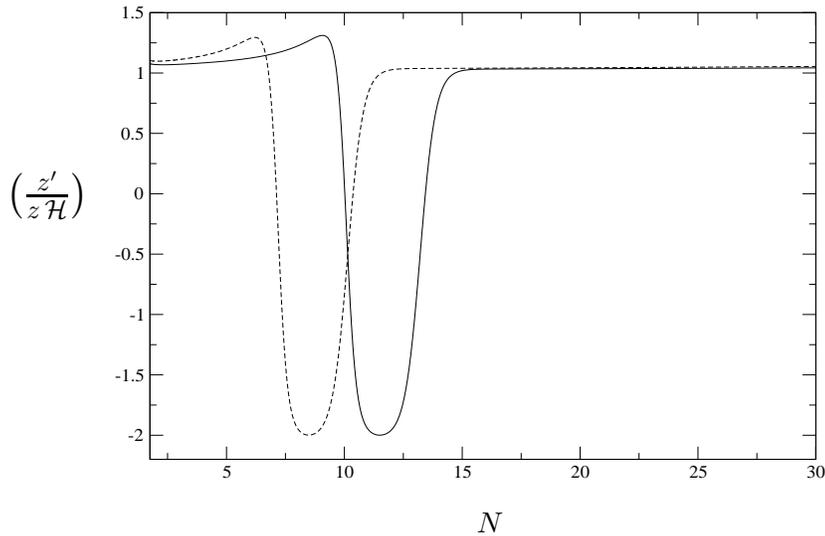


Figure 3. The background quantity $(z'/z\mathcal{H})$ has been plotted as a function of the number of e -folds, say, N , for the cases of $n = 3$ and $n = 4$ in potential (1). The solid line represents the $n = 3$ case with the same values for the potential parameters as in the previous two figures. The dashed line corresponds to the $n = 4$ case with $m = 1.1406 \times 10^{-7}$ and $\lambda = 1.448 \times 10^{-16}$ (corresponding to $\phi_0 = 2.7818$) and, as in the $n = 3$ case, we have chosen these values as they provide the best fit to the WMAP 5-year data. Also, note that we have imposed the following initial conditions for the background field in both the cases: $\phi_{\text{ini}} = 10$ and $\dot{\phi}_{\text{ini}} = 0$. Evidently, the $n = 3$ case departs from slow roll when $7 \lesssim N \lesssim 15$, while the departure occurs during $4 \lesssim N \lesssim 12$ in the case of $n = 4$.

is manifest from these figures that the departure from slow roll occurs roughly between e -folds $7 \lesssim N \lesssim 15$ in the $n = 3$ case and between e -folds $4 \lesssim N \lesssim 12$ for $n = 4$. We should also point out that inflation is actually interrupted for about a e -fold during the fast roll. In Fig. 5, we have plotted the corresponding scalar spectra evaluated numerically. The broad arguments we had presented in the previous subsection are evidently corroborated by these two figures. Note that, in plotting all these figures, we have chosen parameters that eventually provide the best fit to the WMAP 5-year data. Also, in the inset in the top panel of Fig. 5, we have highlighted the difference between the scalar spectra in our model and the power law case (i.e. when $\mathcal{P}_s(k) = A_s k^{n_s - 1}$, with $A_s = 2.1 \times 10^{-9}$ and $n_s \simeq 0.955$). Moreover, we should stress here that the standard sub-Hubble, Bunch-Davies, initial conditions have been imposed on *all* the modes in arriving at these spectra.

The scalar power spectrum with a drop in power at large scales is often approximated by an expression with an exponential cut off of the following form [18,

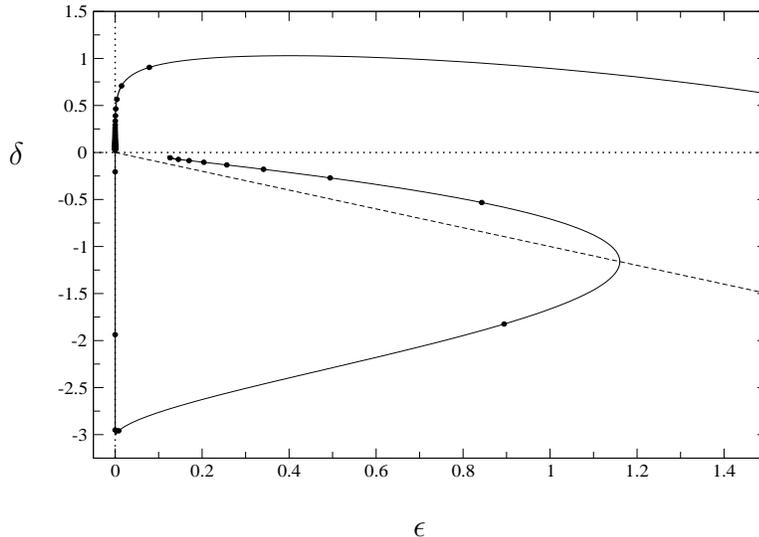


Figure 4. The evolution of the scalar field has been plotted (as the solid black line) in the plane of the first two Hubble slow roll parameters ϵ and δ in the case of $n = 3$ and for the best fit values of the parameters m and λ we have used earlier in Figs. 1 and 2. The black dots have been marked at intervals of one e -fold, while the dashed line corresponds to $\epsilon = -\delta$. Note that $\epsilon > 1$ during $8 < N < 9$. In other words, during fast roll, inflation is actually interrupted for about a e -fold.

19, 25]:

$$\mathcal{P}_s(k) = A_s \left(1 - \exp[-(k/k_*)^\alpha]\right) k^{n_s-1}. \quad (15)$$

In Fig. 5, we have also plotted this expression for values of A_s , n_s , α and k_* that closely approximate the spectra we obtain. It is useful to note that the spectra we obtain correspond to $A_s = 2 \times 10^{-9}$, $n_s \simeq 0.945$, $\alpha = 3.35$ and $k_* = 2.4 \times 10^{-4} \text{ Mpc}^{-1}$ when $n = 3$, while $A_s = 2 \times 10^{-9}$, $n_s \simeq 0.95$, $\alpha = 3.6$ and $k_* = 9.0 \times 10^{-4} \text{ Mpc}^{-1}$ in the $n = 4$ case. We should emphasize here that we have arrived at these values for A_s , n_s , α and k_* by a simple visual comparison of the numerically evaluated result with the above exponentially cut off spectrum.

4. Comparison with the recent WMAP 5-year data

In this section, we shall discuss as to how our model compares with the recent WMAP 5-year data.

4.1. The parameters in our model and the priors we work with

In the standard concordant cosmological model—viz. the Λ CDM model with a power law inflationary perturbation spectrum—six parameters are introduced when comparing the theoretical results with the CMB data (see, for instance, Ref. [67]). Four of them

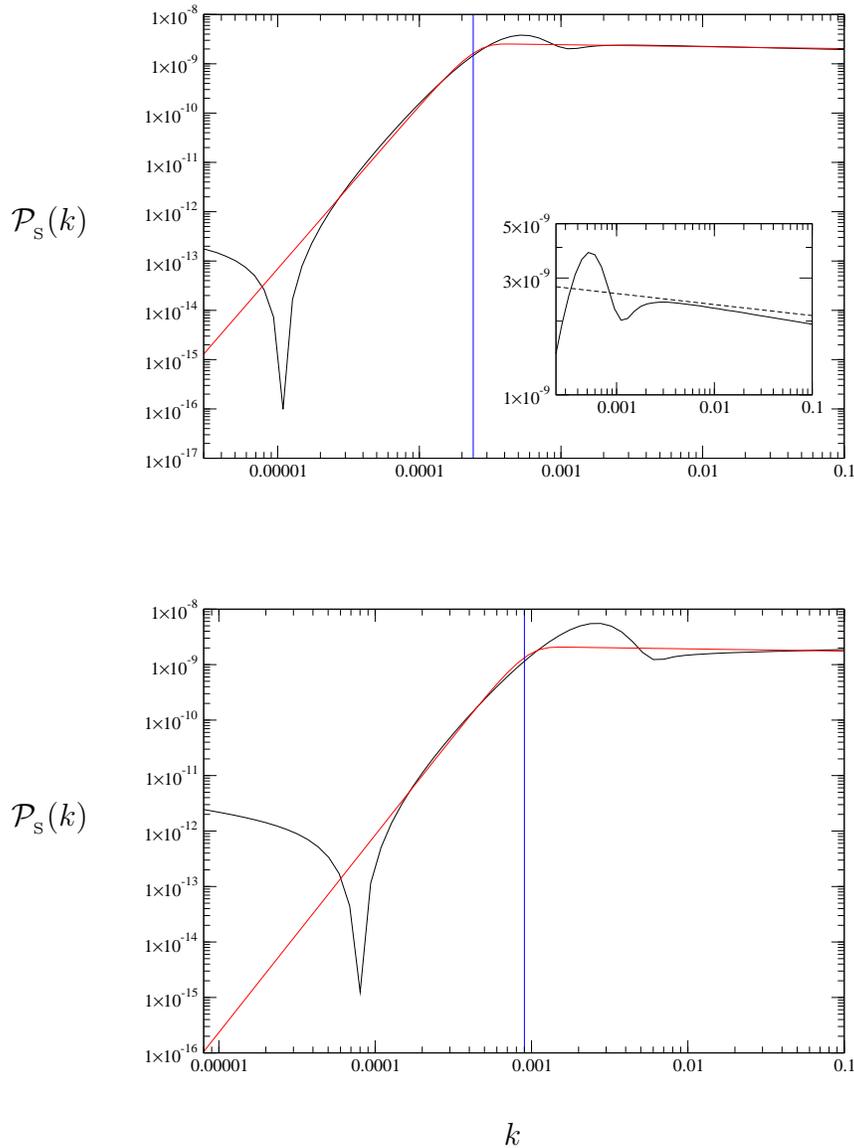


Figure 5. The scalar power spectrum $\mathcal{P}_s(k)$ (the solid black line) have been plotted as a function of the wavenumber k for the cases of $n = 3$ (on top) and $n = 4$ (at the bottom). We have chosen the same values for the potential parameters as in the earlier figures. Moreover, we should emphasize that we have arrived at these spectra by imposing the standard, Bunch-Davies, initial condition on *all* the modes. The red line in these plots is the spectrum (15) with the exponential cut off. It corresponds to $A_s = 2 \times 10^{-9}$, $n_s \simeq 0.945$, $\alpha = 3.35$ and $k_* = 2.4 \times 10^{-4} \text{ Mpc}^{-1}$ in the $n = 3$ case, while $A_s = 2 \times 10^{-9}$, $n_s \simeq 0.95$, $\alpha = 3.6$ and $k_* = 9.0 \times 10^{-4} \text{ Mpc}^{-1}$ in the case of $n = 4$. Note that the vertical blue line denotes k_* . The inset in the top panel illustrates the difference between our model and the standard power law case (i.e. when $\mathcal{P}_s(k) = A_s k^{n_s-1}$, with the best fit values $A_s = 2.1 \times 10^{-9}$ and $n_s \simeq 0.955$) at smaller scales. This disparity leads to a difference in the CMB angular power spectrum at the higher multipoles, which we have highlighted in the inset in Fig. 8.

are the following background parameters: the baryon density ($\Omega_b h^2$), the density of cold dark matter ($\Omega_c h^2$), the angular size of the acoustic horizon θ , and the optical depth τ , with h denoting the Hubble constant today (viz. H_0) expressed in units of $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$. The parameters that are introduced to describe the inflationary perturbation spectrum are the scalar amplitude A_s and the scalar spectral index n_s . The tensor-to-scalar ratio r is also introduced as a parameter provided the ratio is sufficiently large, say, when $r \gtrsim \mathcal{O}(10^{-2})$. However, in the models we consider, the tensor-to-scalar ratio proves to be smaller than 10^{-4} over the scales of cosmological interest. So, we completely ignore the contribution due to the gravitational waves in our analysis. We retain the standard background cosmological parameters, and we introduce the following three parameters to describe the inflationary perturbation spectrum: m , ϕ_0 and a_0 . While m appears explicitly in the potential (1), ϕ_0 has been chosen in place of λ . The quantity a_0 denotes the initial value of the scale factor (i.e. at $N = 0$), and it basically determines the location of the cut-off in the power spectrum. Thus, we have one additional parameter in comparison with the standard case. Essentially, we have traded off the scalar amplitude A_s for m , and the scalar spectral index n_s for ϕ_0 . In Tab. 4.1, we have listed the ranges of uniform priors that we have imposed on the various parameters.

Model	Parameter	Lower limit	Upper limit
Common parameters	$\Omega_b h^2$	0.005	0.1
	$\Omega_c h^2$	0.001	0.99
	θ	0.5	10.0
	τ	0.01	0.8
Reference model	$\log [10^{10} A_s]$	2.7	4.0
	n_s	0.5	1.5
Our model	$\log [10^{10} m^2]$	-9.0	-8.0
	ϕ_0	1.7	2.3
	a_0	0.1	2.0

Table 1. The priors on the various parameters describing the reference Λ CDM model with a power law primordial spectrum and our model. While the first four background cosmological parameters are common for both the models, the fifth and the sixth parameters describe the power law primordial spectrum of the reference model. As discussed in the text, in our model, we have traded off the scalar amplitude A_s for m and the spectral index n_s for ϕ_0 . The additional parameter in our model, viz. a_0 , represents the value of the scale factor at $N = 0$ and it essentially identifies the location of the cut-off in the power spectrum.

4.2. The best fit values and the joint constraints

We have compared the power spectra for the $n = 3$ and the $n = 4$ cases with the recent WMAP 5-year data for the temperature-temperature, the temperature-electric

polarization and the electric-electric polarization angular power spectra of the CMB anisotropies [5]. We have used a modified version of the cosmological Boltzmann code CAMB [68, 69] to calculate the angular power spectra of the CMB anisotropies, with the inflationary perturbation spectrum computed from a separate routine. We have evaluated the likelihood function using the likelihood code that has been made publicly available by the WMAP team [70]. We have obtained the best fit values for the parameters of our model using COSMOMC [71, 72], the publicly available, Markov Chain Monte Carlo (MCMC) code for the parameter estimation of a given cosmological model. The MCMC convergence diagnostics are done on multiple parallel chains using the Gelman and Rubin (“variance of chain means”/“mean of chain variances”) R statistics for each parameter, demanding that $(R-1) < 0.01$, a procedure that essentially looks at the fluctuations amongst the different chains and decides when to terminate the run. We find that while the $n = 3$ case provides a better fit to the data than the reference concordant model [67], the $n = 4$ case leads to such a poor fit to the data that we do not consider it hereafter. We attribute the poor fit by the $n = 4$ case (and also in the cases wherein $n > 4$) to the large bump in the scalar power spectrum that arises just before the spectrum turns scale invariant (cf. Fig. 5). We have plotted the one-dimensional marginalized and mean likelihood curves for the various parameters in the $n = 3$ case in Fig. 6. And, in Fig. 7, we have plotted the corresponding $1\text{-}\sigma$ and $2\text{-}\sigma$ two-dimensional joint constraints on the various parameters. We have listed the best fit values and the $1\text{-}\sigma$ constraints on the various parameters describing the reference model and the $n = 3$ case in Tab. 4.2. We find that the $n = 3$ case provides a much better fit to the data than the reference model with an improvement in χ_{eff}^2 of 6.62. It is clear from

Parameter	Reference model	Our model
$\Omega_b h^2$	$0.02242^{+0.00155}_{-0.00127}$	$0.02146^{+0.00142}_{-0.00108}$
$\Omega_c h^2$	$0.1075^{+0.0169}_{-0.0126}$	$0.12051^{+0.02311}_{-0.02387}$
θ	$1.0395^{+0.0075}_{-0.0076}$	$1.03877^{+0.00979}_{-0.00931}$
τ	$0.08695^{+0.04375}_{-0.03923}$	$0.07220^{+0.04264}_{-0.02201}$
$\log [10^{10} A_s]$	$3.0456^{+0.1093}_{-0.1073}$	—
n_s	$0.9555^{+0.0394}_{-0.0305}$	—
$\log [10^{10} m^2]$	—	$-8.3509^{+0.1509}_{-0.1473}$
ϕ_0	—	$1.9594^{+0.00290}_{-0.00096}$
a_0	—	$0.31439^{+0.02599}_{-0.02105}$

Table 2. The mean values and the $1\text{-}\sigma$ constraints on the various parameters that describe the reference model and our model. As we mentioned in the text, we find that the $n = 3$ case provides a much better fit to the data than the reference model with an improvement in χ_{eff}^2 of 6.62.

Figs. 6 and 7 that the constraint on the parameter m is prior dominated. In our model, it is the parameter m that determines the amplitude of the power spectrum when it is nearly scale invariant. This amplitude, in turn, is essentially determined by the first

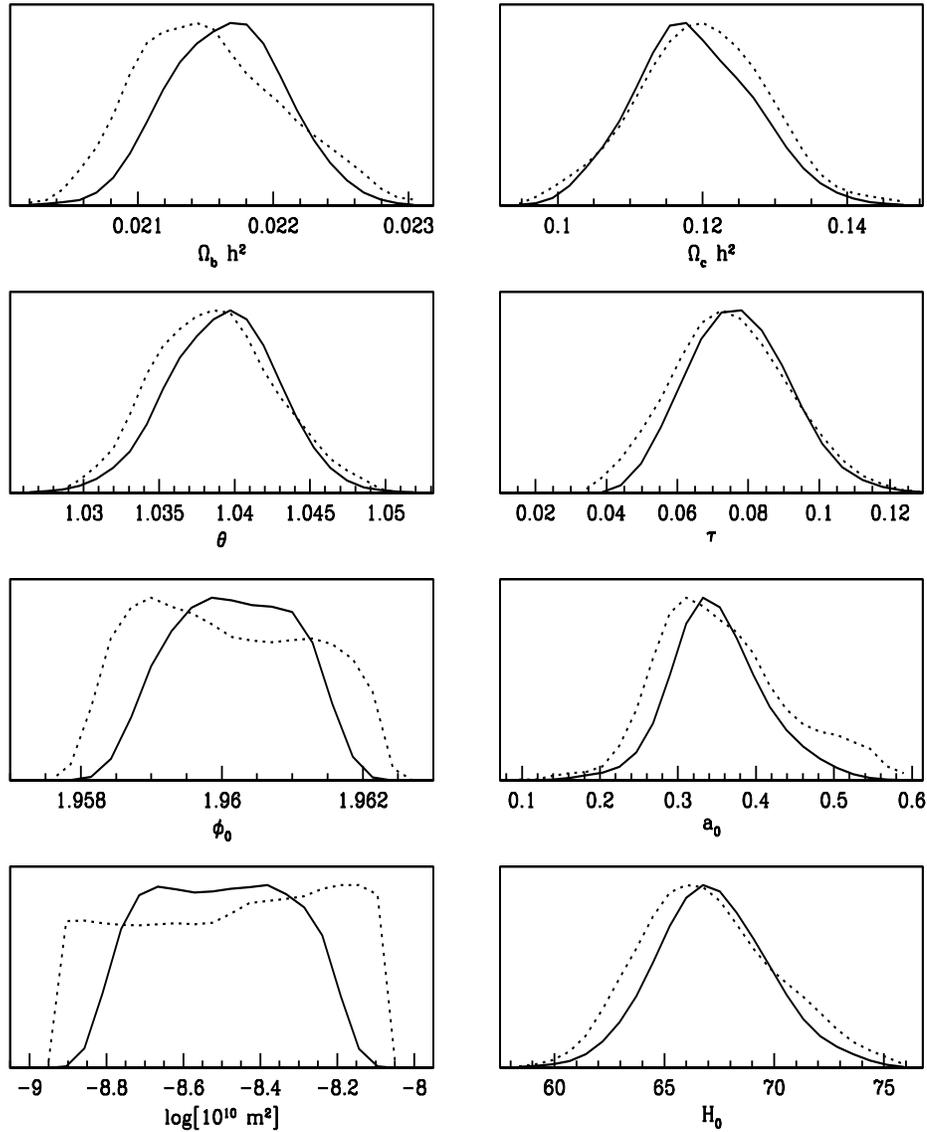


Figure 6. The one-dimensional mean (the solid lines) and marginalized (dashed lines) likelihood curves for all the input parameters (and the derived parameter H_0) in the $n = 3$ case.

peak of the CMB angular power spectrum. We should mention here that our choice of priors for the parameter m has been arrived at by a simple visual fit to the first peak.

4.3. The CMB angular power spectra for the best fit values

In Fig. 8, we have plotted the angular power spectrum of the CMB temperature anisotropies for the best fit values of the parameters for the $n = 3$ case. For comparison, we have also plotted the angular power spectrum for the best fit reference model. It is immediately obvious from the figure that our model fits the lower multipoles much better

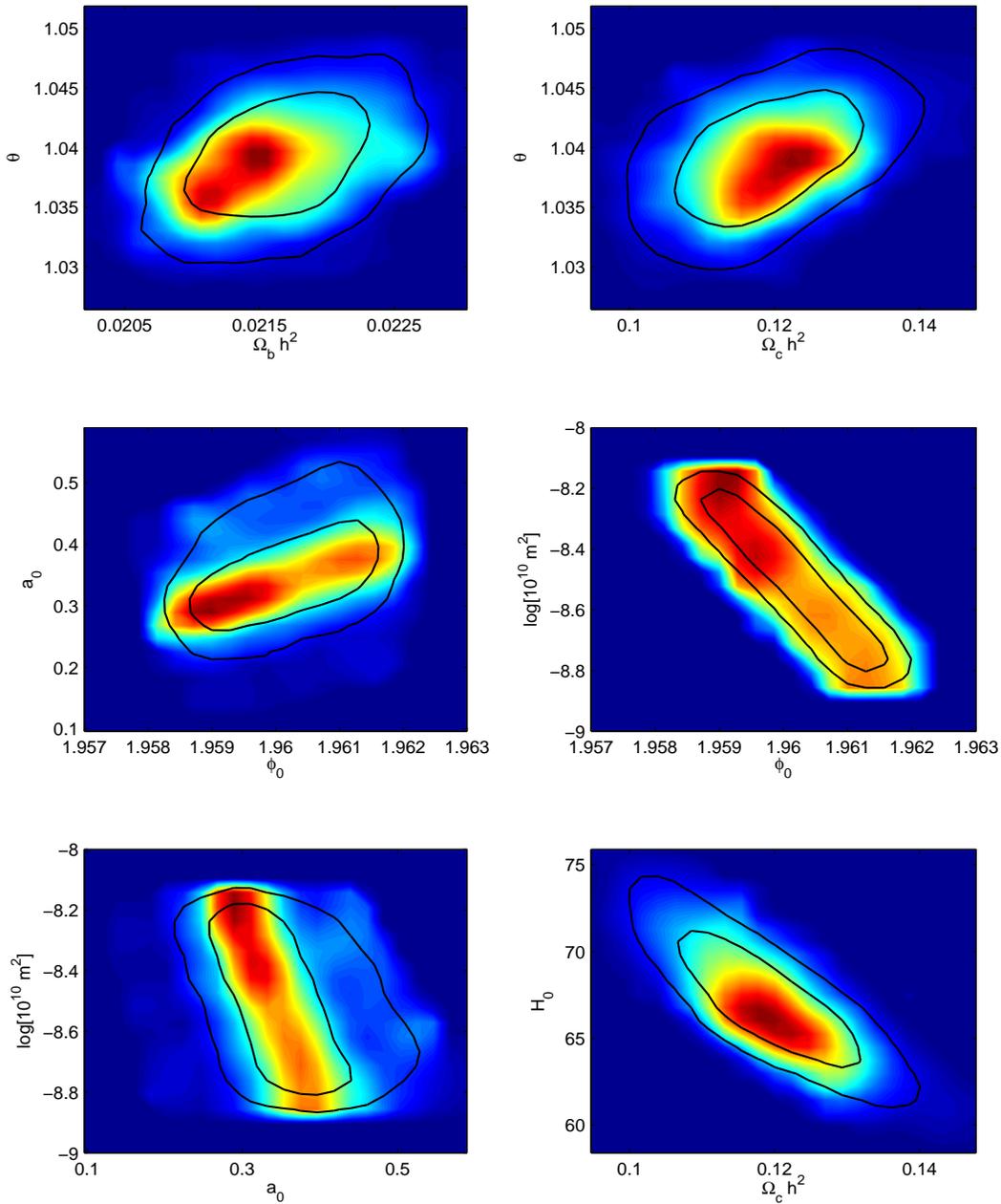


Figure 7. The $1\text{-}\sigma$ and $2\text{-}\sigma$ two-dimensional joint constraints on the different input parameters (and the derived parameter H_0) in the $n = 3$ case.

than the reference model. As we have mentioned above, we obtain an improvement in χ_{eff}^2 of 6.62 at the cost of introducing just one additional parameter when compared to the standard power law case. We should also emphasize here that the improvement in the fit that we have achieved is not only due to the cut-off in the scalar power spectrum, but also because of the presence of the oscillations at the top of the spectrum, just before it turns scale invariant. Also, note the difference in the angular power spectrum for our model and the standard power law spectrum at the higher multipoles, which we have

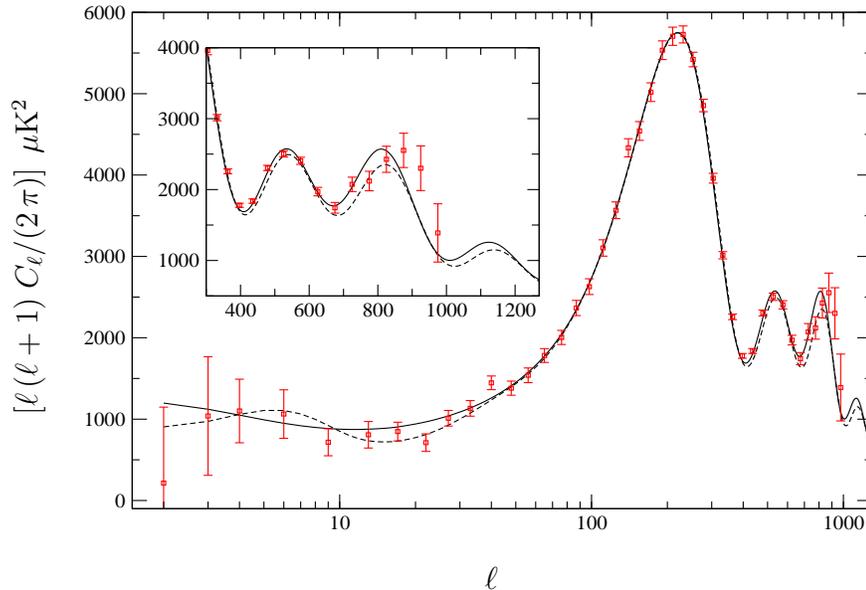


Figure 8. The CMB angular power spectrum for the best fit values of the $n = 3$ case (dashed line) and the best fit power law, reference model (solid line) (cf. Tab. 4.2). Visually, it is evident that our model fits the data much better than the standard power law case at the lower multipoles. The inset highlights the difference between our model and the power law spectrum at the higher multipoles. This difference arises due to the fact that, while the spectral index in the power law case is about $n_s \simeq 0.955$, the asymptotic spectral index in our case turns out to be $n_s \simeq 0.945$.

illustrated in the inset in Fig. 8. This disparity essentially arises due to the difference in the asymptotic spectral index in our model (which proves to be about $n_s \simeq 0.945$) and the spectral index in the power law case (which is about $n_s \simeq 0.955$, cf. Tab. 4.2). The PLANCK mission [66] is expected to provide more accurate data at these higher multipoles and, therefore, may aid us discriminate between these models better.

5. Summary and discussion

In this section, after a quick summary of our results, we compare the results we obtain with those obtained in another single scalar field model that has been considered earlier. We emphasize the fact that the difference between these models has immediate observational consequences.

5.1. Summary

In this work, we have investigated a two stage slow roll inflationary scenario sandwiching an intermediate period of deviation from inflation, driven by potentials that are similar in shape to certain MSSM potentials [49]. In the MSSM case, inflation occurs when the field values are much smaller than the Planck scale [49, 50, 51]. However, in our

case, since we demand two epochs of slow roll, it necessarily requires that the initial values of the field (assuming, say, about 60 e -folds of inflation) be greater than M_{Pl} . The period of fast roll produces a sharp drop in the scalar power spectrum for the modes that leave the Hubble radius just before the second slow roll phase. We choose our scales such that the drop in power corresponds to the largest cosmological scales observable today. We find that the resulting scalar power spectrum provides a much better fit to the recent WMAP data than the canonical, nearly scale invariant, power law, primordial spectrum.

5.2. Discussion

At this stage, it is important that we compare our results with those obtained in another single field model that has been studied before. As we had mentioned in the introduction, an initial kinetic dominated (i.e. fast roll) stage preceding slow roll inflation, driven by a quadratic potential has been considered earlier to provide a sharp drop in the scalar power spectrum at large scales [18, 30]. At first glance, one may be tempted to conclude that the model we have studied here is equivalent to such a scenario if we disregard the first slow roll stage, since we have a kinetic dominated phase preceding a period of slow roll inflation. However, there are crucial differences between the two models which we have outlined below.

To begin with, in the model we consider, there is no freedom to choose the type of fast roll (say, the equation of state during the epoch of fast roll). It is fixed once we have chosen the parameters so as to fit the observations. Secondly, in the scenario considered earlier, the modes which are outside the Hubble radius during the kinetic dominated phase *would have always remained so* in the past [18, 30]. The authors assume that somehow there may have been a previous phase of inflation, during which they were inside the Hubble radius and began life in the Bunch-Davies vacuum. While it is not impossible to think of situations where there may have been a previous inflationary epoch—for instance, it can be achieved by invoking another scalar field [54, 55, 56]—the consequences can be quite different. In contrast, in the scenario that we have considered here, the standard, sub-Hubble, Bunch-Davies, initial conditions have been imposed on *all* the modes. Thirdly, it was argued that, since the suppression of power for the scalar spectrum proves to be sharper than that of the tensor, the tensor-to-scalar ratio r displays a sharp rise towards large physical scales [30], a feature that may possibly be detected by upcoming missions such as, for instance, PLANCK [66]. However, in the models that we consider, the tensor amplitude on scales of cosmological interest proves to be too small ($r < 10^{-4}$) to be detectable in the very near future. In conclusion, we would like to mention that a detection of the C_ℓ^{BB} modes corresponding to, say, $r > 10^{-4}$, can rule out the class of models that we have considered in this work.

Acknowledgments

RKJ and LS would like to thank the Inter-University Centre for Astronomy and Astrophysics (IUCAA), Pune, India, and the Korea Institute for Advanced Study (KIAS), Seoul, Korea, for hospitality, where part of this work was carried out. We would also like to acknowledge the use of the high performance computing facilities at the Harish-Chandra Research Institute, Allahabad, India, as well as at IUCAA and KIAS. We wish to thank Anupam Mazumdar, Biswarup Mukhopadhyaya and Arman Shafieloo for discussions. We also wish to thank Hiranya Peiris and Alexei Starobinsky for valuable comments on our results. We would also like to thank Tuhin Ghosh, Minu Joy, Juhan Kim, Subharthi Ray and Arman Shafieloo for help with the numerical codes at various stages. JG is partly supported by the Korea Research Foundation Grant KRF-2007-357-C00014 funded by the Korean Government. Finally, we acknowledge the use of the COSMOMC package [72] and the data products provided by the WMAP science team [70].

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