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Isoperimetric clustering-based network partitioning algorithm for voltage–apparent power coupled areas

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Abstract: This work proposes a novel relative electrical distance measure that provides information of coupling between voltage and apparent power between two buses in power systems. Relative electrical distance measure is derived from the bus admittance matrix which can be obtained in real time using Phasor Measurement Units. Based on the relative electrical distance measure, in this work, an isoperimetric clustering based algorithm for partitioning power systems into voltage–apparent power coupled areas is proposed. The advantage of the partitioning algorithm proposed in this work is that large networks can be represented as a weighted graph with number of vertices equal to number of generators in the system which is much lesser than the size of system, thereby reducing the computational effort for partitioning. Isoperimetric clustering technique along with k-means is then applied to the graph to obtain voltage–apparent power coupled areas. Simulations carried out on New England 39-bus system and IEEE 118-bus system demonstrate the effectiveness of the proposed methodology for partitioning the system into voltage–apparent power coupled areas, subject to changes in the operating condition of the system. The quality of clustering is analysed and compared with Cheeger inequality bounds, which ensures that power system is well partitioned.

1 Introduction

In power systems, reactive power and voltage control services are provided locally, wherein critical locations in the system are identified through voltage stability index. Identification of weak bus with respect to reactive power, complex power and voltage sensitivity has been made using continuation load flow and voltage collapse proximity [1, 2]. A large power system network is then partitioned into subsystems considering the electrical distance between generator and load buses, which is quantified using bus admittance matrix or the Jacobian matrix of converged load flow solutions. The voltage control mechanism is implemented in the subsystems, which also provides information regarding the reactive power support required in the system. Partitioning of the large system into subsystems as voltage control areas (VCAs) is found to be effective for voltage control, reactive power support and planning.

The electrical distance-based partitioning of the power system network has been widely used in the literature. The electrical distance between two buses can be quantified directly by the bus admittance matrix of the system. The Jacobian matrix of converged load flow solution is also considered as a measure of electrical distance between two buses to determine the real power and reactive power interaction with voltage. However, the computation of Jacobian matrix for real-time applications may become tedious for certain operating conditions of the system. The VCAs identified through electrical distance method were modified through contingency analysis based on voltage security index [3]. Clustering of buses based on electrical distance [4], heuristic methods [5, 6] was used for the identification of VCAs. Monte Carlo simulation-based framework which considered wind power uncertainties in the identification of VCAs was proposed in [7]. Graph theory-based [8] and bifurcation system-based [9] identifications of VCAs were also studied. A comparison of five methodologies for partitioning power systems into VCAs was presented in [10].

The energy sensitivity matrix-based method for VCA identification was proposed in [11]. Heuristic algorithms like shuffled frog-leaping algorithm and particle swarm optimisation for identification of VCA were compared in [12]. Real-time zonal identification of VCAs based on electrical distance and bus admittance matrix analysis was proposed in [13]. Coupled two-port network analysis-based voltage stability analysis using Phasor Measurement Unit (PMU) data was presented in [14]. The voltage coupling relative gain method has been used to partition the system into voltage stability critical areas [15]. Volt/Var interaction based on relative gain was studied in [16]. The dynamic relative gain method was also used for partitioning the system to VCAs [17]. 2D visualisation of electrical network suitable for voltage stability analysis was proposed [18] which may be useful for online identification of VCAs. However, in these system partitioning schemes, the electrical distance between buses quantifies volt/var interactions assuming that real power is constant. Thus, these measures of electrical distance will not depict the actual coupling between buses in the system when both real and reactive power vary due to real-time market mechanisms.

Under deregulation, reactive power support in the system has been identified as an ancillary market service, where generation companies trade reactive power. The supply of reactive power is considered as one of the most important ancillary services for system security by the Federal Energy Regulatory Commission Order No. 888 [19]. If it is assumed that the real power market is settled, the localised reactive power market can be implemented by partitioning the large power network based on the coupling between bus voltages and reactive power in the system [20]. However, due to the inherent coupling between real and reactive power requirements in the system, real-time pricing mechanism of reactive power is affected by the pricing mechanism of real power as well. Hence partitioning the system based on VCAs may not be effective for applications like real-time pricing mechanisms for reactive power.

In this paper, a new relative electrical distance measure is derived from the bus admittance matrix. The proposed electrical distance depicts the voltage–apparent power coupling between load and generator bus (N_{LG}) and voltage–apparent power coupling between load buses (M_{LL}). The computational effort for calculating the proposed relative electrical distance measure is much less when compared to a calculation of the Jacobian matrix. Another advantage of the proposed measure is that the input for calculating the relative electrical distance is the bus admittance matrix derived from voltage and current phasor measurements from PMUs. Thus, real-time data of the power system can be utilised for identifying

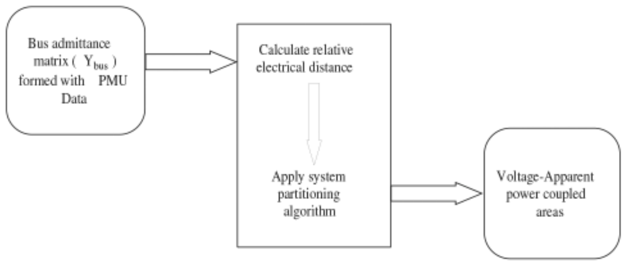

Fig. 1 *Scheme for partitioning the system into voltage–apparent power coupled areas*

localised markets. Hence the proposed measure for electrical distance is suitable for real-time applications.

Also, an algorithm based on isoperimetric graph clustering along with K-means is proposed for partitioning the power systems into the optimal number of voltage–apparent power coupled areas. In the proposed algorithm, the large power systems are reduced to a weighted complete graph, wherein the number of vertices is equal to the number of generator buses in the system. Thus the graph theory-based clustering techniques like spectral clustering, isoperimetric clustering etc. can be applied to the lesser computational effort, as the number of generator buses in a system is lesser than the total number of buses in the system. Among various graph clustering techniques available in the literature, isoperimetric clustering technique is found to be faster than spectral clustering techniques as isoperimetric clustering technique is based on solving a system of linear equations while spectral clustering techniques involve computation of eigenvectors [21]. A schematic representation of the technique to partition the system into voltage–apparent power coupled areas, proposed in this paper is shown in Fig. 1. The proposed algorithm is then verified in New England 39-bus system and IEEE 118-bus system. The quality of clustered graph is compared with Cheeger inequality, which proves that the system is well partitioned.

The main contributions of this paper are two-fold; relative electrical distance measure and isoperimetric clustering-based system partitioning. The relative electrical distance measure proposed in this work quantifies the coupling between buses in the system with respect to voltage and apparent power. In prior works referred in this paper, voltage–reactive power coupling between generator and load buses were quantified for measuring relative electric distance while considering real power to be constant. Such relative electrical distance measures that do not consider real power interactions may not be suitable in applications where the coupling between real and reactive power is significant. Such applications include real-time pricing of real and reactive power, simultaneous active–reactive power markets wherein the coupling of real and reactive power is significant and cannot be ignored as in the active power market has a direct influence on the reactive power market and vice versa in real time. Also, the proposed relative electrical distance measure is based on bus admittance matrix and not Jacobian matrix-based measure as in previous works. Thus even during critical system conditions in which convergence of Jacobian matrix is an issue, the relative electrical distance measure can be calculated. Using the relative electrical distance measure proposed the system is then partitioned using isoperimetric clustering-based algorithm. In comparison with previous works, it has been observed that the algorithm is fast and the quality of clustering is improved.

The paper is organised as follows: Derivation of the proposed relative electrical distance measure to quantify the coupling between bus voltages and apparent power ($\boldsymbol{N}_{\rm G}$ and $\boldsymbol{M}_{\rm L}$) based on the bus admittance matrix is presented in Section 2. Section 3 describes the isoperimetric clustering-based algorithm for partitioning the system. Simulation results and analysis of the proposed clustering algorithm on New England 39-bus system and IEEE 118-bus system are detailed in Section 4 and conclusions are drawn in Section 5.

2 Relative electrical distance

Consider a system with g number of generator buses and $(b-g)$ number of load buses, where b is the total number of buses in the system. Then for the given system,

$$\begin{bmatrix} I_{\rm L} \\ I_{\rm G} \end{bmatrix} = Y_{\rm bus} \begin{bmatrix} V_{\rm L} \\ V_{\rm G} \end{bmatrix} \tag{1}$$

where

$$Y_{\rm bus} = \begin{bmatrix} Y_{\rm LL} & Y_{\rm LG} \\ Y_{\rm GL} & Y_{\rm GG} \end{bmatrix}$$

$\boldsymbol{I}_{\rm L}$ and $\boldsymbol{V}_{\rm L}$ are the vectors that represent current and voltage phasors of load buses, respectively, and $\boldsymbol{I}_{\rm G}$ and $\boldsymbol{V}_{\rm G}$ are vectors that represent the current and voltage phasors at generator buses, respectively.

It is assumed that the bus admittance matrix is known, either directly from the system topology or estimated using PMU-based state estimation techniques in cases where system topology is not known especially post fault conditions. The optimal number of PMUs required for minimising error in state estimation depends on the observability of the system.

The bus admittance matrix can be analytically obtained or statistically estimated from the PMU data. If the current phasors and voltage phasors at all buses are obtained in real time using optimal placement of PMUs and state estimation techniques, the bus admittance matrix can be calculated analytically. However, in cases where interconnection information is not known, the bus admittance matrix can be statistically estimated using PMU measurements [22].

The aforementioned systems of equations have been rearranged in [23] as

$$\begin{bmatrix} V_{\rm L} \\ I_{\rm G} \end{bmatrix} = \begin{bmatrix} Z_{\rm LL} & F_{\rm LG} \\ K_{\rm GL} & Y_{\rm GM} \end{bmatrix} \begin{bmatrix} I_{\rm L} \\ V_{\rm G} \end{bmatrix} \tag{2}$$

where

$$\boldsymbol{Z}_{LL} = \boldsymbol{Y}_{LL}^{-1}$$

$$\boldsymbol{F}_{\rm LG} = -\boldsymbol{Y}_{\rm LL}^{-1}\boldsymbol{Y}_{\rm LG}$$

$$\boldsymbol{K}_{\rm GL} = \boldsymbol{Y}_{\rm GL}\boldsymbol{Y}_{\rm LL}^{-1}$$

$$\boldsymbol{Y}_{\rm GM} = \boldsymbol{Y}_{\rm GG} - \boldsymbol{Y}_{\rm GL}\boldsymbol{Y}_{\rm LL}^{-1}\boldsymbol{Y}_{\rm LG}$$

A new coupling factor is presented in this section that provides the coupling between voltage and apparent power requirements between two buses in the system.

Premultiplying (2) with $\begin{bmatrix} I_{\rm LL}^* & 0 \\ 0 & V_{\rm GG}^* \end{bmatrix}$, where $\boldsymbol{I}_{\rm LL}^*$ is defined as $(b-g) \times (b-g)$ diagonal matrix with elements of $\boldsymbol{I}_{\rm L}^*$ as the diagonal entry, $\boldsymbol{V}_{\rm GG}^*$ is defined as $g \times g$ diagonal matrix with elements of $\boldsymbol{V}_{\rm G}^*$ as the diagonal entry.

We obtain

$$\begin{bmatrix} I_{\rm LL}^* V_{\rm L} \\ S_{\rm G}^* \end{bmatrix} = \begin{bmatrix} I_{\rm LL}^* Z_{\rm LL} I_{\rm L} + I_{\rm LL}^* F_{\rm LG} V_{\rm G} \\ V_{\rm GG}^* K_{\rm GL} I_{\rm L} + V_{\rm GG}^* Y_{\rm GM} V_{\rm G} \end{bmatrix} \tag{3}$$

where $S_{\rm G}^* = V_{\rm GG}^* I_{\rm G}$ which implies that

$$V_{\rm G} = Y_{\rm GM}^{-1} V_{\rm GG}^{*-1} S_{\rm G}^* - Y_{\rm GM}^{-1} K_{\rm GL} I_{\rm L} \tag{4}$$

$$V_{\rm L} = [Z_{\rm LL} - F_{\rm LG} Y_{\rm GM}^{-1} K_{\rm GL}] I_{\rm L} + [F_{\rm LG} Y_{\rm GM}^{-1} V_{\rm GG}^{*-1}] S_{\rm G}^* \tag{5}$$

$$V_{\rm L} = M_{\rm L} V_{\rm LL}^{*-1} S_{\rm L}^* + N_{\rm G} V_{\rm GG}^{*-1} S_{\rm G}^* = V_{\rm L}^{\rm I} + V_{\rm L}^{\rm II} \tag{6}$$

where

$$\boldsymbol{M}_{\rm L} = \boldsymbol{Z}_{\rm LL} - \boldsymbol{F}_{\rm LG}\boldsymbol{Y}_{\rm GM}^{-1}\boldsymbol{K}_{\rm GL}$$

$$\boldsymbol{N}_{\rm G} = \boldsymbol{F}_{\rm LG}\boldsymbol{Y}_{\rm GM}^{-1}$$

$\boldsymbol{M}_{\rm L}$ and $\boldsymbol{N}_{\rm G}$ are complex and measure the relative electrical distances between load bus-load bus and load bus-generator bus, respectively. From (6), it is noted that the no-load voltage at a particular load bus is influenced by the apparent power at other load buses ($\boldsymbol{V}_{\rm L}^{\rm I}$) and by the apparent power at generator buses ($\boldsymbol{V}_{\rm L}^{\rm II}$). $\boldsymbol{N}_{\rm G}$ provides the coupling between ith generator with the kth load which is given by

$$\left|V_{lk}^{\rm II}\right| = \left|N_{G_{ki}}\right|\left|S_{gi}\right|/\left|V_{gi}\right| \tag{7}$$

where $N_{G_{ki}}$ is the (k, i)th element of $\boldsymbol{N}_{\rm G}$ matrix.

Also, $\boldsymbol{M}_{\rm L}$ provides the coupling between jth load with kth load and is given by

$$\left|V_{lk}^{\rm I}\right| = \left|M_{L_{jk}}\right|\left|S_{lj}\right|/\left|V_{lj}\right| \tag{8}$$

where $M_{L_{jk}}$ is the (j, k)th element of $\boldsymbol{M}_{\rm L}$ matrix.

The voltage–apparent power relation between the ith generator and kth load is defined as

$$\left|S_{gi}\right|/[\left|V_{lk}^{\rm II}\right|\left|V_{gi}\right|] = 1/\left|N_{G_{ki}}\right| \tag{9}$$

Similarly, the voltage–apparent power relation between jth load and kth load is defined as

$$\left|S_{lj}/[\left|V_{lk}^{\rm I}\right|\left|V_{lj}\right|] = 1/\left|M_{L_{jk}}\right|\right. \tag{10}$$

The higher the value of $\left|M_{L_{jk}}\right|$ (or $\left|N_{G_{ki}}\right|$), the lesser will be the voltage–apparent power coupling between the buses. This implies that voltage–apparent power coupling between the buses will be less when the electrical distance between the buses is high. Thus, as shown in (9) and (10), the relative electrical distance between buses in the power system given in $\boldsymbol{M}_{\rm L}$ and $\boldsymbol{N}_{\rm G}$ provide the coupling factor between bus voltages and apparent power. The advantage of the proposed measure for relative electrical distance is that, it can be calculated using real-time PMU data, as the measure is based on the bus admittance matrix. Thus the computational effort, when compared to the calculation of Jacobian matrix from converged load flow solutions, is much less and hence more suitable for real-time applications. Another advantage of the derived relative electrical distance measure is that, it reflects the topological proximity of the buses in the systems, thereby making the measure suitable for identifying localised areas in the system.

3 Isoperimetric clustering-based network partitioning

The relative electrical distance detailed in the previous section and given by (9) and (10), is utilised for system partitioning to define voltage–apparent power coupled subsystems, and the algorithm for partitioning the system is presented in this section.

Let matrix $\boldsymbol{N}_{\rm LG}$ represent the coupling factor between generator buses and load buses in the system, such that the (i, j)th element of the matrix $\boldsymbol{N}_{\rm LG}$ is

$$N_{LG_{ij}} = \frac{1}{\left|N_{G_{ij}}\right|} \tag{11}$$

where $N_{G_{ij}}$ is the (i, j)th element of $\boldsymbol{N}_{\rm G}$

Let $\boldsymbol{M}_{\rm LL}$ represent the coupling factor between load buses in the system, such that the (i, j)th element of the matrix $\boldsymbol{M}_{\rm LL}$ is

$$M_{LL_{ij}} = \frac{1}{\left|M_{L_{ij}}\right|} \tag{12}$$

where $M_{L_{ikj}}$ is the (i, j)th element of $\boldsymbol{M}_{\rm L}$.

The ith column of $\boldsymbol{N}_{\rm LG}$ matrix provides the list of loads that are highly coupled to the ith generator bus. Mean cut is applied to the ith column of $\boldsymbol{N}_{\rm LG}$ matrix and the set of loads with coupling above the mean value is grouped under the ith generator bus. The set of load buses that are highly coupled to a generator bus forms a generator-loads set.

Sorting and mean cut technique is applied to each column of $\boldsymbol{N}_{\rm LG}$ resulting in g number of generator-loads sets in the system. It may be noted that, the number of load buses above mean value under the ith generator bus may not always be same as that under other generator buses. Let n be the number of load buses above mean value under the ith generator bus and let m be the number of load buses above mean value under the jth generator bus. Then the ith generator-loads set has the ith generator bus and n number of loads. Similarly, the jth generator-loads set has the jth generator bus and m number of loads.

Now, to find the coupling between the ith generator-loads set and the jth generator-loads set, a factor X_{ij} is defined as

$$X_{ij} = \max_{i = 1, 2, \ldots, n} [\max_{j = 1, 2, \ldots, m} M_{LL_{ij}}] \tag{13}$$

A symmetric matrix $\boldsymbol{X}$ (of order $g \times g$) that represents the coupling between generator-loads sets is defined, with non-diagonal entries obtained from (13), and diagonal entries equal to zero

$$X = \begin{cases} X_{ij} = \max_{i = 1, 2, \ldots, n} [\max_{j = 1, 2, \ldots, m} M_{LL_{ij}}] \\ X_{ii} = 0 \end{cases} \tag{14}$$

Consider a complete weighted graph $\boldsymbol{G}(\boldsymbol{V},\boldsymbol{E})$, where V is the set of vertices and E is the set of edges connecting the vertices in the graph. The set of vertices is given by g number of generator-loads sets and the weight of the edges in set E is defined by the elements of matrix $\boldsymbol{X}$. The advantage of defining $\boldsymbol{X}$ matrix is that, the dimension of the large system has been reduced to the number of generators in the system, which is generally much lesser than the number of buses in the system. Graph partitioning techniques can be applied to this weighted graph to obtain the subsystems.

Partitioning of a graph into k subgraphs is an NP-complete problem. The objective of graph partitioning problem is to minimise the total cut and the exact algorithms that solve the optimisation are based on branch and bound framework which have immense running time as the size of the graph increases [24]. Many graph partitioning techniques have been proposed in the literature like knapsack, max-flow min-cut [25], spectral clustering [26, 27] and isoperimetric clustering [28]. Spectral decomposition techniques are applied to the Laplace matrix defined for a graph for partitioning. The advantage of applying spectral decomposition-based techniques is that Cheeger inequality [29] provides a bound for the quality of the optimal cut. Spectral clustering techniques are based on the evaluation of eigenvalues and eigenvectors of the Laplacian of the graph. Isoperimetric clustering technique is based on the isoperimetric constant of the graph which results in solving a set of linear simultaneous equations. Thus when compared with spectral clustering technique, isoperimetric clustering has improved speed and numerical stability. Hence isoperimetric clustering technique is applied in this work to solve the graph partitioning problem.

3.1 Laplacian of a graph and isoperimetric clustering

The adjacency matrix $\boldsymbol{A}(\boldsymbol{G})$ of a weighted graph $\boldsymbol{G}(\boldsymbol{V},\boldsymbol{E})$ is defined as [24]

$$\boldsymbol{A}(\boldsymbol{G}) = \begin{cases} a_{vw} > 0 & \text{if } vw \in E \\ a_{vw} = 0 & \text{if } vw \notin E \end{cases} \tag{15}$$

where $a_{vw} = a_{wv}$.

The matrix $\boldsymbol{D}(\boldsymbol{G})$ is defined as

$$\boldsymbol{D}(\boldsymbol{G}) = \mathrm{diag}(\mathrm{deg}_v : v \in V) \tag{16}$$

where

$$\boldsymbol{d} = \mathrm{deg}_v = \sum_{w \in V} a_{vw} \tag{17}$$

$\boldsymbol{d}$ is called the degree vector of the graph. The Laplacian of a graph, $\boldsymbol{L}$ is a positive semi-definite matrix and is defined as

$$\boldsymbol{L} = \boldsymbol{D}(\boldsymbol{G}) - \boldsymbol{A}(\boldsymbol{G}) \tag{18}$$

The isoperimetric number of graph G [29] (h_G) cut into G_1 and $G_2 = G - G_1$ is given by

$$h_{\mathrm{G}} = \frac{\mathrm{Cut}(G_1, G_2)}{\min(\mathrm{Mass}(G_1), \mathrm{Mass}(G_2))} \tag{19}$$

where $\mathrm{Cut}(G_1, G_2)$ is the sum of weights of edges between G_1 and G_2. $\mathrm{Mass}(G_1)$ is the sum of weights of edges incident to each vertex of the cluster G_1 and $\mathrm{Mass}(G_2)$ is the sum of weights of edges incident to each vertex of the cluster G_2. A well-partitioned graph implies that the isoperimetric number of graphs is minimum. Thus the graph partitioning problem reduces to optimising the isoperimetric number, i.e. the minimising the cut of the subgraph.

Defining an indicator vector $\boldsymbol{x}$, the minimisation of cut translates to minimisation of $x^T\boldsymbol{L}\boldsymbol{x}$ [29], such that the volume of each subgraph (i.e. $x^T\boldsymbol{d}$) is a constant(K). The Lagrangian of the optimisation problem is

$$Q(x) = x^T\boldsymbol{L}\boldsymbol{x} - \lambda(x^T\boldsymbol{d} - K) \tag{20}$$

where λ is the Lagrange multiplier.

The optimisation problem thus reduces to solving a system of linear equations derived from the Lagrangian, i.e.

$$\boldsymbol{L}\boldsymbol{x} = \boldsymbol{d} \tag{21}$$

If $\boldsymbol{L}$ is singular, the node with the maximum degree is considered as the ground node(gn) [28, 30]. The ground node is deleted from $\boldsymbol{L}$ and $\boldsymbol{d}$, with $\boldsymbol{x}(\boldsymbol{gn}) = 0$, resulting in another set of linear equations as

$$L_0 x_0 = d_0 \tag{22}$$

The solution $\boldsymbol{x}_0$ assigns real values to each node which can be then clustered using K-means or other vector clustering techniques.

In the weighted graph $\boldsymbol{G}(\boldsymbol{V},\boldsymbol{E})$ defined earlier in this section with g number of vertices, the adjacency matrix of the graph is the same as the $\boldsymbol{X}$ matrix defined in (14), from which $\boldsymbol{D}(\boldsymbol{G})$ of the graph is derived. Hence the Laplacian of the graph $\boldsymbol{G}(\boldsymbol{V},\boldsymbol{E})$, $\boldsymbol{L}$ is defined as $\boldsymbol{L} = \boldsymbol{D}(\boldsymbol{G}) - \boldsymbol{X}$.

Since the graph is complete, any node can be considered as the ground node for applying isoperimetric clustering. For convenience, the first node is considered as the ground node and the modified $\boldsymbol{L}_0$ is then applied to (22). On solving (22), $\boldsymbol{x}_0$ is obtained, which assigns real values to each node in the graph. K-means clustering technique is applied to $\boldsymbol{x}_0$ where the optimal number of clusters is obtained through Elbow method [31].

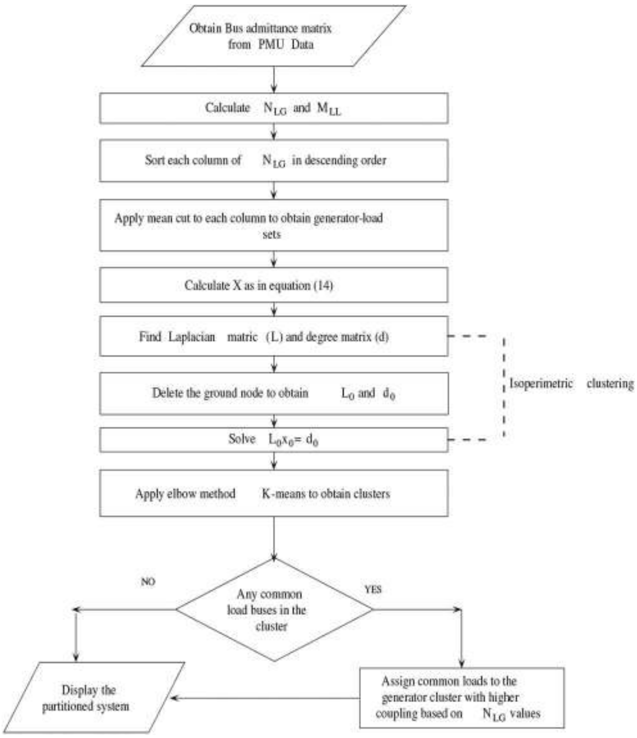

Fig. 2 *Flowchart for partitioning system into voltage–apparent power coupled areas*

3.2 Procedure for isoperimetric clustering-based network partitioning

The concept of relative electrical distance and the algorithm for isoperimetric clustering of the system into voltage–apparent power coupled areas have been discussed in the preceding sections.

The detailed flowchart of the proposed partitioning algorithm is provided in Fig. 2. The major steps involved in the partitioning of the system into voltage–apparent power coupled areas, proposed in this work is summarised below:

1. Calculate $\boldsymbol{N}_{\mathrm{LG}}$ and $\boldsymbol{M}_{\mathrm{LL}}$ matrices from the bus admittance matrix.
2. Sort columns of $\boldsymbol{N}_{\mathrm{LG}}$ in descending order and assign those loads above mean value of the column of the matrix to the corresponding generator to form generator-loads sets.
3. Find the coupling between such generator-loads sets, to form $\boldsymbol{X}$ given by (14).
4. Obtain the Laplacian matrix, $\boldsymbol{L}$ and degree vector $\boldsymbol{d}$.
5. Apply isoperimetric clustering technique to obtain $\boldsymbol{x}_0$ as in (22).
6. Find the optimal number of clusters in $\boldsymbol{x}_0$ using the Elbow method.
7. Apply K-means clustering to $\boldsymbol{x}_0$ to obtain voltage–apparent power coupled areas in the system.

3.3 Quality of clustering

As mentioned earlier, the partitioning of a graph is an NP-hard problem and thus approximate solutions are obtained using partitioning techniques one of which is an isoperimetric clustering technique. The quality of partitioned graph is measured through Cheeger inequality, which is the main theoretical justification for spectral decomposition techniques [29]. This provides a bound for the quality of graph given by

$$\frac{\lambda_k}{2} \le \rho(k) \le O(k^2)\sqrt{\lambda_k} \tag{23}$$

where k is the number of clusters. λ_k is the kth eigenvalue of the Laplacian matrix, $\boldsymbol{L}$ $\rho(k)$ is the quality factor of the graph.

The quality of a cluster ($\boldsymbol{G}_1$) in graph $\boldsymbol{G}$ is defined by its expansion $\phi(G_1)$, which is given by

$$\phi(G_1) = \frac{\mathrm{Cut}(G_1, G - G_1)}{\mathrm{Vol}(G_1)} \tag{24}$$

where

$$\mathrm{Cut}(G_1, G - G_1) = \sum_{i \in G_1, j \in G - G_1} X_{ij} \tag{25}$$

$$\mathrm{Vol}(G_1) = \sum_{i \in G_1} D(G)_{ii} \tag{26}$$

Then the quality factor $\rho(k)$ of the graph with clusters $\boldsymbol{G}_1, \boldsymbol{G}_2, \ldots \boldsymbol{G}_k$, also called as k-way expansion constant of the graph is defined as

$$\rho(k) = \max_{i = 1, 2, \ldots, k} \phi(G_i) \tag{27}$$

The objective of graph partitioning algorithm is to minimise the quality factor of the graph and hence if the quality factor of the graph is closer to the lower bound of Cheeger inequality given by (23), the clusters are said to be fairly balanced resulting in a well-partitioned system.

4 Case studies

In this section, the proposed relative distance electrical measure and algorithm for partitioning the system into voltage–apparent power coupled areas is demonstrated in detail on the New England 39-bus system. The partition results for IEEE 118-bus system are also provided along with the cluster quality analysis. The algorithm was implemented in Matlab platform.

4.1 Case study 1: New England 39-bus system

Two scenarios have been considered in the system, to demonstrate the proposed methodology.

Case 1: Base case of New England 39-bus system.
Case 2: Outage of line 15-16.

4.1.1 Case 1: The bus admittance matrix is obtained from the detailed data of the system [32]. Accordingly, $\boldsymbol{N}_{\mathrm{LG}}$ matrix and $\boldsymbol{M}_{\mathrm{LL}}$ matrix are derived from $\boldsymbol{N}_{\mathrm{G}}$ and $\boldsymbol{M}_{\mathrm{L}}$ matrices, respectively, which are calculated using (2), (5) and (6) from the bus admittance matrix. The $\boldsymbol{X}$ matrix which represents the graph with nodes as generator-load sets is then obtained, which is of order 10×10 as there are ten generator buses in the system. The matrix $\boldsymbol{X}$ of the system is (see equation below). Thus the weighted graph $\boldsymbol{G}(\boldsymbol{V},\boldsymbol{E})$ has $|V| = 10$ and weight of edges given by the matrix $\boldsymbol{X}$, which also indicates that the graph is complete. From $\boldsymbol{X}$, the Laplacian matrix and the degree vector are obtained. The isoperimetric clustering technique is then applied by solving (22), considering the first node of the graph as the ground node. The indicator vector ($\boldsymbol{x}_0$) obtained from isoperimetric clustering is plotted and shown in Fig. 3.

Fig. 3 shows that the optimal number of clusters for the base case of the test system is 2. K-means clustering is then applied to the indicator vector $\boldsymbol{x}_0$ and the partitioned system is obtained, which is shown in Fig. 4.

4.1.2 Case 2: The outage of line 15-16 is considered in the system. The proposed relative electrical distance is found and the algorithm for partitioning is applied to the system. Fig. 5 provides the indicator vector of the graph. The optimal number of clusters in this scenario is 2, which can be observed in Fig. 5. The partitioned system after applying K-means to the solution vector of isoperimetric clustering is shown in Fig. 6.

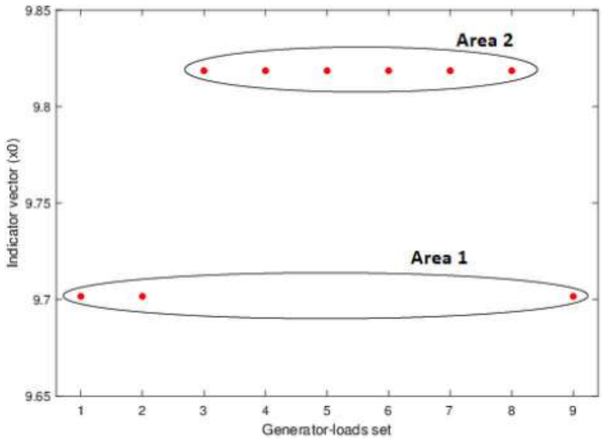

Fig. 3 *Optimal number of clusters for New England 39-bus system-base case*

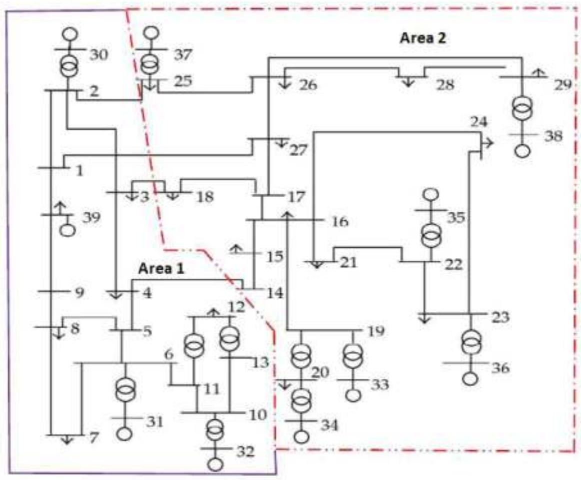

Fig. 4 *Voltage–apparent power partitioning in New England 39-bus system-base case*

Analysis and observations: Comparing Figs. 4 and 6, it is observed that the subsystems obtained in base case are different from those obtained during line outage. This implies that the proposed algorithm for partitioning the system is subject to changes in the operating condition of the system. It may be noted that, the operating condition of the system influences real and reactive power requirements in the system which will also affect real-time market operations. The case studies show that the proposed algorithm to partition the system to voltage–apparent power coupled areas is able to adapt to the operating condition of the system, thereby making it suitable for applications in real time.

The quality of the clustering is compared with the lower bound of the Cheeger inequality as shown in Table 1. To compare the proposed algorithm with spectral clustering algorithm in [27] for partitioning bulk power systems into volt/var control areas, a closeness factor (CF) is defined which is the ratio of the quality factor of graph to the lower bound of Cheeger inequality.

$$\begin{bmatrix} 0 & 10.52 & 9.15 & 9.15 & 9.26 & 9.26 & 9.26 & 9.26 & 13.88 & 10.52 \\ 10.52 & 0 & 10.45 & 10.45 & 9.53 & 9.53 & 9.53 & 9.53 & 10.52 & 12.50 \\ 9.15 & 10.45 & 0 & 11.25 & 10.53 & 10.53 & 10.53 & 10.53 & 10.53 & 11.25 \\ 9.15 & 10.45 & 11.25 & 0 & 10.53 & 10.53 & 10.53 & 10.53 & 10.53 & 11.25 \\ 9.26 & 9.53 & 10.53 & 10.53 & 0 & 13.88 & 13.88 & 13.88 & 13.88 & 9.58 \\ 9.26 & 9.53 & 10.53 & 10.53 & 13.88 & 0 & 13.88 & 13.88 & 13.88 & 9.58 \\ 9.26 & 9.53 & 10.53 & 10.53 & 13.88 & 13.88 & 0 & 13.88 & 13.88 & 9.58 \\ 9.26 & 9.53 & 10.53 & 10.53 & 13.88 & 13.88 & 13.88 & 0 & 13.88 & 9.58 \\ 13.88 & 10.52 & 10.53 & 10.53 & 13.88 & 13.88 & 13.88 & 13.88 & 0 & 10.52 \\ 10.52 & 12.50 & 11.25 & 11.25 & 9.58 & 9.58 & 9.58 & 9.58 & 10.52 & 0 \end{bmatrix}$$

$$\text{Closenessfactor (CF)} = \frac{\rho(k)}{(\lambda_k/2)} \quad (28)$$

An exact partition of the graph will yield a CF equal to 1. Graph partitioning is an NP-hard problem, approximate solutions are obtained through the graph partitioning techniques in the literature and hence a value of CF equal to 1 is not obtained. However, when the value of CF is close to 1 it can be inferred that the graph is well partitioned. The CF of the partitioned graph obtained through the proposed algorithm is compared with that using a spectral clustering algorithm in [27] for the base as well as line outage case. The comparison is tabulated in Table 2.

It is observed that the quality of the partitioned graph is close to the lower bound of the Cheeger inequality, which confirms that the proposed algorithm performs well in system partitioning.

The proposed partitioning technique on the New England 39-bus system is compared with results in [16, 33]. In [16], bulk power system is partitioned considering the volt–var interactions of buses with respect to pilot buses. With this method of partitioning, only buses that have a strong cross relative gain with the pilot buses identified form a cluster. These clusters do not include every bus in the system which makes it not suitable for purposes like real-time localised reactive power market.

The number of clusters obtained in the proposed isoperimetric clustering-based partitioning was 2, whereas in [33] the number of clusters obtained for the base case was 5. On close observation of the clusters formed based on VCAs in [33], there are clusters with single generators serving loads. Such technique for clustering will not be efficient in cases where the partitioning of system aims at forming clusters suitable for localised reactive power market. Single generator clusters will lead to exercising of market power and price volatility. Also such clusters with single generators may also be not suitable for active and reactive power management in real time if generation outage contingencies are to be considered for safe operation of the system.

4.2 Case study 2: IEEE 118-bus system

Similar to the previous case study, two scenarios have been considered in the IEEE 118-bus system to test the proposed algorithm in comparatively larger systems. Detailed data of the system is available in [34]

Case 1: Base case.
Case 2: Outage of line 43-44.

4.2.1 Case 1: The weighted graph $\boldsymbol{G}(\boldsymbol{V},\boldsymbol{E})$ has $|V| = 54$. The isoperimetric clustering algorithm is applied to the system and the optimal number of clusters is found to be 3. The loads buses and generator buses in the three voltage apparent power coupled areas obtained after partitioning is tabulated in Table 3. The optimal number of clusters is found to be 3 and the partitioned system is shown in Fig. 7.

4.2.2 Case 2: The outage of line 43-44 is considered in the IEEE 118-bus system. This line connects Area 1 and Area 3 of the base case partitioned system. Upon outage of the tie line, the voltage–apparent power coupling between the buses will change in Area 1 and Area 3. The partitioned system after applying K-means to the solution vector of isoperimetric clustering is shown in Fig. 8.

The generator and load buses in the three-voltage apparent power coupled areas obtained after partitioning the system are provided in Table 4. It can be observed that an outage of tie-line between Area 1 and Area 3 of the base case partitioned system affects the generator bus and the load bus distribution among Area 1 and Area 3 only. Area 2 is not affected, which shows that the proposed isoperimetric clustering-based partitioning algorithm is suitable for defining localised real-time operation like control and market mechanism. The case studies show that the proposed relative electrical distance measure and the partitioning algorithm are suitable for comparatively larger systems also and adapts to the operating condition of the system. The quality factor of the graph is compared with the lower bound of the Cheeger inequality as shown

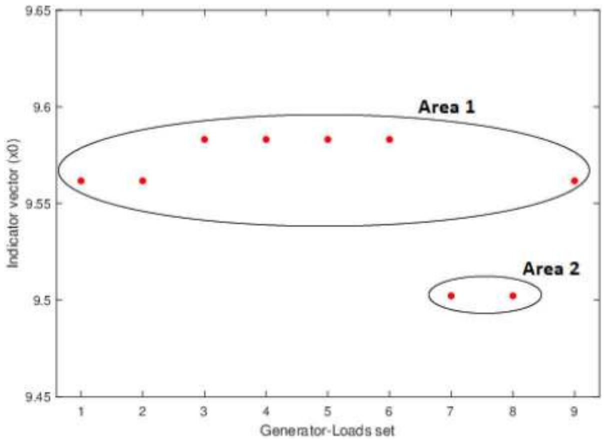

Fig. 5 *Optimal number of clusters for New England 39-bus system-outage of line 15-16*

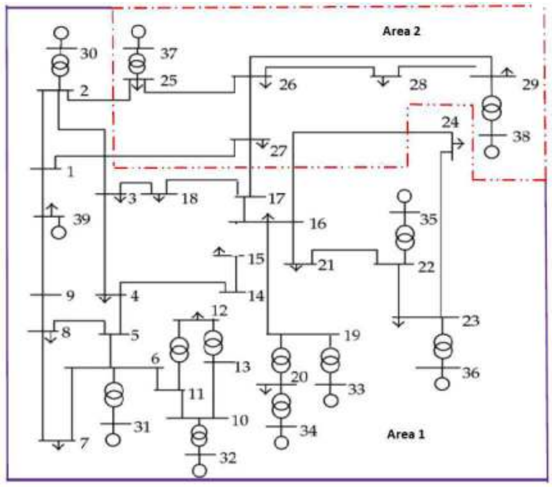

Fig. 6 *Voltage–apparent power partitioning in New England 39-bus system-outage of line 15-16*

Table 1 Quality factor of clustering in the New England 39-bus system

Case	Quality factor $\rho(k)$	Cheeger inequality $\frac{\lambda_k}{2}$
base case	0.6667	0.5300
line outage	0.8889	0.5158

Table 2 Comparing isoperimetric clustering algorithm with spectral clustering algorithm [26] for the New England 39-bus system

Case	Isoperimetric clustering CF	Spectral clustering [27] CF
base case	1.26	5.52
line outage case	1.72	3.03

Table 3 Voltage–apparent power coupled areas in IEEE 118-bus system-base case

Area	Generator buses in the area	Load buses in the area
1	4, 6, 8, 10, 12, 16, 18, 19, 24, 25, 26, 27, 31, 32, 34, 36	1, 2, 3, 6, 7, 11, 13, 14, 20, 21, 22, 23, 28, 29, 33, 35, 43, 114, 115, 117
2	65, 66, 69, 70, 74, 76, 77, 80, 82, 85, 87, 89, 90–92, 99 100, 103–105, 107, 110–112, 116	75,78, 79, 83 84, 86, 88, 93–98, 101, 102, 106, 108, 109, 118
3	40, 42, 45, 49 54–56, 59, 61, 62 72, 75, 115	17, 39, 41, 44, 46, 47, 48, 50–53, 57, 58, 60, 67

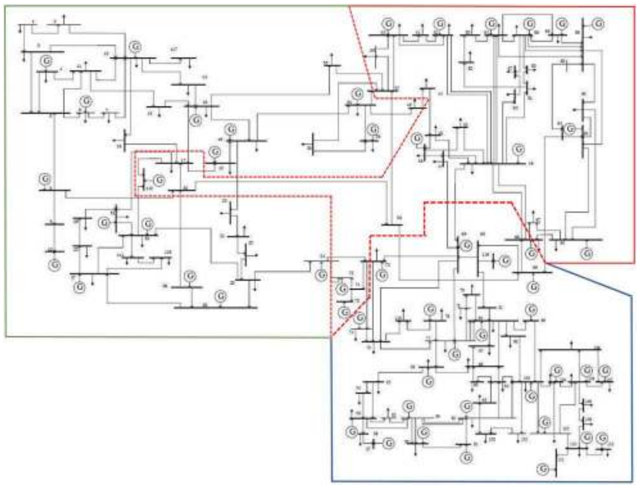

Fig. 7 *Voltage–apparent power partitioning in IEEE 118-bus system-base case*

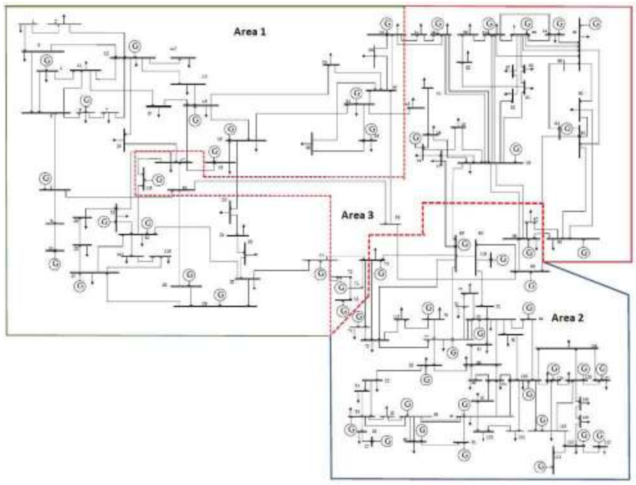

Fig. 8 *Voltage–apparent power partitioning in IEEE 118-bus system-outage of line 43-44*

in Table 5. The computation time and the CF of the proposed partitioned system are compared with that obtained through spectral clustering technique for partitioning the system into volt/var control areas [27] and is tabulated in Table 6. The proposed algorithm is faster than the spectral clustering technique. Also, the quality of graph is well close to the lower bound of the Cheeger inequality, confirming that the proposed algorithm performs well in partitioning the system into voltage–apparent power coupled areas subject to changes in operating conditions.

5 Conclusion

In this paper, a relative electrical distance measure that reveals the voltage–apparent power coupling between the buses in large power systems is proposed. Based on the derived voltage–apparent power coupling factor, New England 39-bus system and IEEE 118-bus system have been partitioned into voltage–apparent power coupled areas, by applying isoperimetric clustering-based partitioning algorithm that is proposed in this work. The contribution of this work is the relative electrical distance measure that provides information about the coupling of voltage and apparent power requirement in the system and an algorithm based on isoperimetric clustering to partition the system into the optimal number of voltage–apparent power coupled areas. The advantage of the proposed measure for relative electrical distance is that, it is based on the bus admittance matrix of the system, which can be obtained using real-time PMU data. Since calculation of Jacobian matrix is not involved, the computation of the proposed voltage–apparent power coupling factor is much faster. For partitioning the system into voltage–apparent power coupled areas, isoperimetric

Table 4 Voltage–apparent power coupled areas in IEEE 118-bus system-outage of line 43-44

Area	Generator buses in the area	Load buses in the area
1	4, 6, 8, 10, 12, 16, 18, 19, 24, 25, 26, 27, 31, 32, 34, 36, 40	1, 2, 3, 6, 7, 11, 13, 14, 20, 21, 22, 23, 28, 29, 33, 35, 39, 43, 114, 115, 117
2	65, 66, 69, 70, 74, 76, 77, 80, 82,85,87,89,90–92,99 100, 103–105, 107, 110–112, 116	75,78, 79, 83 84, 86, 88, 93–98, 101, 102, 106, 108, 109, 118
3	42, 45, 49 54–56, 59, 61, 62 72, 75, 115	17, 41, 44, 46, 47, 48, 50–53, 57, 58, 60, 67

Table 5 Quality factor of clustering in the IEEE 118-bus system

Case	Quality factor $\rho(k)$	Cheeger inequality $\frac{\lambda_k}{2}$	CF
base case	0.5706	0.4755	1.20
line outage	0.6532	0.4820	1.35

Table 6 Comparing isoperimetric clustering algorithm with spectral clustering [26] for the IEEE 118-bus system-base case

	Isoperimetric clustering	Spectral clustering [27]
computation time	0.2483s	0.4456s
CF	1.2	13.6

clustering-based algorithm is proposed in this work which is faster and more numerically stable when compared to widely used spectral clustering technique for partitioning. The proposed algorithm is found to be adaptive to operating condition that reflects the change in apparent power requirement in the system. The quality of the proposed partitioning technique is compared with Cheeger inequality which ensures that each area has strong voltage–apparent power coupling within the area and weak coupling with other areas in the system. The proposed algorithm is thus suitable for defining localised real and reactive power markets in real time.

6 References

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