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3	INTERACTION DIAGRAMS FOR FRP STRENGTHENED RC
4	RECTANGULAR COLUMNS WITH LARGE ASPECT RATIO
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26 Abstract

The load carrying capacity of reinforced concrete columns (RC) shall be enhanced by 27 providing lateral confinement using fiber reinforced polymer (FRP). As per ACI 440.2R-08 28 29 [1], external strengthening using FRP composites is more effective for circular columns than rectangular columns having an aspect ratio greater than 2.0. The main objective is to develop 30 column interaction diagrams for RC rectangular columns having an aspect ratio greater than 31 2.0 externally strengthened using FRP composites without any shape modification and 32 subjected to combined axial load (P_n) and bending moment (M_n) due to lateral loading. Tests 33 34 are conducted and semi-empirical equations developed for FRP strengthened RC rectangular columns with large aspect ratio. Pn - Mn interactions diagrams are developed using the semi-35 empirical solutions and validated with 3D finite element analysis. The column interaction 36 37 diagrams are developed by varying the unconfined compressive strength of concrete, yield strength of steel, percentage of steel reinforcement and thickness of FRP composite. These 38 diagrams shall be used for the design of FRP strengthened RC rectangular columns with large 39 aspect ratio. 40

Key words: P_n - M_n interaction diagrams; Semi-empirical solutions; Finite element analysis;
 RC rectangular columns; FRP composites; Large aspect ratio; shape modification.

43 1. Introduction

The performance of a building is defined by its structural stability, life safety and operation. The majority of existing buildings and bridges were designed using the old codal provisions. In accordance to the revised standards, certain structures are not satisfying the requirement and need to be strengthened. Columns being the primary load bearing members in structures, might require to be strengthened to increase the flexural and shear strength, deformation capacity, rotation at the beam-column junction and strengthen the regions of non-standard splicing of longitudinal bars. External strengthening using FRP composite is more effective than steel and concrete jacketing due to its high strength to weight ratio, high stiffness to weight ratio and excellent corrosion resistance. Due to the uniform distribution of lateral confining pressure in circular sections, circular columns can be effectively strengthened using FRP composites compared to square and rectangular columns. The confining pressure is maximum at the corners and varying non-uniformly in between the corners in square and rectangular sections.

Extensive research work has been reported on axially loaded RC circular columns and 57 rectangular columns having aspect ratio less than 2.0 externally strengthened with FRP 58 composites. Parvin and Jamwal [6] studied influence of wrap thickness, wrap ply angle 59 configuration and concrete strength on the axial capacity of columns through nonlinear finite 60 element analysis. Pedro et al. [7] studied the effect of shape of cross section and corner radii 61 on the strength of column under axial compression. Chen and Togay [4] and Mohammad et 62 63 al. [10] tested square and rectangular FRP tubes filled with concrete under axial loading. Alsayed et al. [2], Prota et al. [8], Tan [13] and Tanwongsval et al. [14] predicted the 64 confinement effect of FRP composites on small scale wall-like RC columns with rectangular 65 and elliptical cross sections having high aspect ratio 3.65 under axial and sustained axial 66 loading. Lam and Teng [5] and Teng and Lam [15, 16] developed a stress-strain model based 67 on the experiments and existing studies to predict the behavior of FRP confined square and 68 elliptical RC columns under axial loading. Challal et. al. [3] and Mimiran and Shahawy [9] 69 developed analytical models to predict the axial load carrying capacity of small scale FRP 70 71 confined RC columns. Seible et al. [11] and Wang and Hsu [17] proposed a design method to evaluate the load carrying capacity of square and rectangular RC columns with aspect ratio 72 less than 2.0 confined with FRP composites under axial and combined axial and lateral 73 loading with shape modification. As can be observed, test data, semi-empirical solutions, 74 finite element models and Pn - Mn interactions diagrams are not available for FRP 75

strengthened RC rectangular columns having aspect ratio greater than 2.0 without any
 modification of cross section and subjected to combined axial and lateral loading.

78 **2. Objective and Scope**

The main objective of the study is to develop $P_n - M_n$ interaction diagrams for RC rectangular columns having an aspect ratio greater than 2.0 externally strengthened using FRP composites without any shape modification and subjected to combined axial and lateral loading.

The scope of work is limited to (i) developing finite element (FE) models for FRP strengthened RC rectangular columns and validating with the test data and (ii) comparing the proposed semi-empirical solutions with FE models and (iii) developing $P_n - M_n$ interaction diagrams using semi-empirical solutions and FE models by varying the unconfined compressive strength of concrete, yield strength of steel, percentage of steel reinforcement and thickness of FRP composite.

89 **3. Experimental Study**

Six prototype RC rectangular columns of length (*l*) 3000 mm, breadth (*b*) 600 mm and depth 90 (D) 230 mm designated as CCA1 (control column under axial load), RCA2 (retrofitted 91 column under axial load), CCL3, RCL4, CCC5 and RCC6 were cast. The strengthening 92 scheme and loading pattern of tested columns are given in Table 1. The column RCC6 was 93 clamped at one end and the other end kept free. An axial compressive load up to 600 kN was 94 applied at the free end and maintained constant. An incremental lateral load was also applied 95 at the free end in such a way that the column was subjected to weak axis bending. Based on 96 the strain measurements, it is noted that the failure of column RCC6 was initiated by cracking 97 98 of CFRP composite perpendicular to the longitudinal axis of the column and followed by yielding of steel reinforcement. No major cracks were observed in the column. 99

Table 1 Details of Tested Specimens

Specimen	Strengthening scheme	Loading pattern
CCA1	-	Axial load
RCA2	Two layers of carbon fibre fabric of uni-directional cloth (UDC) 430 gsm $(t_{frp} = 1.2mm)$	Axial load
CCL3	-	Lateral load (weak axis bending)
RCL4	Two layers of carbon fibre fabric of UDC 430 gsm ($t_{frp} = 1.2$ mm)	Lateral load (weak axis bending)
CCC5		Combined loading
RCC6	Two layers of carbon fibre fabric of UDC 430 gsm (t_{frp} = 1.2mm)	Combined loading

The axial and lateral load carrying capacity of tested columns is calculated using the semiempirical solutions available in the literature for columns with aspect ratio less than 2.0 without partial safety factors and compared with the test data. It is observed that the semiempirical solutions are under estimating and over estimating the axial and lateral load carrying capacity of FRP strengthened RC rectangular columns with aspect ratio greater than 2.0 up to 22% and 17% respectively.

107 4. Semi-Empirical Solutions

The procedure given in ACI 440.2R-08 is modified for FRP strengthened RC rectangular columns having aspect ratio greater than 2.0 without shape modification of cross section. New semi-empirical solutions are proposed to calculate the load carrying capacity of these columns subjected to axial, lateral and combined axial and lateral loading.

The axial capacity of FRP strengthened RC columns without any partial safety factors iscalculated using Eqn. (1).

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$$P_n = [f'_{co} + k_1 k_a f_l] (A_g - A_{st}) + f_y A_{st}$$
(1)

In which, f'_{co} is the unconfined compressive strength of concrete, f_y is the yield strength of steel, A_g is the gross sectional area of column, A_{st} is the area of steel reinforcement, f_l is the lateral confining pressure and k_1 and k_a are the shape factors. The value of ' k_l ' obtained is 1.78 from test data by trial and error method. k_a can be calculated using the equations given in Sreelatha and Alagusundaramoorthy [12].

The representative $P_n - M_n$ interaction diagram is shown in Fig. 1. Point 'A' represents the column under pure axial compression, point 'B' corresponds to ultimate compressive strain (ε'_{ccu}) at the extreme compression fiber and zero strain at the extreme layer of tensile reinforcement, point 'C' corresponds to ultimate compressive strain at the extreme compression fiber (ε'_{ccu}) and yielding tensile strain (ε_{sy}) at the extreme layer of tensile reinforcement and point 'D' represents pure bending.



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Fig. 1. Representative Interaction Diagram



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$$P_{n(B,C)} = [A(y_t)^3 + B(y_t)^2 + C(y_t) + D] + \sum A_{si} f_{si}$$
(2)

130
$$M_{n(B,C)} = [E(y_t)^4 + F(y_t)^3 + G(y_t)^2 + H(y_t) + kI] + \sum A_{si} f_{si} d_i$$
(3)

The parameters *A*, *B*, *C*, *D*, *E*, *F*, *G*, *H*, *I*, y_t , A_{si} , f_{si} and d_i are calculated as per the equations given in ACI 440.2R-08. Values of constant '*k*' are obtained from the test data as 1.12 and 1.55 for lateral load and combined axial and lateral loading respectively by trial and error method. The semi-empirical solutions are validated with the test data and the uncertainty parameters such as mean value, standard deviation and coefficient of variation (COV) are calculated as 1.000, 0.035 and 3.550 respectively.

137 **5. Finite Element Analysis**

Unstrengthened and strengthened 3D models of RC columns are generated using the finite 138 element analysis (FEA) software ABAQUS. Solid C3D8R elements, Truss T3D2 elements 139 and Shell S4R elements are used to model concrete, steel reinforcement and FRP composite. 140 The steel reinforcement cage is embedded into concrete using EMBED constraint. FRP 141 142 composite is modeled separately and connected to concrete substrate using TIE and SHELL SOLID COUPLING constraints. The shell-to-solid coupling is enforced by the automatic 143 creation of an internal set of distributing coupling constraints between the nodes on the shell 144 edge and nodes on the solid surface. An optimum mesh size of 100 mm is arrived from the 145 convergence study. Static Riks analysis with arc length control is used for non linear analysis 146 147 of RC columns.

The developed FE models for all columns is analyzed and compared with the test data and the explanation is provided for column RCC6. Fixed boundary condition is ensured at one end by arresting the translations U_X , U_Y and U_Z and rotations θ_X , θ_Y , θ_Z at and the other end is kept free. The specimen RCC6 is strengthened with 1.2 mm thick CFRP composite and S4R shell element is used to model 1.2 mm thick FRP composite. Axial load up to 600 kN is applied on the free end in one single load step using static general analysis and kept constant. An incremental lateral load is also applied on the free end in the second step using static Riks analysis till yield strain 4940 μ m/m is reached in the longitudinal steel reinforcement and the analysis terminates automatically. The failure pattern of column RCC6 obtained from the finite element model closely predicts the experimental failure pattern (Fig. 2). The lateral load/lateral deflection is obtained from FEA and compared with the corresponding experimental curve (Fig. 3).









Fig. 3. Lateral Load/Lateral Deflection Curves of Column RCC6

FE models are developed for two standard FRP strengthened RC rectangular columns of 168 cross section 600 mm (b) \times 230 mm (d) to validate the P_n - M_n interaction diagrams. The first 169 column modeled with concrete of strength 20 N/mm² reinforced with six 25 mm dia. steel 170 rebars having yield strength 415 N/mm² distributed on two longer faces. The percentage of 171 steel reinforcement (ρ_s) is 2.1. The column is strengthened with 1.8 mm thick CFRP 172 composite which consist of three layers. The second column is developed for concrete of 173 strength 30 N/mm² reinforced with eight 20 mm dia. steel rebars of yield strength 500 N/mm² 174 distributed equally on four sides. The percentage of steel reinforcement (ρ_s) is 1.8. The 175 column is strengthened with 1.2 mm thick CFRP composite which consist of two layers. 176

P_n - M_n interaction diagrams are generated from the salient points A, B, C and D (Fig. 3) obtained through FE models and compared with the corresponding curves developed by semi-empirical equations (Fig. 4). It is observed that the interaction diagrams developed by FEM validated well with the curves generated by semi-empirical solutions.







Fig. 4. Comparison of P_n - M_n interaction diagram

186 6. Pn - Mn Interaction Diagrams

P_n - M_n interaction diagrams are developed using the proposed semi-empirical solutions and 3D finite element models by varying the unconfined compressive strength of concrete (f'_{co}) , yield strength of steel (f_y) , percentage of steel reinforcement (ρ_s) and thickness of FRP composite (t_{frp}) for FRP strengthened RC columns with aspect ratio greater than 2.0 and without any shape modification. The values of $P_n/f'_{co}bd$ and $M_n/f'_{co}bd^2$ are obtained at four different points (Fig. 1).

Influence of $f_{co}^{'}$, f_{y} and t_{frp} on lateral load carrying capacity of FRP strengthened RC 193 rectangular column subjected to combined axial and lateral loading is studied for various 194 percentage of steel reinforcement as 0.6 %, 1.1%, 1.8%, 2.1% and 2.6% and assuming that 195 the steel reinforcement is distributed equally on two opposite sides. The value of 196 $f_{co}^{'}$ considered in this study is 20 N/mm², 30 N/mm² and 40 N/mm². The yield strength of 197 steel reinforcement considered is 415 N/mm² and 500 N/mm². The thickness of FRP 198 composite is varied from 0.6 mm (1 layer of FRP composite), 1.2 mm (2 layers of FRP 199 composite) and 1.8 mm (3 layers of FRP composite) and the Pn - Mn interaction diagrams for 200 f_{co}^{\prime} 20 N/mm² and f_y 415 N/mm² and 500 N/mm² are shown in Figs. 5 and 6. P_n - M_n 201 interaction diagrams for FRP strengthened RC columns with f_{co}' 30 N/mm² and 40 N/mm² 202 are shown in Figs. 7 to 10. From the interaction diagrams, it is observed that the lateral load 203 carrying capacity is directly proportional to the unconfined compressive strength of concrete, 204 yield strength of steel, thickness of FRP composite and percentage of steel reinforcement. 205





(b)



Fig. 5. P_n - M_n interaction diagrams for f'_{co} 20 N/mm² and f_y 415 N/mm² (a) t_{frp} 0.6 mm,















(b)



Fig. 7. P_n - M_n interaction diagrams for f'_{co} 30 N/mm² and f_y 415 N/mm² (a) t_{frp} 0.6 mm,







 $\begin{aligned} f_{co}^{i} &= 30 \text{ N/mm}^{2} \\ f_{y}^{i} &= 415 \text{ N/mm}^{2} \\ t_{frp}^{i} &= 1.8 \text{ mm} \end{aligned}$

0.4

233

234



1

0.5

0

0





Fig. 8. P_n - M_n interaction diagrams for f'_{co} 30 N/mm² and f_y 500 N/mm² (a) t_{frp} 0.6 mm, (b) t_{frp} 1.2 mm and (c) t_{frp} 1.8 mm

1.8 %

0.1

2.1 %

0.2

(c)

 $M_n/f'_{co}bd^2$

2.6 %

0.3











Fig. 9. P_n - M_n interaction diagrams for f'_{co} 40 N/mm² and f_y 415 N/mm² (a) t_{frp} 0.6 mm,

(b) t_{frp} 1.2 mm and (c) t_{frp} 1.8 mm













256 7. Analysis Problem

- 257 Calculate the moment carrying capacity of a RC rectangular column strengthened with one
- 258 layer of CFRP composite without shape modification subjected to uni axial bending with
- following data. Size of column 200 mm × 500 mm, $f'_{co} = 25 \text{ N/mm}^2$, $f_y = 415 \text{ N/mm}^2$, Axial
- load 1300 kN, Reinforcement Details 4 no's of 20 mm dia. steel bars on two longer sides.
- Assuming cover of 40 mm, Effective depth d= 160 mm
- 262 Percentage of steel reinforcement, $\rho_s = 2.51\%$
- 263 Thickness of FRP for one layer = 0.6 mm
- 264 For $f'_{co} = 20 \text{ N/mm}^2$, $P_n / f'_{co} bd = 0.812$
- 265 From Fig. 5(a), $M_n / f'_{co} bd^2 = 0.22$
- 266 Therefore, $M_n = 56.3$ kN m
- 267 Similarly, For $f'_{co} = 30 \text{ N/mm}^2$, $P_n / f'_{co} bd = 0.54$
- 268 From Fig. 7(a), $M_n / f'_{co} bd^2 = 0.37$
- 269 Therefore, $M_n = 142$ kN m
- From linear interpolation, For $f'_{co} = 25 \text{ N/mm}^2$, $M_n = 100 \text{ kN m}^2$
- For the same column details moment carrying capacity is obtained as 110 kN m from semiempirical equations. So, the load carrying capacity of columns for different values other than the specified values of parameters given in interaction diagrams can be obtained by linear interpolation.

275 8. Summary and Conclusions

Semi-empirical solutions proposed by ACI 440.2R-08 for columns with an aspect ratio less than 2.0 are modified to predict the lateral capacities of RC rectangular columns having aspect ratio greater than 2.0 strengthened using FRP composites and without any shape modification and subjected to combined axial and lateral loading. 3D finite element models for RC rectangular columns strengthened with FRP composites are generated using FEA software ABAQUS. The failure pattern and load/deflection curve obtained from FEA is compared and validated with the corresponding tested specimens. $P_n - M_n$ interaction diagrams are generated from the FE models and semi-empirical solutions and compared. $P_n M_n$ interaction diagrams are developed using the proposed semi-empirical solutions by varying parameters such as unconfined compressive strength of concrete (f'_{co}) and yield strength of steel (f_y), percentage of steel reinforcement (ρ_s) and thickness of FRP composite (f_{trp}).

288 The following major conclusions are drawn from this study.

- The proposed semi-empirical solutions shall be used to predict the lateral load
 carrying capacity of FRP strengthened RC rectangular columns having aspect ratio
 greater than 2.0 and without shape modification of cross section.
- 292 2. Finite element models are validated well with the semi-empirical solutions and can 293 be used for developing $P_n - M_n$ interaction diagrams.
- 294 3. $P_n M_n$ interaction diagrams obtained from this study shall be used for the design of 295 strengthening of RC rectangular columns with large aspect ratio and without shape 296 modification.

9. Suggestions for Future Work

Prototype FRP strengthened RC rectangular columns having different aspect ratios greater than 2.0 shall be tested under combined axial and lateral loading and the developed $P_n - M_n$ interaction diagrams can be further validated

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- 355 Figure Captions
- 356 **Fig. 1.** Representative Interaction Diagram
- 357 Fig. 2. Failure Pattern of Column RCC6 (a) Test (b) FEA
- 358 Fig. 3. Lateral Load/Lateral Deflection Curves of Column RCC6
- **Fig. 4.** Comparison of P_n M_n interaction diagram
- 360 **Fig. 5.** P_n M_n interaction diagrams for f'_{co} 20 N/mm² and f_y 415 N/mm² (a) t_{frp} 0.6 mm,
- 361 (b) t_{frp} 1.2 mm and (c) t_{frp} 1.8 mm
- 362 **Fig. 6.** P_n M_n interaction diagrams for f_{co} 20 N/mm² and f_y 500 N/mm² (a) t_{frp} 0.6 mm,
- 363 (b) t_{frp} 1.2 mm and (c) t_{frp} 1.8 mm
- 364 **Fig. 7.** P_n M_n interaction diagrams for f_{co} 30 N/mm² and f_y 415 N/mm² (a) t_{frp} 0.6 mm,
- 365 (b) t_{frp} 1.2 mm and (c) t_{frp} 1.8 mm
- Fig. 8. P_n M_n interaction diagrams for f_{co} 30 N/mm² and f_y 500 N/mm² (a) t_{frp} 0.6 mm, (b) t_{frp} 1.2 mm and (c) t_{frp} 1.8 mm
- 368 **Fig. 9.** $P_n M_n$ interaction diagrams for f'_{co} 40 N/mm² and f_y 415 N/mm² (a) t_{frp} 0.6 mm,
- 369 (b) t_{frp} 1.2 mm and (c) t_{frp} 1.8 mm
- **Fig. 10.** $P_n M_n$ interaction diagrams for f_{co} 40 N/mm² and f_y 500 N/mm² (a) t_{frp} 0.6 mm,
- 371 (b) t_{frp} 1.2 mm and (c) t_{frp} 1.8 mm