written as

$$B_n = \begin{bmatrix} 1 & -\alpha & & & 0 \\ -\alpha & 1+\alpha & \ddots & & \\ & \ddots & \ddots & \ddots & \\ 0 & & \ddots & 1+\alpha & -\alpha \\ & & & -\alpha & 1 \end{bmatrix}. \quad (A.6)$$

Applying an orthogonal transformation

$$x = PYQ^T$$

where P and Q are two orthogonal square matrices of order m and n, respectively, to the matrix equation (A.4) gives

$$\ddot{Y} + (P^T B_m P)Y + Y(Q^T B_n Q) = \epsilon \dot{Y} - (1/3)\epsilon P^T \dot{Z} Q \quad (A.7)$$

which is the fundamental mode equation.

The matrices P and Q which diagonalize matrix B can be determined as an eigenvalue problem of matrices by solving the difference equation

$$-\alpha p_{(i-1)j} + (1+\alpha)p_{ij} - \alpha p_{(i+1)j} = {}_m\lambda_j p_{ij} \quad (A.8)$$

with boundary conditions $p_{0j} = p_{mj}$; $p_{1j} = P_{(m+1)j}$ where p_{ij} is an element of P and ${}_m\lambda_j$ is the eigenvalue of B_m. Similarly, the eigenvalues ${}_n\lambda_i$ and q_{ij} element can be determined for the B_n matrix.

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On Arbitrary-Level IIR and FIR Filters

R. D. KOILPILLAI, P. P. VAIDYANATHAN, AND S. K. MITRA

***Abstract*—A recently published method for designing IIR digital filters with multi-level magnitude response is re-interpreted from a different viewpoint. Based on this interpretation, techniques for extending these results to the case of FIR filters are then developed.**

I. Review of the IIR Case

Ansari [1] has described a procedure of designing multi-level IIR digital filters, i.e., filters whose magnitude response is piecewise constant with the levels at arbitrary values. This procedure extends the well-known IIR filter design techniques, which are for designing filters with the standard two-level response (i.e., the normalized passband response around unity and the stopband response around zero). The results in [1] enable us to design and implement filters such that we have the advantage of having a tuning parameter by which we can generate a family of filters (with the same transition edges) with desired response at arbitrary levels.

Let $[H(z), G(z)]$ be a doubly complementary pair of transfer functions, i.e., a pair satisfying

$$|H(e^{j\omega})|^2 + |G(e^{j\omega})|^2 = 1 \qquad \forall\omega \quad (1)$$

and

$$H(z) + G(z) = A_0(z) \quad (2)$$

where $A_0(z)$ is stable allpass. Then, the following equation is also automatically satisfied:

$$H(z) - G(z) = A_1(z) \quad (3)$$

where $A_1(z)$ is also stable allpass. In matrix form, we can express these as (Fig. 1)

$$\begin{bmatrix} H(z) \\ G(z) \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} A_0(z) \\ A_1(z) \end{bmatrix} = \boldsymbol{S}\boldsymbol{a}(z) \quad (4)$$

where

$$\boldsymbol{S} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

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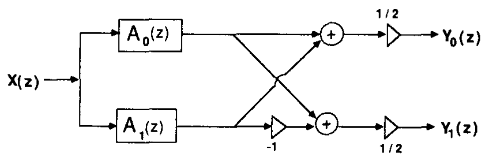

Fig. 1. Realizing the doubly complementary pair.

and

$$a(z)=\frac{1}{\sqrt{2}}\begin{bmatrix}A_0(z)\\A_1(z)\end{bmatrix}.$$

We know from [2] and [3] the conditions under which a digital IIR transfer function can be implemented as a sum of two allpass functions. Assuming these conditions are satisfied, we will now study how this result can be modified to design arbitrary level IIR filters. Let $H(z)$ be a conventional low-pass filter. Then $G(z)$, satisfying (1), is the power-complementary high-pass filter. On the unit circle, we can write (4) as

$$H(e^{j\omega})=\frac{1}{2}\left[A_0(e^{j\omega})+A_1(e^{j\omega})\right]. \quad (5)$$

Since $A_0(z)$ and $A_1(z)$ are allpass functions, they can be expressed as $A_0(e^{j\omega})=e^{j\phi_0(\omega)}$ and $A_1(e^{j\omega})=e^{j\phi_1(\omega)}$. Substituting these in (5), we get

$$|H(e^{j\omega})|^2=\frac{1}{2}\left[1+\cos(\phi_0(\omega)-\phi_1(\omega))\right]. \quad (6)$$

In the passband, the magnitude response $|H(e^{j\omega})|$ is close to unity. This is possible if and only if the two allpass functions are approximately in phase, i.e., $\phi_0(\omega)=\phi_1(\omega)$. In the stopband of $H(z)$, the magnitude response is close to zero. This implies that the allpass functions are approximately 180 degrees out of phase, i.e., $\phi_0(\omega)-\phi_1(\omega)=\pi$ (or any odd multiple of π). In other words,

$$|H|_{\max}\approx 1,\quad 0\leqslant\omega\leqslant\omega_p\ \textbf{(region 1)},\ \phi_0(\omega)=\phi_1(\omega)$$

$$|H|_{\min}\approx 0,\quad \omega_s\leqslant\omega\leqslant\pi\ \textbf{(region 2)},\ \phi_0(\omega)-\phi_1(\omega)=\pi.$$

In (4) we note that the scaling matrix $\boldsymbol{S}$ is orthogonal, and this yields the transfer functions $H(z)$ and $G(z)$ with the standard two level response. The orthogonality of $\boldsymbol{S}$ ensures power complementarity of $H(z)$ and $G(z)$. The generalized filters in [1] can be obtained by replacing $\boldsymbol{S}$ with the most general form of a 2×2 orthogonal matrix, viz., a planar rotation.

$$\boldsymbol{R}=\begin{bmatrix}\cos\theta & \sin\theta\\ \sin\theta & -\cos\theta\end{bmatrix}. \quad (7)$$

Let the new filters obtained be $H'(z)$ and $G'(z)$, i.e.,

$$\begin{bmatrix}H'(z)\\G'(z)\end{bmatrix}=\frac{1}{\sqrt{2}}\begin{bmatrix}\cos\theta & \sin\theta\\ \sin\theta & -\cos\theta\end{bmatrix}\begin{bmatrix}A_0(z)\\A_1(z)\end{bmatrix}. \quad (8)$$

Clearly, (4) is a special case of (8) with $\theta=45°$. We now study the response of the new filters.

$$H'(z)=\frac{1}{\sqrt{2}}\left[\cos(\theta)A_0(z)+\sin(\theta)A_1(z)\right] \quad (9)$$

In **region 1** where the allpass functions are in phase,

$$|H'(e^{j\omega})|\approx\frac{1}{\sqrt{2}}|\cos\theta+\sin\theta|=p. \quad (10)$$

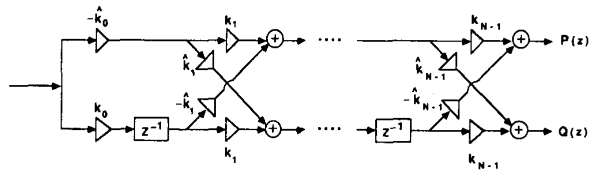

Fig. 2. Cascaded lattice structure.

In **region 2** where the allpass functions are 180° out of phase,

$$|H'(e^{j\omega})|\approx\frac{1}{\sqrt{2}}|\cos\theta-\sin\theta|=q. \quad (11)$$

This shows how we can obtain a filter $H'(z)$ with passband response level p and stopband response level q. The levels of the response can be varied by changing θ. Only one of the two levels p,q can be arbitrarily chosen. The other level is then fixed since they satisfy the relation

$$p^2+q^2=1. \quad (12)$$

The complementary response $G'(z)$ clearly satisfies

$$|G'(e^{j\omega})|=\begin{cases}q & \text{in region 1}\\ p & \text{in region 2.}\end{cases}$$

A typical design of such filters would proceed as follows: Starting with an odd-order elliptic filter $H(z)$, we find the two allpass functions $A_0(z)$ and $A_1(z)$ using the technique in [3]. We then use the parameter θ as in (8) to obtain the arbitrary-level responses. The detailed shapes of the passband ripples and stopband ripples of $H'(z)$ are the same as those of the passband ripples of $H(z)$ and $G(z)$, respectively. The parameter θ changes only the average levels. An advantage of this implementation is that since the scaling matrix $\boldsymbol{R}$ is orthogonal, the power-complementarity and the low-sensitivity properties are preserved, as required in many applications [3], [7], [8].

II. FIR Case

In the FIR case, we can design filters with desired magnitude response at arbitrary levels by using the well-known techniques, such as [9]. A direct approach like this would require us to re-design the filter if different response levels are required. As shown in [1], for the particular case of symmetric even-order linear-phase filters, we can generate any arbitrary level filter from the same prototype. This method, though simple, elegant, and attractive, works only for the restricted case of even-order linear-phase FIR filters. Moreover, if we are interested in obtaining a *pair* of power complementary transfer functions (as is required in filter-bank applications [4], [5], [8]), this method does not retain the power complementary property while the levels are being adjusted in this manner.

In this section we introduce a technique for adjustable FIR filter design which works for filters of arbitrary order and arbitrary phase response, and in addition gives rise to a pair of transfer functions which remain power complementary inspite of the level adjustment. As in [1], there is again no need to redesign the entire filter in order to obtain the adjustable levels. In the IIR case, we had a power-complementary pair of transfer functions, namely, $H(z)$ and $G(z)$. After implementing them in a structurally power-complementary way, we then added a generalized orthogonal matrix (a 2×2 rotation matrix) and obtained the necessary shifting and scaling of the magnitude response. In the FIR case, the lossless-lattice implementation of filters [4] is the only known way of structurally enforcing the power complementary property. From [4], we know that any bounded-real (BR)

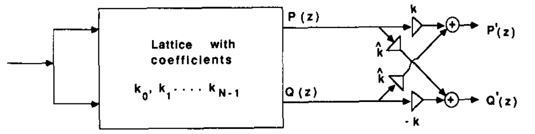

Fig. 3. Lattice structure with scaling section ($k = \cos\phi$, $\hat{k} = \sin\phi$).

FIR transfer function can be implemented as a passive cascaded lattice structure. So we will focus on achieving arbitrary level FIR filters with lattice implementations.

Let $P(z)$ be a FIRBR transfer function and let $Q(z)$ be such that $|P(e^{j\omega})|^2 + |Q(e^{j\omega})|^2 = 1$. Then the FIR allpass vector $\boldsymbol{R}(z) = [P(z) \quad Q(z)]^T$ can be implemented as a lattice as in Fig. 2 (see [4] for details). Then we add a scaling section to the lattice structure, as shown in Fig. 3. The new filters can be written as

$$\begin{bmatrix} P'(z) \\ Q'(z) \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi \\ \sin\phi & -\cos\phi \end{bmatrix} \begin{bmatrix} P(z) \\ Q(z) \end{bmatrix}. \tag{13}$$

In the implementations, we use the multipliers k and $\hat{k}$ for the scaling section ($k = \cos\phi$, $\hat{k} = \sin\phi$). Assume the $P(z)$ and $Q(z)$ are power complementary filters with standard two-level response. So the passband(s) of $P(z)$ coincides with the stopband(s) of $Q(z)$ and vice versa. Hence,

$$|P'(e^{j\omega})| \approx \begin{cases} |\cos\phi| \text{ in passband of } P(z) \\ |\sin\phi| \text{ in stopband of } P(z). \end{cases} \tag{14}$$

Similarly,

$$|Q'(e^{j\omega})| \approx \begin{cases} |\sin\phi| \text{ in passband of } P(z) \\ |\cos\phi| \text{ in stopband of } P(z). \end{cases} \tag{15}$$

Thus, the arbitrary levels are $|\cos\phi|$ and $|\sin\phi|$. In summary, the structure of Fig. 3 ensures that $[P'(z), Q'(z)]$ is a power complementary pair while at the same time achieving arbitrary specification levels. The levels can clearly be controlled by changing k and $\hat{k}$. We note that we can replace the normalized four-multiplier scaling section with a denormalized, two-multiplier scaling section such that we have a single tuning parameter $\alpha = \hat{k}/k$.

Design Example I: In this example, $P(z)$ is a linear-phase low-pass equiripple FIR filter designed with the McClellan–Parks algorithm to meet the following specifications: passband edge $\omega_p = 0.196\pi$, stopband edge $\omega_s = 0.27\pi$, passband peak ripple $\delta_1 \leqslant 0.00042$, and stopband attenuation $A_s \geqslant 27$ dB. The required filter order turns out to be $N - 1 = 60$. Once the BR transfer function $P(z)$ is obtained, the power complementary filter $Q(z)$ can be obtained by solving for a spectral factor of $[1 - |P(e^{j\omega})|^2]$. This is done using the procedure outlined by Mian and Nainer [6]. Fig. 4(a) shows the magnitude responses of the filters $P(z)$ and $Q(z)$. Fig. 4(b) shows the frequency response of the filters $P'(z)$ and $Q'(z)$ with $\phi = \pi/6$. The results of (14) and (15) are readily verified. We notice that there are peaks in the transition band, i.e., the response in the transition band exceeds the passband level. This is a disadvantage even though the transition bands may be treated as "*don't care*" bands. The reason why this occurs in the FIR case is that, in the lattice implementation, the magnitude response of the filter is bounded by unity and not by the response levels p (or q, whichever is larger). In contrast, in the IIR case (see (8)), the responses $|H'(e^{j\omega})|$ and $|G'(e^{j\omega})|$ are bounded above by their respective passband levels, so that the transition band responses are less than the passband levels.

We now discuss another approach to design arbitrary level FIR filters, having an even closer analogy to the IIR case. Assume that we have implemented an FIR power-complementary pair $\boldsymbol{R}(z) = [P(z) \quad Q(z)]^T$ as a lattice [4]. We then add an orthogonal scaling section with $\phi = \pi/4$ and obtain a new FIR allpass vector $\boldsymbol{R}'(z) = [A(z) \quad B(z)]^T$. Then

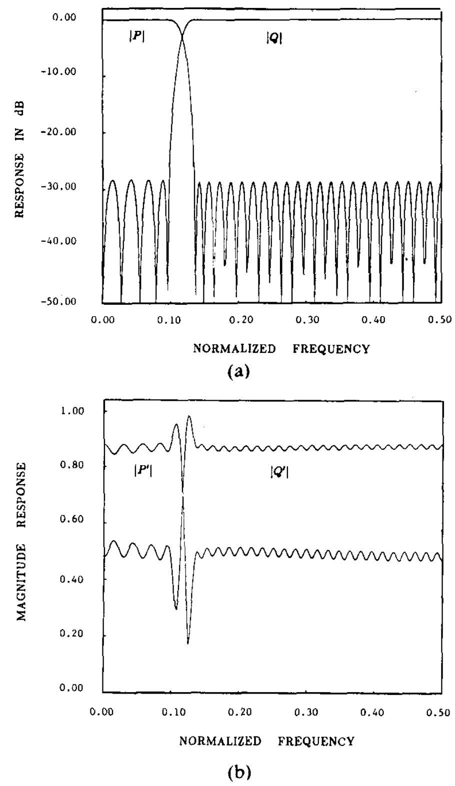

Fig. 4. (a) Magnitude response of P and Q. (b) Magnitude response of P' and Q' (with $\phi = \pi/6$).

$$\begin{bmatrix} A(z) \\ B(z) \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} P(z) \\ Q(z) \end{bmatrix}. \tag{16}$$

Hence,

$$\begin{bmatrix} P(z) \\ Q(z) \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} A(z) \\ B(z) \end{bmatrix} \tag{17}$$

and

$$P(z) = \frac{1}{\sqrt{2}} [A(z) + B(z)].$$

We know that $|A(e^{j\omega})|^2 + |B(e^{j\omega})|^2 = 1$ because $[P(z), Q(z)]$ is a power complementary pair. It can be shown that in the passband of $P(z)$, where $|P(e^{j\omega})| \approx 1$, we will have $|A(e^{j\omega})| \approx |B(e^{j\omega})| \approx 1/\sqrt{2}$ and moreover the phases of $A(e^{j\omega})$ and $B(e^{j\omega})$ are approximately aligned. The stopbands of $P(z)$ correspond to the passbands of $Q(z)$ where $|Q(e^{j\omega})| \approx 1$. In this region we have $|A(e^{j\omega})| \approx |B(e^{j\omega})| \approx 1/\sqrt{2}$ and the phases of $A(e^{j\omega})$ and $B(e^{j\omega})$ differ approximately by π.

We can thus conclude that $A(z)$ and $B(z)$ are approximate FIR allpass functions whose phases align in the passband of $P(z)$ and differ by π in the stopband of $P(z)$. We have now created a situation similar to the IIR case, where the implementation was done with two allpass functions. In order to obtain arbitrary level filters, we use the 2×2 rotation matrix and get the

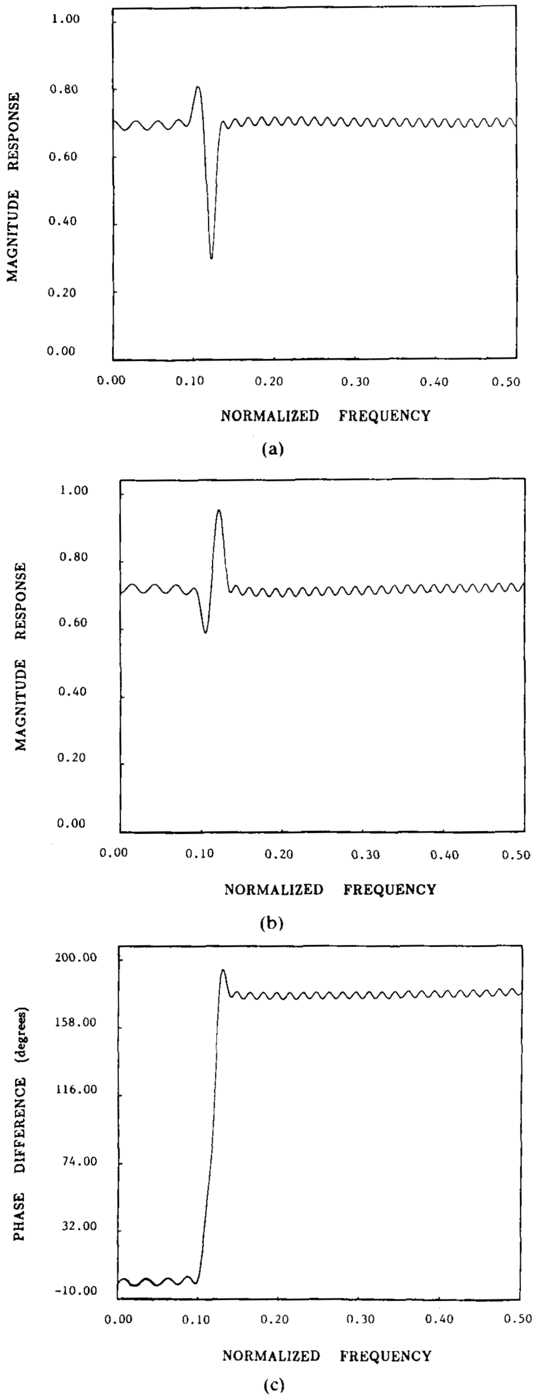

Fig. 5. (a) Magnitude response of A. (b) Magnitude response of B. (c) Phase difference $\arg[A(e^{j\omega})]-\arg[B(e^{j\omega})]$.

filters $P'(z)$ and $Q'(z)$.

$$\begin{bmatrix} P'(z) \\ Q'(z) \end{bmatrix} = \begin{bmatrix} \cos\psi & \sin\psi \\ \sin\psi & -\cos\psi \end{bmatrix} \begin{bmatrix} A(z) \\ B(z) \end{bmatrix}. \tag{18}$$

The levels of the magnitude response p and q are obtained just as in the IIR case, viz., $p = 1\sqrt{2}\,|\cos\psi+\sin\psi|$ and $q = 1\sqrt{2}\,|\sin\psi - \cos\psi|$. By varying ψ (i.e., by varying $k=\cos\psi$ and $\hat{k}=\sin\psi$ in the implementation), we can generate an entire family of arbitrary level filters.

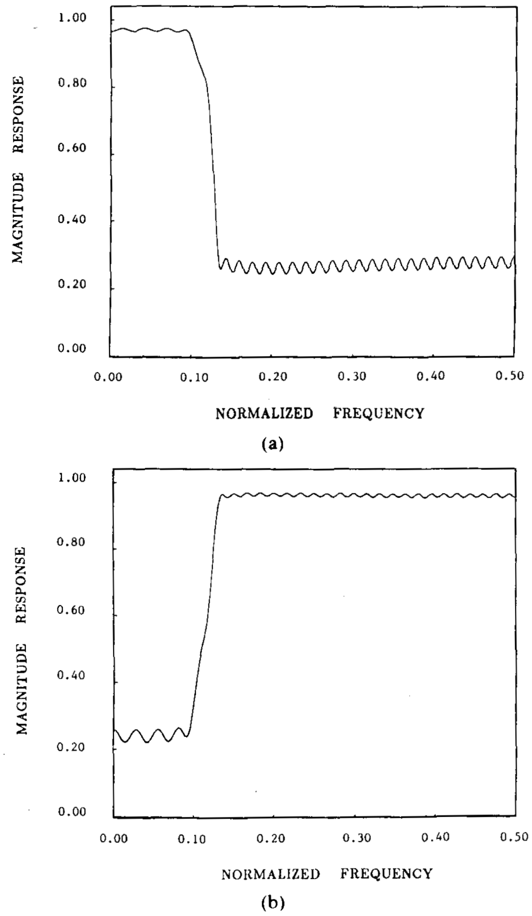

Fig. 6. (a) Magnitude response of P'. (b) Magnitude response of Q'.

Design Example 2: We continue with the filters $P(z)$ and $Q(z)$ designed in Example 1. By choosing $\phi = \pi/4$, we first generate the auxilliary approximate allpass functions $A(z)$ and $B(z)$. A plot of their magnitude responses is shown in Fig. 5(a) and (b), and their phase difference is shown in Fig. 5(c). It can be verified that the phases are approximately equal in the passband of P (stopband of Q) and they differ by π in the stopband of P (passband of Q). In Fig. 6(a) and (b) we have the magnitude responses of the shifted level filters P' and Q', (with $\psi = \pi/6$, $p \approx 0.966$, and $q \approx 0.259$).

III. Concluding Remarks

We have shown how to design arbitrary-level FIR filters. An advantage of our method is that when we implement the arbitrary-level filter, we simultaneously obtain its power-complementary filter, which may be required in specific applications. Also by means of a tuning factor (a parameter of the scaling matrix) we can generate a whole family of arbitrary-level filters. In the IIR case, the allpass filters can be implemented in a *structurally lossless* manner. As a result, for a fixed value of θ in (8), the magnitude responses $|H'(e^{j\omega})|$ and $|G'(e^{j\omega})|$ are structurally bounded above by p (or q, whichever is larger). In other words, for a given value of θ (i.e., for given p and q), any perturbation of the parameters internal to the structure implementing $A_0(z)$ or $A_1(z)$ does not increase the passband level of $H'(z)$. In the FIR case, however, this situation is not true because the functions $A(z)$ and $B(z)$ are not *structurally allpass* (however assuming that $P(z)$ and $Q(z)$ are designed to have "good" passbands and stopbands, this is approximately so). The size of the ripples in the arbitrary-level filters depend on the ripples of $A(z)$ and $B(z)$, which in turn depend on the stopband attenuations of the original filter and its power complementary filter. It must be noted that we have the freedom to choose only one of the two magnitude levels and that the accuracy of the spectral factorization technique directly affects the design.

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Reduced Order Strip Kalman Filtering Using Singular Perturbation Method

M. R. AZIMI-SADJADI AND K. KHORASANI

Abstract—**Strip Kalman filtering for restoration of images degraded by linear shift invariant (LSI) blur and additive white Gaussian (WG) noise is considered. The image process is modeled by a 1-D vector autoregressive (AR) model in each strip. It is shown that the composite dynamic model that is obtained by combining the image model and the blur model takes the form of a singularly perturbed system owing to the strong-weak correlation effects within a window. The time scale property of the singularly perturbed system is then utilized to decompose the original system into reduced order subsystems which closely capture the behavior of the full order system. For these subsystems the relevant Kalman filtering equations are given which provide the suboptimal filtered estimates of the image and the one-step prediction estimates of the blur needed for the next stage. Simulation results are also provided.**

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I. Introduction

Parametric representation of digital images have found numerous applications in image restoration [1]–[3], image data compression [4], and texture analysis [5]. An image is modeled by a finite-order autoregressive (AR) or autoregressive moving average (ARMA) representation which closely match the autocorrelation function or equivalently the spectral density function (SDF) of the image field. Even though high-order models may take more correlations into account, in general they are not necessarily capable of better image representation. Additionally, the principle of parsimony precludes the use of a large order model for this representation.

Suresh and Shenoi [2] proposed a strip Kalman filtering process which makes use of a vector scanning scheme. The image process is modeled by a finite-order vector AR model which relates a column of pixels to the past columns in a certain region within the strip. The effect of an LSI blur is modeled by a 2-D state-space structure [6] implemented by a 1-D structure with intrastrip and interstrip recursion characteristics [2]. The size of the state-vector in the composite dynamic model and hence the computational effort of the filtering process are dependent on (i) the size of the blur model and (ii) the order of the AR model used to generate the image. If the width of each strip and the order of the vector AR model are denoted by W and M, respectively, the size of the state-vector in the composite dynamic model is shown [2] to be $WM+W+1$ which may be large even for moderate values of W and M.

The contribution of this paper is to employ singular perturbation methodology for decomposing a given image model into reduced order models whereby the image restoration can be performed effectively. To this effect, a singularly perturbed model of the original system in [2] is obtained by expressing the state variables into a set of slow and fast variables. Using the model reduction capabilities of the singular perturbation technique the full order model is decomposed into reduced order sub-models corresponding to strong-weak correlation areas. The utility of the singular perturbation method lies in the fact that the aggregated effects of the weakly correlated states are taken into account in the reduced order models. These models can be used in the strip Kalman filtering process without losing significant accuracy in estimation.

II. Modeling the Image Process

Consider an $N \times N$ image which is vector scanned horizontally in strips of size $W \times N$. The direction of scanning is assumed to be from left-to-right and top-to-bottom. Each strip is processed independently with an overlap between the adjacent strips to reduce the edge effects. The image is assumed to be represented by a vector Markovian field and modeled, within each strip by an Lth-order vector AR model with causal quarter-plane region of support. If the support region of this model is denoted by $\mathbf{R}$, the following AR model of order L can be written for the process

$$Z(k)=\hat{\phi}_1^t Z(k-1)+\hat{\phi}_2^t Z(k-2)+\cdots+\hat{\phi}_L^t Z(k-L)+U(k) \tag{1}$$

where the superscript t denotes matrix transposition; $\hat{\phi}_1,\cdots,\hat{\phi}_L$ are constant $W \times W$ matrices which constitute the autoregressive parameters; and $Z(k)$ represents a $W \times 1$ vector with elements that are the pixels intensity values in the kth column of a given