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Fatigue damage in randomly vibrating Jack-up platforms under non-Gaussian loads

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ABSTRACT: The problem of damage estimation due to fatigue in ageing jack-up platforms is considered, using theories of random processes. The sea-wave excitations are modeled as stationary, Gaussian random processes, with specified power spectral density function. The well known Morison's equation is used to calculate the load acting on the structure due to sea-waves. Thus, the force is a non-Gaussian process, whose probability density function (pdf) is not known. Analytical expressions are developed to approximate the pdf of the load. The probabilistic characteristics of the structure response are obtained from a random vibration analysis. The simple peak counting method is adopted for estimating the mean fatigue damage. This requires knowledge of the joint pdf of the peaks for the structure response and their first and second time derivatives, at the same time instant. An analytical formulation is developed to approximate this joint pdf. However, closed form approximations may not always be possible. Instead, we demonstrate the use of a recently developed computational algorithm to approximate the peak distributions. The proposed formulation provides an alternative and computationally cheaper technique for estimating the fatigue damage in comparison to the time consuming Monte Carlo simulations. The results are of use in predicting the remaining lifetime of ageing jack-up platforms.

Keywords: random fatigue, offshore structures, random vibrations, peak distributions, non-Gaussian processes

1 INTRODUCTION

Jack-up platforms are used for exploration and extraction of hydrocarbons from the ocean beds. These are large movable structures, which have been designed to operate at various locations with differing sea-bed conditions, great water depths and under various sea conditions. Over the last few decades, the offshore industry has ventured into deeper waters and more severe sea conditions. This has subjected the jack-up platforms to severe environments. Consequently, this has led to increased scrutiny about the safety of these structures. Usually built at massive costs, there is therefore considerable interest in estimating the lifetime of jack-up platforms and devising efficient maintenance schedules which increase their life span without compromising on their usefulness or safety. This necessitates efficient tools for quantifying the ageing process of these structures.

Structural ageing is a phenomenon characterized by structural degradation due to various phenomena, arising from effects such as corrosion from the sea water, differences in temperatures, creep effects, fatigue damage due to the dynamic effects of sea waves and wind and mechanical damage due to accidental impacts with ships and other marine vessels. In this study, we focus our attention in estimating the fatigue damage on jack-up platforms due to the impact of the sea-waves.

Estimating the fatigue damage in offshore structures is a complicated problem, requiring accurate dynamic analysis of these structures, under sea-wave loadings. The analysis usually relies on a number of simplifying assumptions on the forces caused by the sea waves, the structural behavior and the models for the structures. Thus, often, simple linear wave theories are used where the sea elevations are assumed to be stationary, Gaussian processes and the structure behavior is assumed to be linear. However, the increased loads on the offshore structures due to the venturing out of the industry to deeper waters has necessitated studies which address the complicating features arising from nonlinear waves and structure nonlinearities. Thus, studies on nonlinear sea-conditions (Hasselman 1962, Longuet-Higgins 1963, Langley 1987, Hu and Zhao 1992, Machado 2003) and consequent structural behavior (Grigoriu 1984, Winterstein 1988, Ochi and Anh 1994, Naess 2001, Moarefzadeh and Melchers 1996, 2005) have been discussed in the literature. Questions arising from the nonlinear structural behavior have been addressed using methods based on equivalent linearization and frequency domain expansions using transfer functions (Malhotra and Penzien 1970, Donley and Spanos 1990, Naess and

Ness 1992, Winterstein et al. 1994, Li et al. 1995, Paik and Roesset 1996).

As in many areas of random vibrations, estimates of the expected fatigue damage may be obtained via the Monte Carlo method (Grigoriu 1995, Marczyk 1999). The Monte Carlo method basically involves digital generation of an ensemble of random loads, repeated deterministic analyses of a structure subjected to each sample loading and statistical processing of the results obtained from each deterministic analysis. It is well established that the Monte Carlo method offers the best accuracy subject to the limitations of the sample size considered in the analysis. However, high computational costs and large data storage requirements often, prove to be major limitations in implementation of this technique. This is particularly true in random fatigue analysis, which usually involves repeated analysis of long time histories. On the other hand, analytical methods, though often built on certain assumptions, provide elegant and easy to compute solutions, which have acceptable levels of accuracy.

In this paper, we develop analytical approximations for the expected fatigue damage in randomly vibrating jack-up platforms. The sea wave elevations are assumed to be modeled as stationary Gaussian random processes. The forces acting on the structure are calculated using well known expressions and are non-Gaussian in nature. Consequently, the stresses developed in the structure are also non-Gaussian. The fatigue damage is assumed to be expressible using the peak counting method (Madsen et al. 1986, Sobczyk and Spencer 1994). This, in turn, requires the knowledge of the distribution of the peaks for the stresses developed in the structure (Lin 1967, Nigam 1983, Lutes and Sarkani 1994). Here, the stresses are modeled as random processes and are inherently non-Gaussian in nature, with unknown probability density function (pdf). The focus of this paper is on developing analytical approximations for the peak distributions for the random stresses. This, in turn, implies the need for developing approximations for the joint pdf for the non-Gaussian process (stress), its first and second time derivatives, at any time instant. Here, we propose an analytical method to develop approximations for the joint pdf of the non-Gaussian process, and its first and second time derivatives. Techniques for approximating these expressions have been developed recently (Gupta and van Gelder, 2005). The underlying principles of these techniques had been originally proposed by Naess (1985) for a limited class of problems and recently, had been extended to include a wider class of problems (Gupta and van Gelder, 2007). The approximations for the joint pdf, in turn, lead to analytical expressions for the pdf of the peaks for the non-Gaussian stresses. These expressions are subsequently used in estimating the expected fatigue damage, when the peak counting principles are employed in estimating fatigue damage due to random loadings.

2 PROBLEM STATEMENT

We consider the mean fatigue damage induced in a jack-up platform, subjected to Gaussian sea excitations. The structure is assumed to exhibit only linear behavior. If the structure is discretized using finite elements, the governing equations of motion can be represented as

$$\mathbf{M}\ddot{\mathbf{Z}}(t)+\mathbf{C}\dot{\mathbf{Z}}(t)+\mathbf{K}\mathbf{Z}(t)=\mathbf{F}(t), \qquad (1)$$

where, $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the structure mass, damping and stiffness matrices of dimensions $n\ x\ n$, $\ddot{Z}(t)$, $\dot{Z}(t)$, & $\mathbf{Z}(t)$ are n-dimensional vectors for the nodal accelerations, velocities and the displacements, t is time and $\mathbf{F}(t) = \mathbf{\Lambda} f(t)$ is the n-dimensional vector for the nodal forces. Here, $\mathbf{\Lambda}$ is a n-dimensional vector of constants, which account for the wave velocity variations with depth and $f(t)$ is a time varying function for the force induced on the structure on account of the wave excitations. Calculating $f(t)$ using the well known Morison's equations (Nigam and Narayanan, 1994), it can be shown that

$$f(t)=f_I(t)+f_D(t), \qquad (2)$$

where, $f_I(t)$ is the inertial force due to the water particle acceleration and $f_D(t)$ is the force arising due to hydrodynamic drag. Mathematically, these are represented as

$$f_I(t)=C_M\rho\frac{\pi D^2}{4}\dot{u}(t), \qquad (3)$$

$$f_D(t)=\frac{1}{2}C_D\rho D\left|u(t)\right|u(t), \qquad (4)$$

where, $u(t)$ and $\dot{u}(t)$ are respectively, the water particle velocity and accelerations, D is the diameter of the cylindrical platform supports, ρ is the density of the water and C_M and C_D are respectively, the inertial and drag coefficients. In calculating $f_D(t)$, we assume that the structure is stiff and ignore the effect of the relative velocity due to the movement of the structural member.

As is well known, fatigue damage occurs due to stress reversals and hence counting the number of cycles for a particular load constitutes an important aspect in fatigue analysis. The incremental fatigue damage due to a loading is proportional to the amplitude of a cyclic load and the total fatigue damage is estimated by assuming a suitable damage accumulation rule (Sobczyk and Spencer, 1992), such as, the Palmgren-Miner's hypothesis. Here, the fatigue damage at time t, denoted by $D(t)$, due to loading $Z(t)$, is given by the relation

$$D(t)=\sum_{j=1}^{Z(t)}\alpha s_j^{\beta}, \tag{5}$$

where, α and β are material properties, determined from experiments and s denotes the stress levels for the counted cycles. The fatigue damage caused by a random load is evaluated after estimating the number of cycles corresponding to various stress levels. It can be shown that the expected fatigue damage per unit time, $E\,[d(t)]$, is given by the relation (Tovo, 2002)

$$E[d(t)]=\nu_a\alpha\int_0^{\infty}s^{\beta}p_a(s)\,ds, \tag{6}$$

when the mean stress effect is neglected. Here, $p_a(\bullet)$ is the probability density function of the random amplitude levels, a, and ν_a is the mean rate of occurrence for the cycles. When the peak counting method is considered, the pdf for the amplitudes may be replaced by the pdf for the peaks of the loading process $Z(t)$. Also, it can be shown that $\nu_a=\nu_p$ (Rychlik, 1993). This implies that Eq. (6) can be written as

$$E[d(t)]=\nu_p\alpha\int_0^{\infty}s^{\beta}p_p(s)\,ds. \tag{7}$$

Here, $p_p(s)$ denotes the pdf for the peaks of the random process $Z(t)$. The mean occurrence rate of the peaks can be calculated from the well-known Rice's formula (Rice, 1954)

$$\nu_p=\int_{-\infty}^{0}\ddot{z}p_{\dot{Z}\ddot{Z}}(0,\dot{z})d\ddot{z}. \tag{8}$$

Here, $p_{\dot{Z}\ddot{Z}}(\dot{z},\ddot{z})$ is the joint pdf for $\dot{Z}(t)$ and $\ddot{Z}(t)$. If $Z(t)$ is assumed to be stationary, the expected fatigue damage for a time duration of T is thus given by

$$E\big[D(t)\big]=TE\big[d(t)\big]. \tag{9}$$

In this study, we assume that the water particle velocity $u(t)$ can be modeled as a stationary, Gaussian random process. Since the time derivative of a Gaussian random process is also Gaussian, $\dot{u}(t)$, and in turn, $f_I(t)$, are Gaussian random processes. However, $f_D(t)$ being a nonlinear function of $u(t)$, is non-Gaussian. This implies that even if the structure behavior is assumed to be linear, the structure response, at the i^{th} location (node), $Z_i(t)$, is non-Gaussian and can be represented as

$$Z_i(t)=g\big[u(t)\big]. \tag{10}$$

where, $g[\bullet]$ is a nonlinear transformation function. As is well known, the probability distribution function (PDF) for the peaks of $Z(t)$, above a threshold level z, is given by (Nigam 1983, Lutes and Sarkani 1996)

$$P_{pz}(z;t)=1-\frac{1}{\psi}\int_z^{\infty}\int_{-\infty}^{0}\ddot{z}p_{z\dot{z}\ddot{z}}(z,0,\ddot{z};t)d\ddot{z}\,dz \tag{11}$$

where, p_Z is the random variable denoting the peaks of the response $Z(t)$,

$$\psi=\int_{-\infty}^{0}\ddot{z}p_{\dot{Z}\ddot{Z}}(0,\ddot{z};t)d\ddot{z}, \tag{12}$$

and $p_{Z\dot{Z}\ddot{Z}}(\bullet)$ is the instantaneous joint pdf for $Z(t)$ and its first and second time derivatives. The corresponding pdf for p_Z is given by

$$p_{p_z}(z;t)=\frac{1}{\psi}\int_{-\infty}^{0}\ddot{z}p_{Z\dot{Z}\ddot{Z}}(z,0,\ddot{z};t)d\ddot{z}. \tag{13}$$

Since $f(t)$ in Eq. (1) is non-Gaussian, the structure response $Z(t)$ is non-Gaussian, whose pdf is difficult to estimate. The knowledge of the joint pdf $p_{Z\dot{Z}\ddot{Z}}(\bullet)$ is not available either. The focus of this study is to first obtain approximations for $p_{Z\dot{Z}\ddot{Z}}(\bullet)$ and subsequently, develop approximations for the expected fatigue damage, $E[D(t)]$. This has been discussed in the following section.

3 FORMULATION

We assume that $u(t)$ is a stationary, Gaussian random process with specified power spectral density function (PSD), $S_{uu}(\omega)$, and with mean velocity u_0. Following the spectral representation of a random process, $u(t)$ can be expressed as

$$u(t)=u_0+\sum_{j=-N}^{N}(X_j+iY_j), \tag{14}$$

where,

$$X_j=\frac{\sigma_j}{2}[U_j\cos(\omega_j t)+V_j\sin(\omega_j t)], \tag{15}$$

$$Y_j=\frac{\sigma_j}{2}[U_j\sin(\omega_j t)-V_j\cos(\omega_j t)]. \tag{16}$$

Here, U_j and V_j are mutually independent standard normal random variables, N denotes the number of discretized segments for the PSD, $S_{uu}(\omega)$, σ_j^2 is the area of the j^{th} segment, corresponding to frequency ω_j and $i=\sqrt{-1}$. It must be noted here that X_j and Y_j are mutually correlated Gaussian random variables, with zero mean and variance equal to σ_j^2. It follows from Eq. (14) that the time derivative $\dot{u}(t)$ can be expressed as

$$\dot{u}(t)=\sum_{j=-N}^{N}(-\omega_j Y_j+i\omega_j X_j). \tag{17}$$

This leads to the expression for $f_I(t)$ as

$$f_I(t)=C_M\rho\frac{\pi D^2}{4}\sum_{j=-N}^{N}\omega_j(-Y_j+iX_j). \tag{18}$$

The corresponding expressions for $f_D(t)$ is given by

$$f_D(t)=\frac{1}{2}C_D\rho D\text{sgn}[u(t)]\Big\{\sum_{j=-N}^{N}\sum_{k=-N}^{N}(X_j+iY_j)\times(X_k+iY_k)+2u_0\sum_{j=-N}^{N}(X_j+iY_j)+u_0^2\Big\}. \tag{19}$$

Here, sgn[•] is the signum function and takes a value of $+1$, -1 or 0 respectively, for positive, negative and zero arguments. The forcing function can now be written as

$$f(t)=f_L(t)+f_Q(t), \tag{20}$$

where, $f_L(t)$ and $f_Q(t)$ respectively, denote the linear and quadratic terms, such that,

$$f_L(t)=\sum_{j=-N}^{N}(A_j+iB_j), \tag{21}$$

$$f_Q(t)=\frac{\text{sgn}[u(t)]}{2}C_D\rho D\sum_{j=-N}^{N}\sum_{k=-N}^{N}(X_j+iY_j)(X_k+iY_k) \tag{22}$$

Here,

$$A_j=\text{sgn}[u(t)]C_D\rho Du_0X_j-C_M\rho\frac{\pi D^2}{4}\omega_jY_j \tag{23}$$

$$B_j=\text{sgn}\big[u(t)\big]C_D\rho Du_0Y_j-C_M\rho\frac{\pi D^2}{4}\omega_jX_j. \tag{24}$$

Consequently, the structure response, at the i^{th} node, can also be spilt into the linear response $Z_L(t)$ and the quadratic correction term $Z_Q(t)$, given by (Rychlik & Gupta, 2006)

$$Z_L(t)=\sum_{j=-N}^{N}H(\omega_j)\big\{A_j+iB_j\big\},$$

$$Z_Q(t)=\frac{\text{sgn}\big[u(t)\big]}{2}C_D\rho D \tag{25}$$

$$\times\sum_{j=-N}^{N}\sum_{k=-N}^{N}H(\omega_j+\omega_k)(X_j+iY_j)(X_k+iY_k). \tag{26}$$

Here, $H(\omega_j)$ is the structure transfer function at the j^{th} node, corresponding to frequency ω_j, and is given by the j^{th} entry of the vector $\mathbf{H}(\omega)$, and is given by the relation

$$H(\omega)=\mathbf{H}_j(\omega)=\Big\{\Big[-\omega^2\mathbf{M}+\mathrm{i}\omega\mathbf{C}+\mathbf{K}\Big]^{-1}\Lambda(\omega)\Big\}_j. \tag{27}$$

Introducing the following relations,

$$L_{1_j}=H(\omega_j)+H(-\omega_j), \tag{28}$$

$$L_{2_j}=H(\omega_j)+H(-\omega_j), \tag{29}$$

$$R_{1_j}=H(-\omega_j-\omega_k)+H(\omega_j+\omega_k)+H(\omega_j-\omega_k)+H(-\omega_j+\omega_k), \tag{30}$$

$$R_{2_j}=H(\omega_j-\omega_k)+H(-\omega_j+\omega_k)-H(-\omega_j-\omega_k)-H(\omega_j+\omega_k), \tag{31}$$

$$R_{3_j}=i\{H(\omega_j+\omega_k)-H(-\omega_j-\omega_k)+H(-\omega_j+\omega_k)-H(\omega_j-\omega_k)\}, \tag{32}$$

$$R_{4_j}=i\big\{H(\omega_j+\omega_k)-H(-\omega_j-\omega_k)-H(-\omega_j+\omega_k)+H(\omega_j-\omega_k)\big\}, \tag{33}$$

it can be shown that

$$Z(t)=Z_L(t)+Z_Q(t), \tag{34}$$

$$Z_L(t)=\sum_{j=1}^{N}\Big\{A_jL_{1_j}-B_jL_{2_j}\Big\}, \tag{35}$$

$$Z_Q(t)=\frac{\text{sgn}\big[u(t)\big]}{2}C_D\rho D\sum_{j=1}^{N}\sum_{k=1}^{N}\Big\{R_{1_{jk}}X_jX_k+R_{2_{jk}}Y_jY_k+R_{3_{jk}}X_jY_k+R_{4_{jk}}Y_jX_k\Big\}. \tag{36}$$

Similarly, the first and second time derivatives of $Z(t)$, can be written as

$$\dot{Z}(t)=\sum_{j=1}^{N}\Big\{\dot{A}_jL_{1_j}-\dot{B}_jL_{2_j}\Big\}+\frac{\text{sgn}\big[u(t)\big]}{2}C_D\rho D\sum_{j=1}^{N}\sum_{k=1}^{N}\times\Big\{R_{1_{jk}}(\dot{X}_jX_k+X_j\dot{X}_k)+R_{2_{jk}}(\dot{Y}_jY_k+Y_j\dot{Y}_k)+R_{3_{jk}}(\dot{X}_jY_k+X_j\dot{Y}_k)+R_{4_{jk}}(\dot{Y}_jX_k+Y_j\dot{X}_k)\Big\}, \tag{37}$$

$$\ddot{Z}(t)=\sum_{j=1}^{N}\left\{\ddot{A}_jL_{1_j}-\ddot{B}_jL_{2_j}\right\}+\frac{\text{sgn}[u(t)]}{2}C_D\rho D\sum_{j=1}^{N}\sum_{k=1}^{N}$$
$$\times\Big\{R_{1_{jk}}(\ddot{X}_jX_k+2\dot{X}_j\dot{X}_k+X_j\ddot{X}_k)$$
$$+R_{2_{jk}}(\ddot{Y}_jY_k+2\dot{Y}_j\dot{Y}_k+Y_j\ddot{Y}_k)$$
$$+R_{3_{jk}}(\ddot{X}_jY_k+2\dot{X}_j\dot{Y}_k+X_j\ddot{Y}_k)$$
$$+R_{4_{jk}}(\ddot{Y}_jX_k+2\dot{Y}_j\dot{Y}_k+Y_j\ddot{X}_k)\Big\}. \quad (38)$$

Here,

$$\dot{A}_j=\frac{\text{sgn}[u(t)]}{2}C_D\rho Du_0\dot{X}_j-C_M\rho\frac{\pi D^2}{4}\omega_j\dot{Y}_j, \quad (39)$$

$$\ddot{A}_j=\frac{\text{sgn}[u(t)]}{2}C_D\rho Du_0\ddot{X}_j-C_M\rho\frac{\pi D^2}{4}\omega_j\ddot{Y}_j, \quad (40)$$

$$\dot{B}_j=\frac{\text{sgn}[u(t)]}{2}C_D\rho Du_0\dot{Y}_j+C_M\rho\frac{\pi D^2}{4}\omega_j\dot{X}_j, \quad (41)$$

$$\ddot{B}_j=\frac{\text{sgn}[u(t)]}{2}C_D\rho Du_0\ddot{Y}_j+C_M\rho\frac{\pi D^2}{4}\omega_j\ddot{X}_j. \quad (42)$$

and

$$\dot{X}_j=\frac{\omega_j\sigma_j}{2}\left\{-U_j\sin(\omega_jt)+V_j\cos(\omega_jt)\right\}, \quad (43)$$

$$\ddot{X}_j=\frac{\omega_j^2\sigma_j}{2}\left\{-U_j\sin(\omega_jt)-V_j\cos(\omega_jt)\right\}, \quad (44)$$

$$\dot{Y}_j=\frac{\omega_j\sigma_j}{2}\{U_j\ \cos(\omega_jt)+V_j\sin(\omega_jt)\}, \quad (45)$$

$$\ddot{Y}_j=\frac{\omega_j^2\sigma_j}{2}\{-U_j\sin(\omega_jt)+V_j\cos(\omega_jt)\}. \quad (46)$$

Let us denote $\mathbf{V}=[\mathbf{X},\mathbf{Y}]$, where $\mathbf{V}$ is $(2N\times 1)$ dimensional vector of random variables. The structure response and the first and second time derivatives, in terms of $\mathbf{V}$, are given by,

$$Z(t)=g[V_1,...,V_{2N}], \quad (47)$$

$$\dot{Z}(t)=\sum_{j=1}^{2N}g_j^{'}\dot{V}_j(t), \quad (48)$$

$$\ddot{Z}(t)=\sum_{j=1}^{2N}\sum_{k=1}^{2N}g_{jk}^{''}\dot{V}_j\dot{V}_k(t)+\sum_{j=1}^{2N}g_j^{'}\ddot{V}_j(t). \quad (49)$$

Here, g'_j and g''_{jk} are, respectively, the first and second derivatives of $g[\bullet]$ with respect to V_j and $V_j\ \&V_k$.

To proceed, we first introduce a set of transformations that had first been introduced in Naess (1985), with relation to the study of extremes of random processes. Using similar principles, we now rewrite

$$p_{Z\dot{Z}\ddot{Z}}(z,0,\ddot{z})=\int_{-\infty}^{\infty}..\int_{-\infty}^{\infty}p_{V_2..V_{2N}\dot{V}_2..\dot{V}_{2N}Z\dot{Z}\ddot{Z}}$$
$$\times(v_2,...,\dot{v}_{2N},z,0,\ddot{z})\text{d}v_2...\text{d}\dot{v}_{2N} \quad (50)$$

Using the standard technique of transformation of random variables, we seek the relationship between the joint pdf $p_{V_2..V_{2N}\dot{V}_2..\dot{V}_{2N}Z\dot{Z}\ddot{Z}}(\bullet)$ and $p_{V_1..V_{2N}\dot{V}_i..\dot{V}_{2N}\ddot{Z}}(\bullet)$. In order to achieve this, we assume that at time t, $\dot{Z}$ and $\dot{Z}$ are functions of V_1 and only, with all the other random variables remaining constant at $V_2=v_2,..,\ V_{2N}=v_{2N}$, $\dot{V}_2=\dot{v}_2,...,\dot{V}_{2N}=\dot{v}_{2N}$. In other words, we write

$$Z|_{V_2..V_{2N}\dot{V}_2..\dot{V}_{2N}}=\tilde{g}[V_1],$$
$$\dot{Z}|_{V_2..V_{2N}\dot{V}_2..\dot{V}_{2N}}=\tilde{g}'\dot{V}_1, \quad (51)$$

where, $\tilde{g}[\bullet]$ is the modified function when $V_2,...,V_{2N}$ are assumed to be fixed at values $v_2,..,v_{2N}$, respectively and $\tilde{g}'$ denotes the derivative of $\tilde{g}[\bullet]$ with respect to V_1. Assuming that for fixed values of $Z=z$ and $V_2=v_2,...,\ V_{2N}=v_{2N}$, there exists r solutions for $(V_1,\dot{V}_1)$ from Eqs. (47)–(48), we rewrite Eq. (50) in the form

$$p_{Z\dot{Z}\ddot{Z}}(z,0,\ddot{z})=\sum_{k=1}^{r}\int_{\Omega_k}..\int\frac{1}{|\mathbf{J}_2|}p_{V_1..V_{2N}\dot{V}_1..\dot{V}_{2N}\ddot{Z}}(v_1^{(k)},v_2,..v_{2N},$$
$$\dot{v}_1^{(k)},\dot{v}_2,..\dot{v}_{2N},\ddot{z})dv_2..dv_{2N}d\dot{v}_2..d\dot{v}_{2N}\ . \quad (52)$$

Here, Ω_k is the domain of integration determined by the permissible set of values $(v_2,..,v_{2N},\ \dot{v}_2,..,\dot{v}_{2N})$ for each solution for $(V_1,\dot{V}_1)$. $\mathbf{J}_2$ is the Jacobian matrix, given by

$$\mathbf{J}_2=\begin{bmatrix}\partial Z/\partial V_1 & \partial Z/\partial\dot{V}_1\\ \partial\dot{Z}/\partial V_1 & \partial\dot{Z}/\partial\dot{V}_1\end{bmatrix}=\begin{bmatrix}\tilde{g}' & 0\\ \tilde{g}'' & \tilde{g}'\end{bmatrix}. \quad (53)$$

Now, the joint pdf $p_{V_1..V_{2N}\dot{V}_i..\dot{V}_{2N}\ddot{Z}}(\bullet)$ can be written as

$$p_{V_1..V_{2N}\dot{V}_1..\dot{V}_{2N}\ddot{Z}}(v_1,.,v_{2N},\dot{v}_1,.,\dot{v}_{2N},\ddot{z})=p_{\ddot{Z}|\mathbf{V}\dot{\mathbf{V}}}(\ddot{z})p_{\mathbf{V}\dot{\mathbf{V}}}(\mathbf{v},\dot{\mathbf{v}}) \quad (54)$$

Since $\mathbf{V}(t)$ is a vector of stationary, Gaussian random processes, it follows that is jointly Gaussian. From Eq. (49), it is seen that $\ddot{Z}|_{\mathbf{V},\dot{\mathbf{V}}}$ is a linear sum of the components of $\ddot{\mathbf{V}}$ and is thus Gaussian with mean and variance, given by

$$\mu=E\left[a+\sum_{j=1}^{2N}b_j\ddot{V}_j\right]=a+\sum_{j=1}^{2N}b_jE\left[\ddot{V}_j\right], \quad (55)$$

$$\sigma^2 = \sum_{j=1}^{2N}\sum_{k=1}^{2N} b_j b_k \left(E\left[\ddot{V}_j \ddot{V}_k\right] - E\left[\ddot{V}_j\right] E\left[\ddot{V}_k\right]\right). \tag{56}$$

Here, $a = \sum_{j=1}^{2N}\sum_{k=1}^{2N} \dot{v}_j \dot{v}_k (g''_{jk} | V_j = v_j, V_k = v_k)$ and $E[\bullet]$ is the expectation operator. Since appropriate linear transformations can transform a vector of non-zero mean, mutually correlated Gaussian processes into a vector of zero-mean, mutually independent Gaussian processes, without loss of generality, it can be assumed that **V** constitutes a vector of zero-mean mutually independent stationary, Gaussian processes. Thus, it can be shown that $\mu = a$ and $\sigma^2 = \sum_{j=1}^{2N} b_j^2 E[\ddot{V}_j^2]$.

It follows that Eq. (54) can be written in the form

$$p_{V_1..V_{2N}\dot{V}_1..\dot{V}_{2N}\ddot{Z}}(\mathbf{v}, \dot{\mathbf{v}}, \ddot{\mathrm{z}}) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(\ddot{z}-a)^2}{2\sigma^2}\right] \times \prod_{j=1}^{2N} p_{V_j \dot{V}_j}(v, \dot{v}) \tag{57}$$

where, $p_{V_i}(v_j)$ and $p_{\dot{V}_i}(\dot{v}_j)$ denote respectively, the pdf of V_j and $\dot{V}_j$. When appropriate transformations have been applied, such that **V** constitutes a vector of zero-mean, mutually independent Gaussian random processes, we write

$$p_{V_j \dot{V}_j}(v_{j,} \dot{v}_j) = \frac{1}{2\pi\sigma_j \tilde{\sigma}_j} \exp\left[-\frac{v^2}{2\sigma_j^2} - \frac{\dot{v}^2}{2\tilde{\sigma}_j^2}\right], \tag{58}$$

where, σ_j and $\tilde{\sigma}_j$ are, respectively, the standard deviations of V_j and $\dot{V}_j$. It must be noted that for stationary, Gaussian processes, $V \perp \dot{V}$. Substituting Eq. (57) in Eq. (52), an approximation for the joint pdf, $p_{Z\dot{Z}\ddot{Z}}(z, 0, \ddot{z})$, is obtained.

The next step in determining the pdf for peaks of $Z(t)$ lies in evaluating expressions in Eqs. (11) – (13). We first focus attention on evaluation of the integral of the type in Eq. (13). Substituting the expressions for the joint pdf for $p_{Z\dot{Z}\ddot{Z}}(z, 0, \ddot{z})$, we get

$$\Upsilon(z) = \sum_{k=1}^{r} \left\{ \int_{\Omega_k} \frac{p_{V_1 \dot{V}_1}(v_1^{(k)}, \dot{v}_1^{(k)})}{|\mathbf{J}_2|} G_k(\mathbf{v}, \dot{\mathbf{v}}) \times \prod_{j=2}^{2N} p_{V_j \dot{V}_j}(v_j, \dot{v}_j)\, dv_2..dv_{2N} d\dot{v}_2..d\dot{v}_{2N} \right\}, \tag{59}$$

where,

$$G_k(\bullet) = \int_{-\infty}^{0} \ddot{z} p_{\ddot{Z}|V\dot{V}}(\ddot{z} | \mathbf{V} = \mathrm{v}, \dot{\mathbf{V}} = \dot{\mathbf{v}}) d\ddot{z} = -\frac{1}{2\sqrt{\pi}} \left[\sqrt{2}\sigma \exp\left(-\frac{a^2}{2\sigma^2}\right) + a\sqrt{\pi}\,\mathrm{erf}\left(\frac{a}{\sqrt{2}\sigma}\right) - a\sqrt{\pi} \right]. \tag{60}$$

Here, $\mathrm{erf}(x) = (2\int_0^x \exp[-t^2/2] dt)/\sqrt{\pi}$.

The constant Ψ in Eq. (12) can be evaluated from

$$\psi = \int_{-\infty}^{\infty} \int_{-\infty}^{0} \ddot{z} p_{Z\dot{Z}\ddot{Z}}(z, 0, \ddot{z}) d\ddot{z} dz = \int_{-\infty}^{\infty} \gamma(z) dz. \tag{61}$$

The primary difficulties involved in evaluating $I(z)$ in Eq. (59) are:

(a) determining the domain of integration Ω_k, defined by the possible set of solutions for $(V_1^{(k)}, \dot{V}_1^{(k)})$, and,
(b) evaluation of the multidimensional integrals, which are of the form

$$\gamma_k = \int_{\Omega_k} .. \int f(v_1^{(k)}, v_2, .., v_{2N}, \dot{v}_1^{(k)}, \dot{v}_2, .., \dot{v}_{2N}) \times \prod_{j=2}^{2N} p_{V_j \dot{V}_j}(v_{j,} \dot{v}_j) dv_2..dv_{2N} d\dot{v}_2..d\dot{v}_{2N}, \tag{62}$$

where, $f(\bullet) = |\mathbf{J}_2|^{-1} p_{V_1 \dot{V}_1}(v_1^{(k)}, \dot{v}_1^{(k)}) G_k(\mathbf{v}, \dot{\mathbf{v}})$. The dimension of the integrals in Eq. (62) is $2(2n - 1)$. In this study, we adopt a numerical strategy to overcome these difficulties. This is discussed in the following section.

4 NUMERICAL ALGORITHM

A crucial step in the above formulation lies in evaluating integrals of the type as in Eq. (62). Closed form solutions for the integrals are possible only for a limited class of problems. Here, we propose the use of Monte Carlo methods, in conjunction with importance sampling, to increase the efficiency, for evaluating these integrals. The integrals in Eq. (62) can be recast as

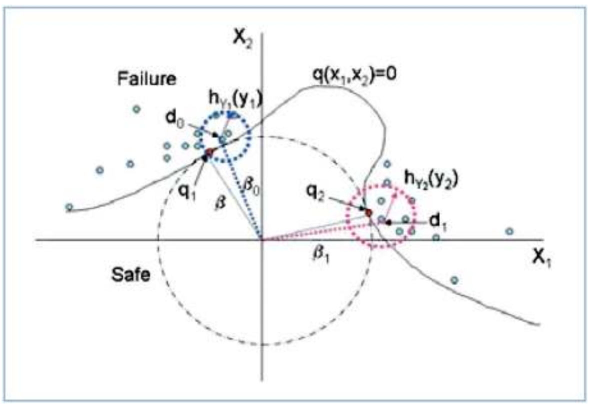

Figure 1. Schematic diagram for numerical algorithm for evaluating multidimensional integrals; $g(x_1, x_2) = 0$ is the limit surface in the $X_1 - X_2$ random variable space; $h_{Y_1}(y_1)$ and $h_{Y_2}(y_2)$ are the two importance sampling pdfs; two design points at distance β from the origin.

$$\Upsilon_k = \int_{-\infty}^{\infty} I[q(\mathbf{X}) \le 0] f(\mathbf{X}) \frac{p_{\mathbf{X}}(\mathbf{x})}{h_{\mathbf{Y}}(\mathbf{x})} h_{\mathbf{Y}}(\mathbf{x}) d\mathbf{x}$$
$$= \frac{1}{N}\sum_{j=1}^{N} I[q(\mathbf{X}_j) \le 0] f(\mathbf{X}_j) \frac{p_{\mathbf{X}}(\mathbf{x}_j)}{h_{\mathbf{Y}}(\mathbf{x}_j)}. \quad (63)$$

where, $h_{\mathbf{Y}}(\mathbf{x})$ is the importance sampling pdf and $I[\bullet]$ is an indicator function taking values of unity if $q(\mathbf{X}) \le 0$, indicating that the sample lies within the domain of integration Ω_k, and zero otherwise. Since the problem is formulated into the standard normal space $\mathbf{X}$, $h_{\mathbf{Y}}(\mathbf{x})$ can be taken to be Gaussian with unit standard deviation and shifted mean. The difficulty, however, lies in determining where should $h_{\mathbf{Y}}(\mathbf{x})$ be centered. An inspection of Eq. (63) reveals that the form of the integrals are similar to reliability integrals, which are of the form

$$\Upsilon_k = \int_{-\infty}^{\infty} I[q(\mathbf{X}) \le 0] p_{\mathbf{x}}(\mathbf{x}) d\mathbf{x}. \quad (64)$$

This implies that for efficient computation of the integrals, the importance sampling pdf $h_{\mathbf{Y}}(\mathbf{x})$ may be centered on the design point for the function $q(\mathbf{X}) = 0$.

If $q(\mathbf{X}) = 0$ is available in explicit form, first order reliability methods (Madsen et al., 1986, Ditlevsen and Madsen 1996, Melchers 1999) can be used to determine the design point. However, in many practical applications, in general, $q(\mathbf{X}) = 0$ is not available explicitly. In such situations, an adaptive importance sampling strategy (Bucher, 1988) can be adopted to determine the design point. In certain problems, the domain of integration, characterized by $q(\mathbf{X}) = 0$, may consist of multiple design points or multiple regions which contribute significantly to Υ_k (Karamchandani et al. 1989, Breitung and Faravelli 1996, Der Kiureghian and Dakessian 1998, Gupta and Manohar 2004). This is especially true when $q(\mathbf{X}) = 0$ is highly nonlinear, irregular or consists of disjointed regions. In these situations, it is necessary to construct a number of importance sampling functions, with each function centered at the various design points.

The steps for implementing the algorithm for numerical evaluation of integrals of the type in Eq. (62) has been developed and discussed (Gupta and van Gelder, 2005a,b). The sequential steps for implementing the algorithm are detailed below, with reference to the schematic diagram in Fig. 1:

1. Carry out pilot Monte Carlo simulations in the standard normal space. If there are too few samples in the failure domain, we carry out Monte Carlo simulations with a Gaussian importance sampling function with mean zero and a higher variance. On the other hand, if there are too few samples in the safe region, the variance of the importance sampling function is taken to be smaller. Repeat this step, till we have a reasonable number of samples in the failure and the safe regions.
2. We sort the samples lying in the failure domain according to their distance from the origin.
3. A Gaussian importance sampling pdf is constructed which is centered at the sample in the failure domain lying closest to the origin. Let this point be denoted by d_0 and its distance from the origin be denoted by β_0.
4. We check for samples in the failure domain, within a hyper-sphere of radius β_1, $\beta_1 - \beta_0 = \varepsilon$, where ε is a positive number.
5. For samples lying within this hyper-sphere, we check for the sample d_1, which lie closest to the origin but are not located in the vicinity of d_0. This is checked by comparing the direction cosines of d_1 and d_0.
6. By comparing the direction cosines of all samples lying within the hyper-sphere of radius β_1, we can identify the number of design points. We construct importance sampling pdfs at each of these design points. If there exists no samples with direction cosines distinctly different from d_0, there is only one design point and a single importance sampling pdf is sufficient.
7. During importance sampling procedure correspon-ding to a design point, for each sample realization, we check if v_1 and $\dot{v}_1$ are real. The indicator function is assigned a value of unity if real, and zero otherwise.
8. An estimate of Υ_k is obtained from Eq. (63).

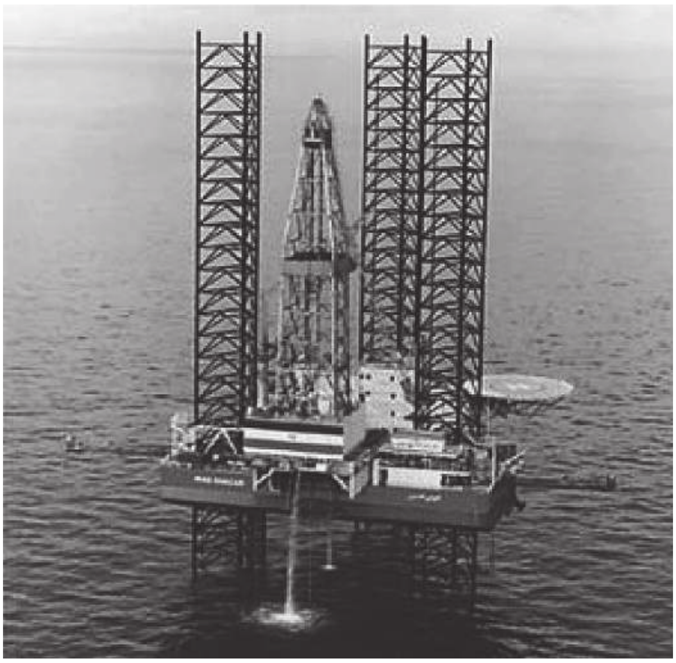

Figure 2. Jack-up platform considered in numerical example.

In the following section, we implement the formulation and the proposed algorithm to predict the probability density function and the probability distribution function for the response of a offshore platform subjected to sea-wave random excitations.

5 NUMERICAL EXAMPLE

We use the proposed formulation to estimate the fatigue damage in one of the structural members of a randomly vibrating jack-up platform. The jack-up platform considered is shown in Fig. 2. Details about the jack-up platform are available in Shabakhty (2004). The excitation loads are assumed to be only due to the sea-waves acting on the structure and are taken to be random dynamic loads.

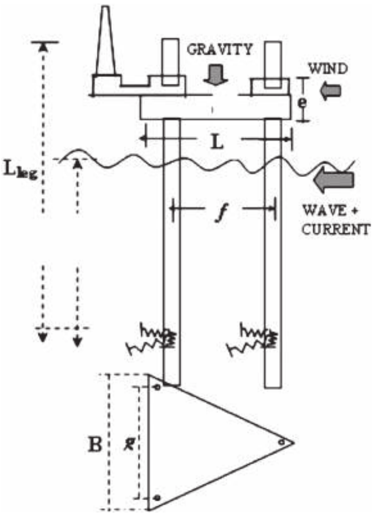

Figure 3. Schematic diagram for the elevation and the plan views for the jack-up platform.

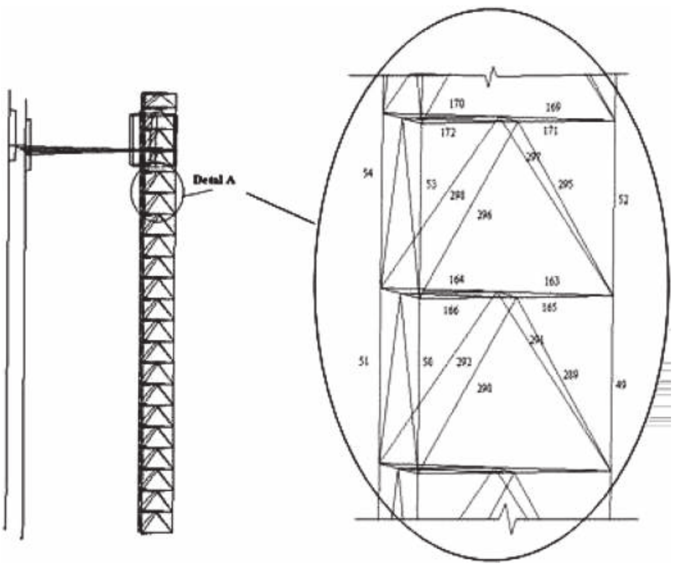

Figure 4. Finite element model for one of the legs for the jackup platform; a close up of the FE model is provided for the region around the element being studied (element number 295).

The water particle velocity of the sea waves are modeled as stationary Gaussian random processes and are characterized in terms of the PSD of the form

$$S_{UU}(\omega) = \frac{\omega^2 \cosh^2\left[2\pi(d+\eta)/L_w\right]}{\sinh^2[2\pi d/L_w]} S_w(\omega), \tag{65}$$

where, d is the water depth, L_w is the wavelength determined from the recursive relation

$$\omega^2 = \frac{2\pi g}{L_w} \tanh\left(\frac{2\pi d}{L_w}\right), \tag{66}$$

η denotes the depth under consideration, which is zero at the mean water surface level and negative below, measured vertically along the legs of the platform, and $S_w(\omega)$ is the PSD of the water surface elevations. Here, we adopt the Pierson-Moskowitz (PM) model for $S_w(\omega)$, given by

$$S_w(\omega) = \frac{4\pi^3 H_s^2}{T_z^4} \omega^{-5} \exp\left[-\frac{16\pi^3}{T_Z^4}\omega^{-4}\right], \tag{67}$$

where, H_s is the significant wave height and T_Z is the zero-crossing period. A schematic diagram of the jack-up platform is shown in Fig. 3.

A finite element (FE) model for the structure has been used for the vibration analysis; see Fig. 4 for the FE model for one of the legs. More details about the FE model are available in Shabakhty (2004). The axial, bending and the shear stresses are computed for the element numbered 295 shown in Fig. 4, as this element is found to be the most stressed element.

The numerical values considered in this example are $H_S = 1.75$ m, $T_Z = 7.4$ s, η is taken to vary from 0 to -95 m, L_w is taken to vary from 358.61 to 6.8679, $D = 2.05$ m, $\rho = 1250$ kg/m^3, $C_D = 2.777$ and $C_M = 1.824$. The mean velocity of the current is taken to be 0.10 m/s. The PSD spectrum $S_{UU}(\omega)$ is

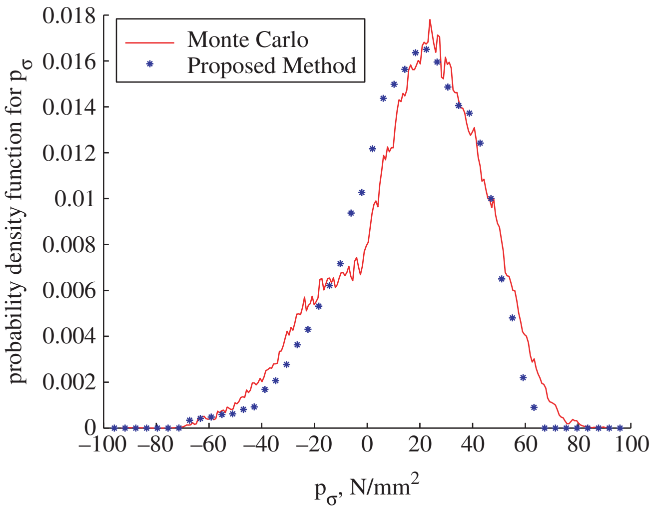

Figure 5. Probability density function (pdf) for the peaks for $\sigma(t)$.

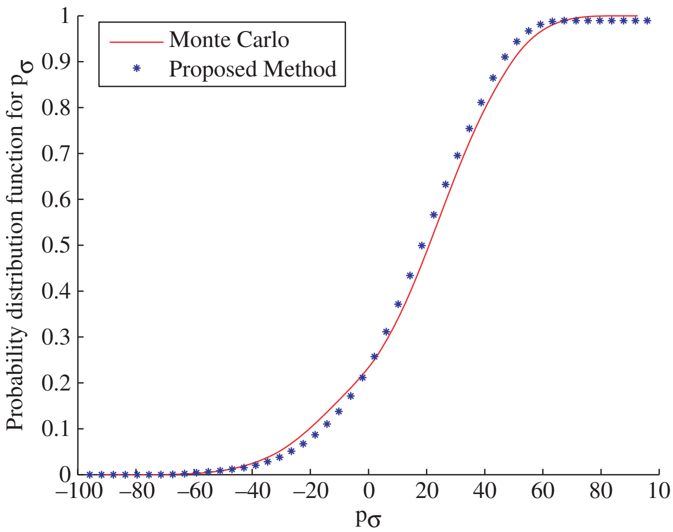

Figure 6. Probability distribution function (PDF) for the peaks for $\sigma(t)$.

discretized into 10 segments and hence $N=10$ in Eq. (14). Thus, the dimensions of the vector of random variables **V**, is 20.

We first carry out a frequency domain vibration analysis on the finite element model. The frequency response function (FRF) for the structure at the desired location of the structure is computed. Subsequently, the parameters defined in Eqs. (28)–(33), are evaluated and stored.

Following Eqs. (34)–(36), it is evident the structure response displacements are non-Gaussian, whose pdf are unknown. We focus our attention only on the stress generated due to the joint action of the axial and bending force, in the element. Thus, the stresses are obtained as linear functions of the displacements and are expressed as $\sigma(t) = \kappa Z(t)$, where, κ is a constant dependent on the material properties and geometric dimensions of the structural member under study. Therefore, it is obvious that $\sigma(t)$ is also non-Gaussian, and has the same distributional properties as $Z(t)$.

The pdf and the corresponding PDF for the peaks of $\sigma(t)$, denoted by the random variable p_σ, using the proposed method, are respectively shown in Figs. 5–6. In implementing the numerical algorithm discussed in the previous section, we consider a sample size of 2×10^5 random variables. It is to be noted that, at any instant of time t, the form of the performance function $q(\mathbf{X}) = 0$ depends on the signum function. Thus, the problem can be viewed as the conjunction of two different problems– depending on the signum function taking values equal to $+1$ or -1. This implies that the contribution to Υ_k in Eq. (63) has contributions corresponding to two different performance function, either of which could have multiple design points. However, it has been found that for this example, there exists only one design point each, corresponding to the two performance functions, subject to the tolerances defined in the algorithm.

The validity of these predictions is compared with the predictions obtained from full scale Monte Carlo simulations, which are assumed to be the benchmark. In the Monte Carlo procedure, we first determine the PSD for $\sigma(t)$, using random vibration theories. Next, we digitally generate an ensemble of time histories for $\sigma(t)$, using the well known spectral decomposition method (Shinozuka and Deodatis, 1991, Nigam and Narayanan, 1994). Using the WAFO toolbox (Brodtkorb et al., 2000), we next identify the peaks corresponding to each sample time history for $\sigma(t)$. Subsequently, we construct the pdf and the corresponding PDF for the peaks of $\sigma(t)$, by statistically processing of the peaks for the entire ensemble. These constructs for the pdf and PDF serve as the benchmark with which we compare the analytical predictions obtained from this study. As can be seen from Figs. 5–6, the predictions compare reasonably well with those obtained from Monte Carlo simulations.

Next, we predict the expected fatigue damage using the peak counting method. The analytical predictions, using Eq. (7), is obtained as 0.0899 for a time duration of 20 s. Here, we assume that the parameters take the numerical values $\alpha = 10^{-4}$ and $\beta = 2.0$. It is to be noted here that these parameters have been arbitrarily assumed for the sake of illustrating the proposed method. The corresponding fatigue damage obtained from Monte Carlo simulations, when the peak counting method is employed, turn out to be 0.1141. However, the computational effort required in making the predictions using the proposed formulation is less than 10% of the computational time required in the Monte Carlo procedure. The saving in computational effort can be significant when analyzing the fatigue damage in large structures for long time durations.

6 CONCLUSIONS

A methodology has been developed for approximating the probability density and distribution function for the peaks of the stresses developed in a randomly vibrating jack-up platform, subjected to non-Gaussian time varying loads. The water surface elevations are modeled as stationary, Gaussian loads. The resulting forces acting on the structure, calculated using the well-known Morison's equations, are non-Gaussian. The structure behavior is assumed to be linear. Random vibration principles, in conjunction with the finite element method, are used to determine the probabilistic characteristics of the response quantities, such as stress and displacements, in a particular location of the structure. These response quantities are modeled as random processes. Using theories of random processes, analytical expressions are developed for the distribution of the peaks of these random quantities. These information, in turn, is subsequently used to estimate the expected fatigue damage, according to the peak counting method. The analytical predictions are compared with those obtained from Monte Carlo simulations which serve as the benchmark. The computational effort involved in the proposed analytical method is significantly less than the Monte Carlo procedure.

A key feature in the development of the proposed method lies in the assumption, that for high thresholds, the number of level crossings of a non-Gaussian process can be modeled as a Poisson point process. The assumption of the out crossings being Poisson distributed has been proved to be mathematically valid for Gaussian processes when the thresholds approach infinity. (Cramer 1966). However, it has been pointed out that for threshold levels of practical interest, this assumption results in errors whose size and effect depend on the bandwidth of the processes (Vanmarcke 1972). While it can be heuristically argued that for high thresholds, the out crossings of non-Gaussian processes can be viewed to be statistically independent and hence can indeed be modeled as a Poisson point process, to the best of the authors' knowledge, studies on the validity of this assumption, do not exist in the literature. The approximations developed in this paper, is thus expected to inherit the associated inaccuracies and limitations, due to this assumption.

It is to be emphasized here that expected fatigue damage can be calculated using various techniques proposed in the literature. It has been shown in the literature that the peak counting method overestimates the fatigue damage and serves as an upper bound for the fatigue damage. Moreover, in realistic structures, the fatigue damage usually takes place due to multiaxial stresses. This however, requires the use of alternative and more involved techniques, such as, the critical plane approaches (Socie 1987, 1993, You and Lee 1996). Some studies involving predicting the fatigue damage due to hot spot stresses or the Von Mises stresses have been reported in the literature (Preumont and Piefort 1994, Pitoiset and Preumont 2000, Rychlik and Gupta 2006). Extension of some these techniques to the present problem are possible and are currently being undertaken by the present authors.

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