Design of *M*-Channel IIR Uniform DFT Filter Banks Using Recursive Digital Filters

M.J. Dehghani, R. Aravind, and K.M.M. Prabhu

In this paper, we propose a method for designing a class of *M*-channel, causal, stable, perfect reconstruction, infinite impulse response (IIR), and parallel uniform discrete Fourier transform (DFT) filter banks. It is based on a previously proposed structure by Martinez et al. [1] for IIR digital filter design for sampling rate reduction. The proposed filter bank has a modular structure and is therefore very well suited for VLSI implementation. Moreover, the current structure is more efficient in terms of computational complexity than the most general IIR DFT filter bank, and this results in a reduced computational complexity by more than 50% in both the critically sampled and oversampled cases. In the polyphase oversampled DFT filter bank case, we get flexible stop-band attenuation, which is also taken care of in the proposed algorithm.

Keywords: DFT filter bank, perfect reconstruction, IIR filter, FIR filter, prototype filter, causal and stable, polyphase, critically sampled, oversampled, computational complexity.

Manuscript received Sept. 30, 2002; revised June 5, 2003.

M.J. Dehghani (phone: + 91 44 257 8361, email: mjdehghani@hotmail.com), R. Aravind (email: aravind@ee.iitm.ernet.in), and K.M.M. Prabhu (email: prabhu_kmm@hotmail.com) are with Electrical Engineering Department, IIT Madras, Chennai-600 036, India.

I. Introduction

In recent years, there has been an increasing trend towards the use of multirate digital signal processing. A filter bank (FB) is a signal processing device that produces *M* signals from a single signal by means of *M* parallel or polyphase filters. There are several designs for perfect reconstruction (PR) digital filter banks, such as parallel uniform bandwidth decomposition [the same as discrete Fourier transform (DFT) FBs], complex-modulated FBs, and cosine-modulated FBs [2]. Although infinite impulse response (IIR) filter banks can potentially offer lower system delays and higher stopband attenuation than their finite impulse response (FIR) counterparts, their design is much more involved, and generally they have been restricted only to the two-channel case [3]-[5]. In fact, the difficulty in designing such filter banks is to satisfy the complicated PR condition and the causality-stability requirements of the filters. Some *M*-channel FBs have been presented [6], [7]. In [6], the design method includes a complicated stabilization procedure of the synthesis filters. Reference [7] proposed the design of a stable, casual, PR, IIR DFT FB, which is based on the geometrical progression expansion of the denominator of the prototype filter and is outlined in section III.

In this paper, we propose a design procedure and the structure for a class of PR IIR FBs, using stable and causal polyphase components. The proposed structure has the same system function denominator for all the polyphase components of its prototype filter. Hence, the PR condition is considerably simplified and it is simpler to satisfy the PR condition and the causality-stability requirements. The proposed work deals with the design of analysis and synthesis filters using a polyphase approach. The main advantages of the proposed method are summarized below:

- These filter banks satisfy both the PR and causality-stability conditions.
- The technique can be used to design filter banks for an arbitrary number of channels *M*, which is a multiple of the decimator/expander factor *D*.
- The analysis and synthesis filters are obtained easily from the prototype filter and can be efficiently implemented, since they have modular structures and are highly parallel. Therefore, they are good candidates for VLSI implementation and suitable for high-bandwidth, complex parallel signal processing.

Section II of the paper details considerations on stability, causality, and PR conditions. Section III gives a general design method with arbitrary frequency specifications for the prototype filter. By allowing the number of polyphase components of the prototype filter to be either a multiple of the decimator/expander factor D or equal to the decimator/expander factor D itself, we have two different structures, namely, the oversampled and the critically sampled filter bank structures. In the same section, we investigate the numerical stability of the proposed technique and also present the realizations of the parallel structures for the critically sampled and oversampled DFT FB. Section IV gives several examples, which are helpful in understanding the proposed method. Finally, section V summarizes the results.

II. Uniform DFT Filter Bank Analysis and PR Conditions

One important class of filter banks is the uniform filter bank, where the input signal is split into equal-width subbands (Fig. 1). In the uniform DFT FB case, the analysis $H_i(z)$ and the synthesis filters $G_i(z)$ are all frequency-translated versions of a prototype lowpass filter [2]. Thus,

$$H_i(z) = H_0(z\,e^{-j2\pi i/M}) \quad \text{and} \tag{1a}$$

$$G_i(z) = G_0(z\,e^{-j2\pi i/M}),\ i = 0, 1, ..., M-1. \tag{1b}$$

Due to non-ideal filters, distortions such as aliasing, imaging, amplitude, and phase are introduced. However, with the proper choice of the prototype filter, the analysis-synthesis system can be free from all these distortions. Such a system satisfies the relation $y(n) = c\,x(n-n_0)$ for any input $x(n)$, and it is said to be a PR system. Using the polyphase decomposition, we can express the analysis and the synthesis filter banks of Fig. 1 [2] as:

$$H_i(z) = \sum_{l=0}^{M-1} z^{-l} E_{il}(z^M) \quad \text{and} \tag{2a}$$

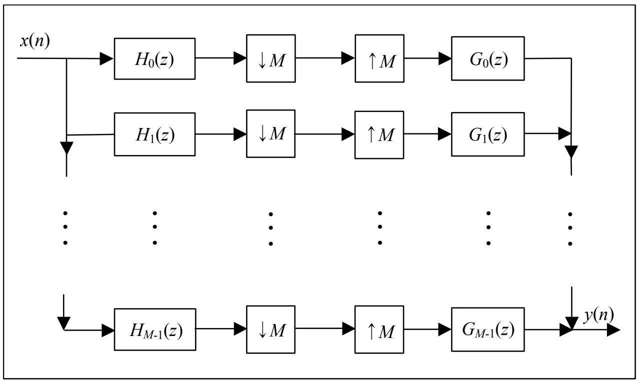

Fig. 1. *M*-channel maximally decimated filter bank.

$$G_i(z) = \sum_{l=0}^{M-1} z^{-(M-1-l)} R_{il}(z^M),\ i = 0, 1, ..., M-1, \tag{2b}$$

where $E_{il}(z^M)$ and $R_{il}(z^M)$ are the polyphase elements of $H_i(z)$ and $G_i(z)$, type I and type II, respectively. It is shown in [2] that the system is PR, in general, if and only if

$$\boldsymbol{R}(z)\boldsymbol{E}(z) = cz^{-m_0}\begin{bmatrix} \boldsymbol{0} & \mathbf{I}_{M-r} \\ z^{-1}\mathbf{I}_r & \boldsymbol{0} \end{bmatrix}, \tag{3}$$

where $\boldsymbol{E}(z) = [E_{il}(z)]$ and $\boldsymbol{R}(z) = [R_{il}(z)]$ are the polyphase matrices, $\mathbf{I}_r$ is an $r \times r$ identity matrix, $0 \le r \le M-1$, m_0 is an integer and *c* is a nonzero constant. Under this constraint, the reconstructed signal is $y(n) = c\,x(n-n_0)$, where $n_0 = Mm_0 + r + M - 1$. In the uniform DFT FB case, the polyphase matrices have, as derived in [2], [7], the following expressions:

$$\boldsymbol{E}(z) = \boldsymbol{W}^* \begin{bmatrix} E_0(z) & 0 & \cdots & 0 \\ 0 & E_1(z) & \cdots & 0 \\ \vdots & \vdots & . & \vdots \\ 0 & 0 & \cdots & E_{M-1}(z) \end{bmatrix} \quad \text{and}$$

$$\boldsymbol{R}(z) = \begin{bmatrix} R_0(z) & 0 & \cdots & 0 \\ 0 & R_1(z) & \cdots & 0 \\ \vdots & \vdots & . & \vdots \\ 0 & 0 & \cdots & R_{M-1}(z) \end{bmatrix} \boldsymbol{W}, \tag{4}$$

where $\boldsymbol{W}$ is the $M \times M$ DFT matrix with (*i*, *l*) elements given by $e^{-j2\pi il/M}/\sqrt{M}$ ($\boldsymbol{W}^*$ is the complex conjugate of $\boldsymbol{W}$). Here, $E_k(z)$ and $R_k(z)$ are the polyphase components of the analysis and the synthesis prototype filters, respectively, and consequently the prototype filters have the following expressions:

$$H_0(z) = \sum_{k=0}^{M-1} z^{-k} E_k(z^M) \quad \text{and} \quad G_0(z) = \sum_{k=0}^{M-1} z^{-(M-1-k)} R_k(z^M). \tag{5}$$

For simplicity, we consider the PR condition with $r = 0$, $m_0 = 0$ and $c = 1$. By substituting (4) in (3), and noting that $\boldsymbol{W}\boldsymbol{W}^* = \mathbf{I}_M$, we get [7]:

$$\boldsymbol{R}(z)\boldsymbol{E}(z) = \begin{bmatrix} R_0(z) & 0 & \cdots & 0 \\ 0 & R_1(z) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & R_{M-1}(z) \end{bmatrix} \cdot \begin{bmatrix} E_o(z) & 0 & \cdots & 0 \\ 0 & E_1(z) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & E_{M-1}(z) \end{bmatrix} = \mathbf{I}_M. \tag{6}$$

Hence, a sufficient condition for PR is $E_k(z) = 1/R_k(z)$, for $k = 0, 1, ..., M-1$. If the prototype analysis filter $H_0(z)$ is causal and stable, then the components $E_k(z)$ are also causal and stable [2]. In the synthesis part, the resulting components $R_k(z)$ are also causal, but their stability requires that $E_k(z)$ be a minimum phase for all k, i.e., all the zeros of $E_k(z)$ are strictly inside the unit circle. Therefore, a straightforward method to design a PR DFT FB is the design of a causal and stable prototype filter (FIR or IIR) $H_0(z)$. In the case of IIR, the only condition for this FB to be stable is that all $E_k(z)$ s must be minimum phase. The main reason for the difficulty in designing a stable PR IIR FB is that there is no simple relationship between the prototype filter and the zeros of its polyphase components. In [7], a design method for an IIR DFT FB having a prototype filter with minimum phase polyphase components was investigated and is summarized in the next section. Figure 2(a) shows the structure of the DFT FB, and we note that the outputs of the inverse discrete Fourier transform (IDFT) operation of Fig. 2(a) are identical to the signals in the corresponding arm of Fig. 1 (after the decimator).

In the above discussion, the filter bank has a maximally decimated or critically sampled structure. Critically sampled refers to the fact that the number of fullband samples per second equals the total number of subband samples per second. In this case, one must carefully control the aliasing in the subband signals through a proper design of the analysis and synthesis filters. We next generalize the FB structure to the oversampled polyphase case, where the decimator/expander factor D is an integer sub-multiple of the number of channels M, i.e., $M = ID$. We refer to I as the oversampling ratio, since it determines the amount of oversampling from the theoretical minimum rate (if $I = 1$, the FB is critically sampled and if $I = 2$, it is oversampled by a factor of two [8]). The general structure shown in Fig.1 becomes an M-channel oversampled FB if we use decimator/expander D instead of M in each channel. Figure 2(b) shows an oversampled DFT FB with the polyphase components $\hat{E}_k(z)$ and $\hat{R}_k(z)$ in the place of $E_k(z)$ and $R_k(z)$. In Appendix A, we have derived a z-domain formulation of the oversampled polyphase decomposition. Based on Appendix A, the system functions $H_0(z)$ and $G_0(z)$ of the prototype analysis and synthesis filters in terms of $\hat{E}_k(z)$ and $\hat{R}_k(z)$, respectively, are:

$$H_0(z) = \sum_{k=0}^{M-1} z^{-k}\hat{E}_k(z^D) \quad \text{and} \tag{7a}$$

$$G_0(z) = \sum_{k=0}^{M-1} z^{-(M-1-k)}\hat{R}_k(z^D). \tag{7b}$$

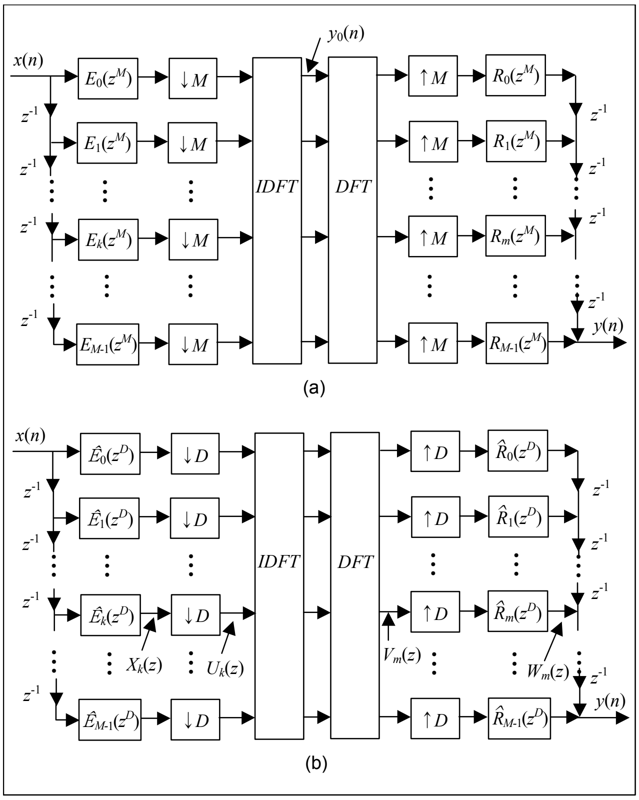

Fig. 2. M-channel DFT filter bank: (a) critically sampled case, (b) oversampled case.

In [8], a time domain approach was used to obtain an oversampled polyphase filter. As we show in the next section, the condition $\hat{E}_k(z)\hat{R}_k(z) = 1$ is sufficient to guarantee PR. The other analysis filters $H_i(z)$ and the synthesis filters $G_i(z)$ are related to the prototype filters as in (1a) and (1b), respectively.

III. M-Channel PR IIR Filter Bank

The proposed filter bank is based on a structure presented in

[1] for IIR filters with z^D in the denominator of the system function for some integer $D > 1$, whose zeros are located on the unit circle, and whose frequency responses have equiripple behavior in the passband and stopband. These system functions are of the form:

$$H_0(z) = \frac{\sum_{k=0}^{M-1} a_k z^{-k}}{1 + \sum_{l=1}^{N} b_l z^{-Dl}}. \tag{8}$$

We observe that such a $H_0(z)$ is suitable for the prototype lowpass filter system function of (7a). From (7a) and (8), we define

$$\hat{E}(z) = \frac{1}{1 + \sum_{l=1}^{N} b_l z^{-l}} \quad \text{and}$$
$$\hat{E}_k(z) = a_k \hat{E}(z), k = 0, 1, ..., M-1. \tag{9}$$

All the polyphase elements $\hat{E}_k(z)$ are thus scaled versions of the same all-pole filter as shown in (9). Hence, the corresponding synthesis polyphase elements $\hat{R}_k(z)$ will be FIR filters with

$$\hat{R}_k(z) = \frac{1}{\hat{E}_k(z)} = \frac{1}{a_k}\left(1 + \sum_{l=1}^{N} b_l z^{-l}\right), \; k = 0, 1, ..., M-1. \tag{10}$$

Since all the poles of $H_0(z)$ are inside the unit circle (causal and stable), the polyphase elements $\hat{E}_k(z)$ are also causal and stable. Therefore, based on the recursive filter design method given in [1], we can design an IIR DFT FB with stable and causal polyphase elements by designing a stable, causal filter consistent with the transfer function (8) as the prototype analysis filter $H_0(z)$ satisfying a given set of specifications.

We now prove the PR property for the filter bank with analysis and synthesis filters as in (7a) and (7b). The signal $X_k(z)$ in the k-th branch shown in Fig. 2(b) is related to the input signal $X(z)$ as

$$X_k(z) = z^{-k} X(z)\, \hat{E}_k(z^D), \quad k = 0, 1, ..., M-1 \tag{11}$$

and the output of the decimator D in the same branch is

$$\begin{aligned} U_k(z) &= \frac{1}{D}\sum_{l=0}^{D-1} X_k(z^{1/D} e^{-j2\pi l/D}) \\ &= \sum_{l=0}^{D-1} z^{-k/D} e^{j2\pi lk/D}\, X(z^{1/D} e^{-j2\pi l/D}) \hat{E}_k(z), \\ & \quad k = 0,1,..., M-1. \end{aligned} \tag{12}$$

The signal $V_m(z)$ in the m-th branch, after the IDFT and DFT shown in Fig. 2(b), is given by

$$\begin{aligned} V_m(z) &= \frac{1}{D} E_m(z) z^{-m/D} \sum_{l=0}^{D-1} e^{j2\pi ml/D} X(z^{1/D} e^{-j2\pi l/D}), \\ & m = 0,1,..., M-1. \end{aligned} \tag{13}$$

After up sampling by D, the output signal of $\hat{R}_m(z^D)$ is

$$\begin{aligned} W_m(z) &= \frac{1}{D} \hat{E}_m(z^D) \hat{R}_m(z^D) z^{-m} \sum_{l=0}^{D-1} e^{j2\pi ml/D} X(z e^{-j2\pi l/D}), \\ & m = 0,1,..., M-1. \end{aligned} \tag{14}$$

The final output is

$$\begin{aligned} Y(z) &= \sum_{m=0}^{M-1} z^{-(M-1-m)} W_m(z) = \frac{1}{D} \sum_{m=0}^{M-1} \hat{R}_m(z^D) \hat{E}_m(z^D) z^{-m} z^{-(M-1-m)} \\ & \times \sum_{l=0}^{D-1} e^{j2\pi ml/D} X(z e^{-j2\pi l/D}). \end{aligned} \tag{15}$$

We now apply the PR condition $\hat{E}_m(z)\hat{R}_m(z) = 1$, for $m = 0, 1, ..., M-1$. Changing the summation order, we get

$$Y(z) = \frac{z^{-(M-1)}}{D} \sum_{l=0}^{D-1} X(z e^{-j2\pi l/D}) \sum_{m=0}^{M-1} e^{j2\pi l m/D}. \tag{16}$$

Since $\sum_{m=0}^{M-1} e^{-j2\pi m l/D} = \frac{1 - e^{-j2\pi l M/D}}{1 - e^{-j2\pi l/D}}$, this expression is nonzero for $l = 1, \ldots, D-1$, unless M is a multiple of D. When $M = I\,D$, this expression is equal to zero for $l \neq 0$ and M for $l = 0$. Hence, the resulting output is

$$Y(z) = \frac{M}{D} z^{-(M-1)} X(z) = I\, z^{-(M-1)} X(z). \tag{17}$$

This means $y(n) = I\, x(n - M + 1)$ and hence the overall structure of Fig. 2(b) with our expressions in (9) and (10) is a valid polyphase representation of a perfect reconstruction IIR DFT FB in the oversampled polyphase case, provided that $M = I D$.

The above reasoning is valid for any oversampled FB as long as $M = ID$ and $\hat{E}_k(z)\hat{R}_k(z) = 1$. In our formulation, we not only satisfy the above requirements but also guarantee the stability of both $\hat{E}_k(z)$ and $\hat{R}_k(z)$, for $k = 0, 1, ..., M-1$. Since the critically sampled FB is a special case of the oversampled FB, i.e., $M = D$, our techniques also can be applied to the critically sampled case.

In [7], a design for a stable, casual, PR, IIR DFT FB was proposed, based on the transformation of a prototype filter into a filter with only powers of z^D in the system function denominator. This is done by making the substitution

$$\frac{1}{z-p_i}=\frac{z^{D-1}+p_i z^{D-2}+\cdots+p_i^{D-1}}{z^D-p_i^D} \tag{18}$$

for all the poles p_i of an arbitrary causal and stable lowpass filter. If the above substitution is applied to an elliptic filter of order *N*, both the numerator and denominator of the system function will be of degree *ND* (with the pole-zero cancellation), the denominator having the form in (8). When this filter is used as the prototype filter of a PR FB, the number of channels is equal to the product of the integers *N* and *D*, with decimator factor *D*, and the PR is achieved. Given the number of channels, this approach results in a reduced flexibility in the choice of *N*. Further, to obtain a critically sampled FB, Klouche-Djedid [7] employs a prototype denominator of the form $1+b_N z^{-N}$ (non-elliptic), which imposes constraints on the prototype poles. In comparison, we do not impose any condition between the number of channels *M* and the integer *N* of (8). Further, we give a common framework for both critically sampled and oversampled FBs.

IV. Structures and Numerical Stability

The polyphase structure of Fig. 2(a) shows a FB structure in the critically sampled case. Hence, we get the *k*-th polyphase branch as depicted in Fig. 3. The decimators can be moved to the left of $a_k E(z)$. The resulting realization is shown in Fig. 4(a) for the critically sampled case. Using the noble identities for polyphase filters, the decimators and the expanders are brought after the all-pole filters $E(z^M)$ and before the all-zero filters $E(z^M)^{-1}$. Hence, a second realization is obtained as shown in Fig. 4(b).

In [8], a DFT FB in the oversampled polyphase structure for $M = ID$ has been described. In the same structure, it could be recognized that the decimator *D* is given by the interconnection of decimator *M* and expander *I*. In Appendix B, we have formulated a *z*-domain approach of the combination mentioned. Hence, we get the *k*-th polyphase branch as depicted in Fig. 5(a). Using the noble identities and rearranging, Fig. 5(a) can be redrawn as in Figs. 5(b), 5(c) and 5(d), respectively. The resulting FB structures in the polyphase oversampled case are shown in Fig. 6 (a). Using the noble identities, the decimators and the expanders are brought after the all-pole filters $\hat{E}(z^D)$ and before the all-zero filters $\hat{E}(z^D)^{-1}$. Hence, a second structure is obtained as shown in Fig. 6(b).

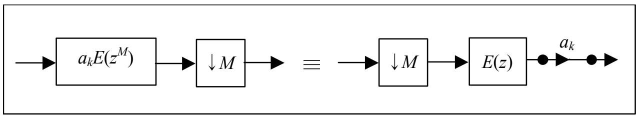

Fig. 3. The *k*-th polyphase branch and its equivalent.

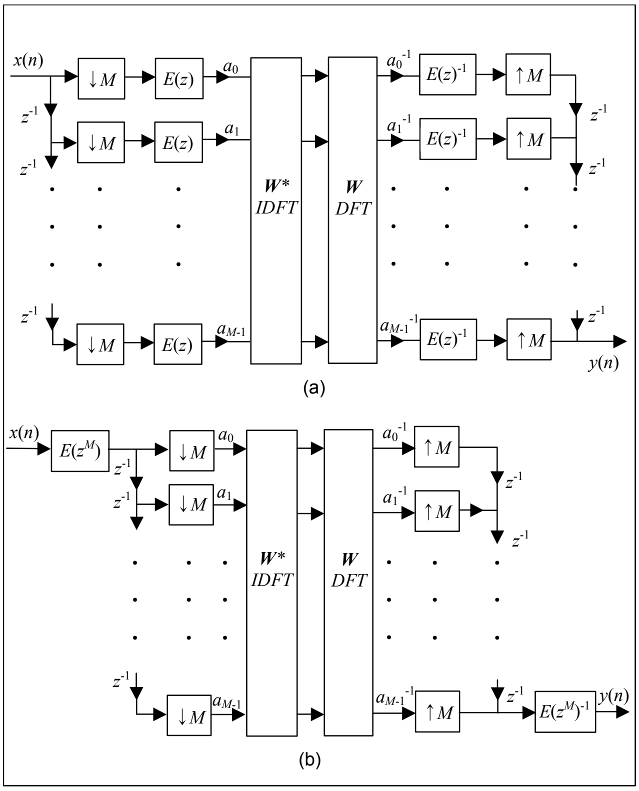

Fig. 4. Realizations of the proposed stable, causal, IIR DFT FB in the critically sampled case: (a) first realization, (b) second realization.

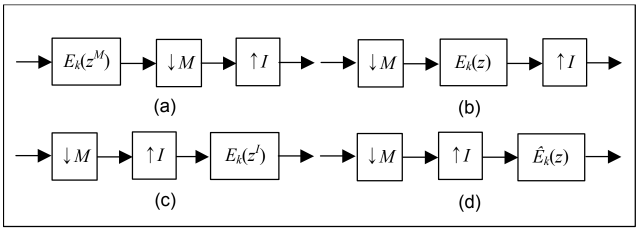

Fig. 5. The equivalent *k*-th polyphase branch in the oversampled case.

The structure shown in Figs. 5(b) and 6(b) are equivalent to placing an IIR filter with system function $E(z^M)$ and $\hat{E}(z^D)$, respectively, in front of a purely FIR DFT filter bank with taps a_k. Consequently, the filter with the system functions $E(z^M)^{-1}$ and $\hat{E}(z^D)^{-1}$ required after the FIR DFT synthesis bank is shown in Figs. 4(b) and 6(b), respectively. An important property of the filter banks, with the prototype filter as given in (8), the polyphase components in (9)

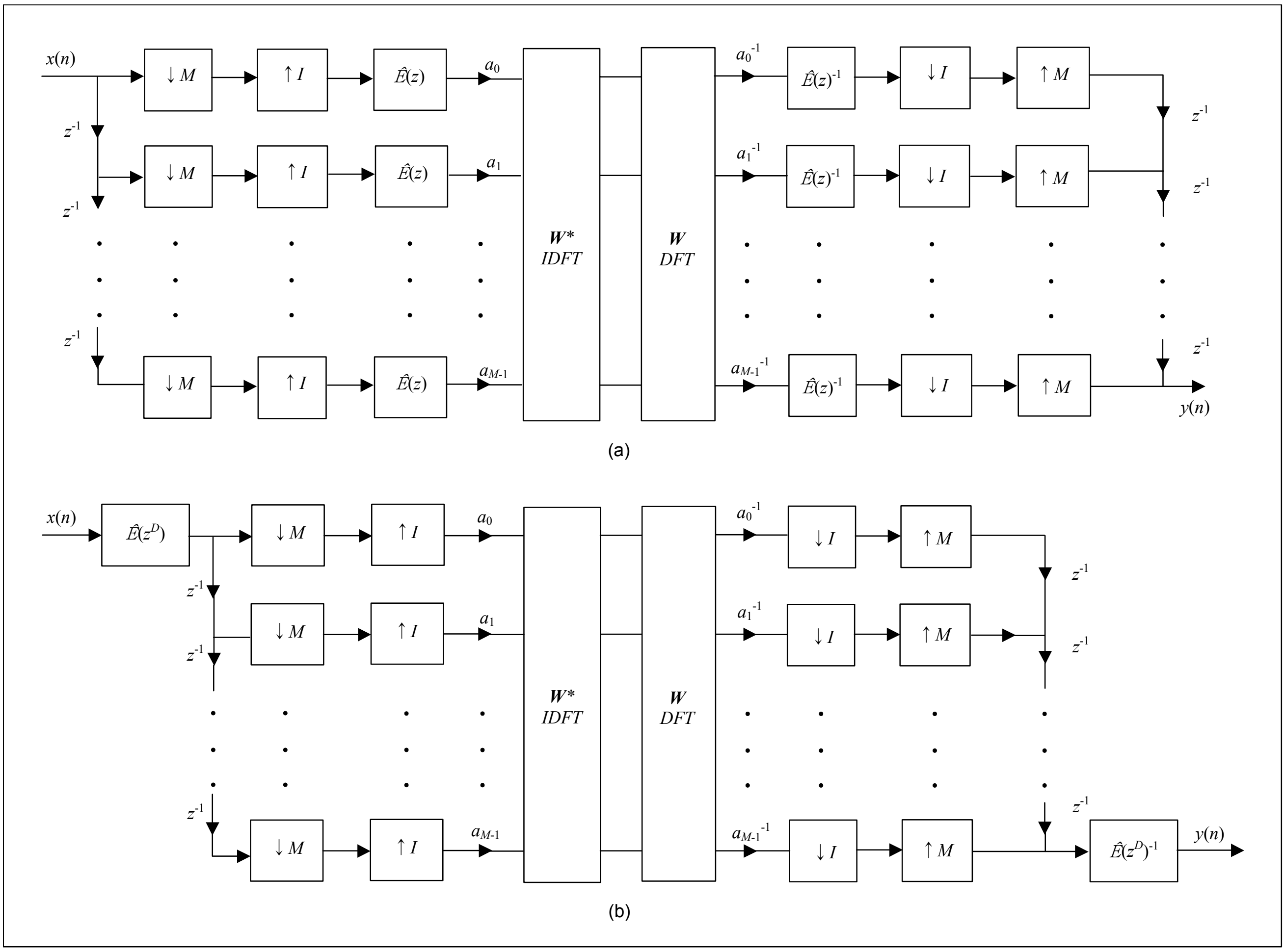

Fig. 6. Structure of the proposed stable, causal, IIR DFT FB in the oversampled case: (a) first realization, (b) second realization.

and (10), and the realizations shown in Figs. 4 and 6, is that they are very simple and modular.

The computational complexity is determined by the number of multiplications per unit time required to implement the IIR DFT FB. In the realization shown in Fig. 6 (a), if we use the IFFT algorithm, the analysis filters need $\frac{M}{2}\log_2(M)$ complex multiplications for the IDFT. Additionally, for the given polyphase components of order N, the multiplication of complex input values with real coefficients requires $2M(N+2)$ real multiplications for the polyphase elements. The statement about the complexity in the analysis filter part is also valid for the synthesis part. Therefore, for the whole FB, the number of real multiplications per unit time is $(3M\log_2(M)+4M(N+2))/D$. The factor D appears in the denominator, due to the reduced rate when compared to the input rate. We have considered three real multiplications for the multiplication of two complex numbers. In the second realization given in Fig. 6(b), the number of real multiplications amounts to $(3M\log_2(M)+4(M+N+1))/D$. We obtain the same expressions for the number of multiplications in the critically sampled structure shown in Figs. 4(a) and 4(b), if we substitute D by M in the above equations. The computational complexity ratio of the structures shown in Figs. 4(b), 6(b) and 4(a), 6(a) for different M and N are summarized in Table 1. The computational complexity ratio (CCR) is defined as

$$CCR = \frac{3M\log_2(M)+4(M+N+1)}{3M\log_2(M)+4M(N+2)}. \tag{19}$$

The results show that the structures proposed in Figs. 4(b) and 6(b) have a reduced amount of complexity, by more than 50% overall, for the critically sampled and oversampled cases. This is due to the same polyphase components in the proposed approach when compared to the general cases shown in Figs. 2(a) and 2(b). In comparison to the conventional FIR [9], the IIR DFT FBs proposed in this paper have a reduced order for the same number of channels. The overall cost is higher and FB delays are much longer for real-time processing, when compared to the delay in the proposed FB.

Table 1. Computational complexity ratio of proposed IIR DFT FB structure.

N \ M	8	16	32	64	256	1024
4	46.9%	47.9%	50%	53.1%	58.5%	63%
5	43.2%	43.7%	45.9%	48.6%	54%	58.7%

In addition to PR, numerical stability is very important in IIR FBs. It can be measured with the help of the Weyl-Heisenberg frame (WHF) theory [10], [11]. The frame bounds A and B in the analysis DFT FB have been derived as the minimum and maximum eigenvalues of the positive definite matrix $\mathbf{E}^*(e^{j\omega})\mathbf{E}(e^{j\omega})$ in [7], [10]. As can be seen from (4), these eigenvalues are equal to the polyphase components of the prototype filter and hence, the frame bounds are given by:

$$\lambda_{\min} = \min_{k,\, 0\le\omega\le\pi} \left|E_k(e^{j\omega})\right|^2 \quad \text{and}$$

$$\lambda_{\max} = \max_{k,\, 0\le\omega\le\pi} \left|E_k(e^{j\omega})\right|^2. \tag{20}$$

The frame ratio, which is defined as $\lambda_{\max}/\lambda_{\min}$, can be used for the description of numerical properties. The frame ratio is bounded by $1<\lambda_{\max}/\lambda_{\min}<\infty$, where the lower bound has the best numerical behavior. Generally, for good numerical behavior, we should have a tight frame ratio, i.e., all $|E_k(e^{j\omega})|$ are constants or all $E_k(z)$ are allpass filters. Stable, causal allpass filters are non-minimum phase, which are not permissible in our case. However, we take into account the WHF approach to numerical properties when designing our PR FBs and try to maintain a tight frame ratio for requiring a smaller dynamic range.

As a check, an important condition is that each of the numerator coefficients a_k of $H_0(z)$ is different from zero. If any a_k is zero, the PR condition is not satisfied, since some synthesis filters $R_k(z)$ would have infinite coefficients [see (10)]. The case of one of the a_k s being zero corresponds to an infinite frame ratio. Further, since our design method places all zeros of $H_0(z)$ on the unit circle, the numerator coefficients a_k will be symmetric, i.e, $a_k = a_{M-1-k}$.

V. Design Examples

In [1], an algorithm for the design of equiripple digital filters is presented. This algorithm finds the equiripple solution by working iteratively with the magnitude of the numerator and the denominator. It works as follows: Given a passband ripple (δ_p), normalized passband edge frequency (f_p), normalized stopband edge frequency (f_s), M, N, and D, the algorithm chooses the value of stopband attenuation (δ_s), such that the resulting filter has equiripple behavior in both the stopband and passband. Refs. [1] and [12] show that this algorithm is optimized in the Chebyshev sense.

According to [1], generally, we should choose an M and N as a starting point and design a filter with these values. If the desired δ_p is not attainable with this M and N, it is necessary to increase N. Once the desired δ_p is obtained, we can check if the desired stopband attenuation is achieved. If not, then by increasing M, the value of δ_s is reduced. Increasing M can make δ_p unattainable, making it necessary to increase N also. In our case, we are not allowed to change the number of channels M, so we have to increase ND. The number of poles of $H_0(z)$ is ND, where only a small number of poles shapes the passband [1]. The rest of the poles increase the attenuation in the stopband. Their effect has to be overcome by the zeros that are restricted to being on the unit circle. Since the number of zeros is also restricted to $M-1$, the number of poles ND must be changed so that the desired passband ripple and stopband attenuation is attained. Since increasing D causes pole repetition in the prototype filter, it is not effective in increasing the attenuation in the stopband. Hence, we have to increase N and choose proper values of D for the desired attenuation in the stopband.

Example 1. Let us consider a four-channel PR IIR DFT FB with normalized stopband and passband edge frequencies $f_p = 0.1$ and $f_s = 0.15$, respectively, and a passband ripple of $\delta_p = 0.025$ dB and stopband attenuation of $\delta_s = 35$ dB. We have applied the algorithm for designing the prototype filter given in (8). With the above considerations, we choose $M = 4$, $N = 5$ and $D = 2$ for the above-mentioned specifications. Figure 7 shows the frequency response of the analysis filters. The passband ripple is about 0.025 dB and the stopband attenuation of the analysis filter is about 38 dB.

The poles of $\hat{E}(z)$ and the coefficients a_k are shown in Table 2 when the frame ratio is about 46.4 dB. As can be seen from Table 2, since the zeros of the prototype low pass filter are located on the unit circle, the coefficients of a_k are symmetric as expected, i.e., $a_k = a_{M-1-k}$. As [7] showed, the typical values obtained for the frame ratio are around 50–60 dB and could be practical for good numerical behavior of the FB. According to (9) and (20), the frame ratio directly depends on the coefficients a_k as well as the poles of the polyphase components of FB. There is a trade-off between the frequency

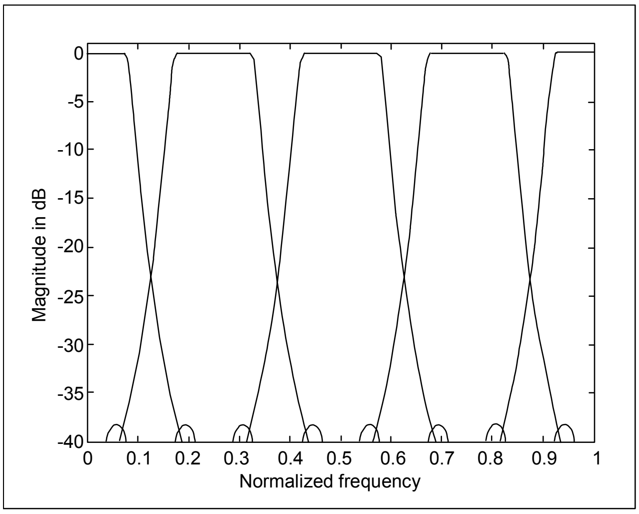

Fig. 7. Frequency response of analysis filters of a 4-channel DFT filter bank.

selectivity of the FB and the frame ratio. By definition, a DFT FB with good frequency selectivity will always have a high frame ratio.

Example 2. Here we design an 8-channel DFT FB with normalized stopband and passband edge frequencies of f_p = 0.0313 and f_s = 0.0938, respectively, a passband ripple of $\delta_p = 0.05$ dB and stopband attenuation of δ_s = 40 dB. Since in the prototype analysis filter the number of zeros is fixed at *M*-1, we have to increase *N* and the decimator factor *D* to get minimum stopband attenuation. We choose *N* = 2 and *D* = 2 as a starting point. For these values, the required stopband attenuation has not been obtained; therefore, it is necessary to increase either *N* or *D*. For *N* = 3 and *D* = 2, the stopband attenuation obtained is about 40 dB and for *N* = 4 and *D* = 2, it is about 55 dB. We observe that for obtaining the desired stopband attenuation, there is no point in increasing *D* because this merely results in repeated poles. For instance, *N* = 2 and *D* = 4 gives a stopband attenuation of only 20 dB. Figure 8 shows the frequency response of all the eight analysis filters designed by the proposed method, for *M* = 8, *N* = 3 and *D* = 2, for which the passband ripple is 0.05 dB and stopband attenuation is about 40 dB. The poles of $\hat{E}(z)$ and coefficients a_k are shown in Table 3. The frame ratio is about 52.7 dB, which is an

Table 2. Specifications of analysis prototype filter of Example 1.

Poles	±0.7716	±0.8028 ±j 0.4436	±0.7691 ±j 0.2415	-
Zeros	–1	–0.8159 ±j 0.5772	-	-
a_k	1	2.6318	2.6318	1

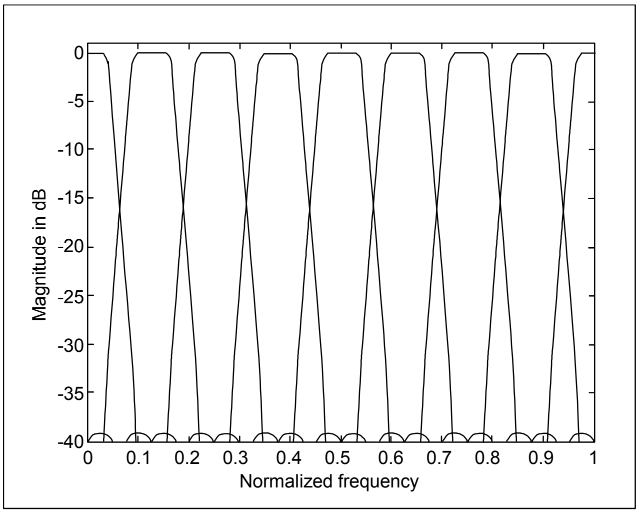

Fig. 8. Frequency response of an 8-channel proposed DFT filter bank.

acceptable value.

Example 3. Let us design a 32-channel DFT FB with normalized stopband and passband edge frequencies of f_p=0.008523 and f_s = 0.022727, respectively; a passband ripple of $\delta_p = 0.05$ dB and a stopband attenuation of δ_s = 60 dB. In this example, we have considered two different cases, ($M = D = 32$, *N* = 3) and (*M* = 32, *D* = 8, *N* = 5). In the first case, the filter order is *ND* = 96, where there are only three effective poles, and each of them is repeated 32 times. In the second case, the filter order is *ND* = 40, where there are five effective poles, and each of them is repeated 8 times. Hence, we expect to get a better frequency response in the second case. Figure 9 shows the frequency response of the four analysis filters designed by the proposed method in the second case. The stopband attenuation obtained in the first case is about 12

Table 3. Specifications of analysis prototype filter of Example 2.

Poles	±0.8141		±0.8930 ±j 0.2171		--		--	
Zeros	–1		0.7952 ±j 0.6063		0.0110 ±j 0.9999		-0.9599 ±j 0.2805	
a_k	1	1.3073	0.2468	0.6212	0.6212	0.2468	1.3073	1

dB and in the second case about 60 dB in spite of a smaller order. This is because of the number of effective poles for shaping the stopband region, which in the second case is more than in the first case.

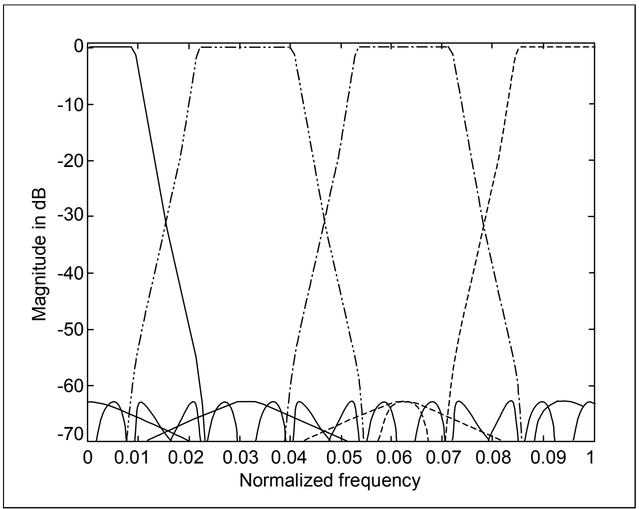

Fig. 9. Frequency response of a 32-channel analysis FB in the oversampled case.

VI. Conclusions

We have proposed a technique for designing an M-channel causal, stable and PR IIR DFT filter bank, which is based on a structure proposed by Martinez et al. [1] for sampling rate reduction in IIR digital filters. The design procedure for an arbitrary channel with prototype filter specifications is included. PR FBs are always possible with stable and causal polyphase components. Two realizations with efficient implementations are discussed. Since the realizations are modular structures, they are very much suitable for VLSI implementation and useful for high-bandwidth, complex parallel signal processing tasks. The results show that the current approach reduces the computational complexity by more than 50% for the critically sampled and oversampled cases. Hence, the proposed technique is very useful for stable, causal, IIR DFT FBs in the critically and oversampled cases. Examples have been included to illustrate the performance of this method.

Appendix

A. Derivation of oversampled polyphase decomposition

In [8], a decomposition of the impulse response of the prototype filter $h_0(n)$ into an oversampled polyphase set of sub-filter $\hat{e}_k(n)$, of the form

$$\hat{e}_k(n) = h_0(nD+k),\ k = 0, 1, ..., M-1, \tag{A1}$$

is given, where D is the decimator/expander factor and M is the number of the polyphase component, which is an integer sub-multiple of D, i.e., $M = I D$.

It can also be noted that all sub-filters $\hat{e}_k(n)$ are not independent and are related within the set. That is, if we define a number of unique components $\hat{e}_m(n)$, $m = 0, 1, ..., D-1$, it can be shown from (A1) that for $k = m + rD$, $r = 0, 1, ..., I-1$,

$$\begin{aligned}\hat{e}_{m+rD}(n) &= h_0(nD+m+rD) \\ &= h_0((n+r)D+m) \\ &= \hat{e}_m(n+r).\end{aligned} \tag{A2}$$

Thus in the oversampled polyphase case, there are D independent components and the rest are delayed versions of them. In the following, we derive a z-domain formulation of the polyphase decomposition.

Let us define $\bar{e}_k(n) = h_0(n+k)$, then

$$\bar{E}_k(z) = z^k H_0(z), \tag{A3}$$

where $\bar{E}_k(z)$ and $H_0(z)$ are the z-transforms of $\bar{e}_k(n)$ and $h_0(n)$, respectively. From (A1), we note that $\hat{e}_k(n) = \bar{e}_k(nD)$. Thus

$$\hat{E}_k(z) = \frac{1}{D}\sum_{l=0}^{D-1}\bar{E}_k(z^{1/D}e^{-j2\pi l/D}),\ k = 0, 1, ..., M-1. \tag{A4}$$

Substituting (A3) in (A4), we get

$$\hat{E}_k(z) = \frac{z^{k/D}}{D}\sum e^{-j2\pi lk/D}H_0(z^{1/D}e^{-j2\pi l/D}), k = 0, 1, ..., M-1. \tag{A5}$$

Equation (A5) can be rewritten as:

$$z^{-k}\hat{E}_k(z^D) = \frac{1}{D}\sum_{l=0}^{D-1}e^{-j2\pi lk/D}H_0(ze^{-j2\pi l/D}). \tag{A6}$$

From (A6), we get

$$\sum_{k=0}^{M-1}z^{-k}\hat{E}_k(z^D) = \frac{1}{D}\sum_{k=0}^{M-1}\sum_{l=0}^{D-1}e^{-j2\pi lk/D}H_0(ze^{-j2\pi l/D}). \tag{A7}$$

Changing the summation order of (A7) and noting that $\sum_{k=0}^{M-1}e^{-j2\pi kl/D} = I\delta(n)$, after normalization, we get

$$H_0(z) = \sum_{k=0}^{M-1}z^{-k}\hat{E}_k(z^D). \tag{A8}$$

B. Derivation of the DFT FB structure in the oversampled case

In [8], a time domain formulation of the oversampled DFT FB is given, which is based on the interconnection of decimator factor M with an expander factor I. We have given the z-domain formulation and applied it to the oversampled polyphase structure shown in Fig. 6(a). Let us consider a critically sampled case. The polyphase components are:

$$e_k(n) = h_0(nM + k),\ k = 0, 1, ..., M-1. \tag{B1}$$

The polyphase components in the oversampled case are the expanders of $e_k(n)$, with expander factor I, i.e.,

$$\hat{e}_k(n) = \begin{cases} e_k(n/I), & n = 0, \pm I, 2I, \cdots \\ 0, & \text{otherwise.} \end{cases} \tag{B2}$$

According to (B1), we get [2]:

$$H_0(z) = \sum_{k=0}^{M-1} z^{-k} E_k(z^M). \tag{B3}$$

From (B2), we get:

$$\hat{E}_k(z) = E_k(z^I). \tag{B4}$$

Substituting (B4) in (B3) and noting that $M = ID$, we can express the prototype filter $H_0(z)$ in terms of the oversampled components $\hat{E}_k(z^D)$ as shown in (A8). Implementation of the k-th branch of the polyphase structure and the rearrangement of its base on the noble identities is shown in Fig. 5. It can be shown that in the proposed oversampled structure shown in Fig. 6, the overall analysis filters $H_i(z)$ and the synthesis filters $G_i(z)$ are related to the prototype filters as in (1a) and (1b).

References

[1] H.G. Martinez and T.W. Parks, "A Class of Infinite-Duration Impulse Response Digital Filter Sampling Rate Reduction", *IEEE Trans. Acoust., Speech, Signal Proc.*, vol. ASSP-27, no.4, Apr. 1979, pp.154-162.

[2] P.P. Vaidyanathan, *Multirate Systems and Filter Banks*, Englewood Cliffs, NJ: Prentice- Hall, 1993.

[3] S.C. Chan, J.S. Muo, and K.L. Ho, "A New Design Method for Two-Channel Perfect Reconstruction IIR Filter Banks", *IEEE Signal Proc. Letters*, vol.7, no.8, Aug. 2000, pp.221-223.

[4] S.M. Phoong, C.W. Kim, P.P. Vaidyanathan, and R. Ansari, "A New Class of Two Channel Biothogonal Filter Banks and Wavelet Bases", *IEEE Trans. Signal Proc.*, vol. 43, no.3, Mar. 1995, pp.649-664.

[5] X. Zhang and T. Yoshikwa, "Design of Two-channel Stable IIR Perfect Reconstruction Filter Banks", *IEICE Trans. Fundamentals*, vol. E81-A, no.8, Aug. 1998, pp.1592-1597.

[6] D.B.H. Tay, "Design of Causal Stable IIR Perfect Reconstruction Filter Banks Using Transfomation in Variables", *Proc. IEEE. Int. Symp. Circuits and Syst.*, vol. 4, 1997, pp. 2425-2427.

[7] A. Klouche-Djedid, "Design of Stable, Causal, Perfect Reconstruction, IIR Uniform DFT Filter Banks", *IEEE Trans. Signal Proc.*, vol.48, no.4, Apr. 2000, pp. 1110-1119.

[8] R. Crochiere and L. Rabiner, *Multirate Digital Signal Processing*, Englewood Cliffs, NJ: Prentice-Hall, Chapter 7, 1983.

[9] K. Swaminathan and P.P. Vaidyanathan, "Theory and Design of Uniform DFT, Parallel, Quadrature Mirror Filter Banks", *IEEE Trans. On Circuits and Systems*, vol. CAS-33, no.12, Dec.1986, pp. 1170-1190.

[10] H. Bolcskei, F. Hlawatsch, and H. G. Feichtinger, "Oversampled FIR and IIR DFT Filter Banks and Weyl-Heisenberg Frames", *Proc IEEE Int. Conf. Acoust., Speech, Signal Proc.*, vol. 3, May 1996, pp.1391-1394.

[11] Z. Cvetkovic and M. Vetterli, "Tight Weyl-Heisenberg Frames in $l^2(Z)$", *IEEE Trans. Signal Proc.*, vol. 46, no.5, May 1998, pp. 1256-1259.

[12] H.G. Martinez and T.W. Parks, "Design of Recursive Digital Filters with Optimum Magnitude and Attenuation Poles on the Unit Circle", *IEEE Trans. Acoust., Speech, Signal Proc.*, vol. ASSP-26, no.2, Apr. 1978, pp.150-157.

M.J. Dehghani received the BS and MS degrees in electrical engineering from the Isfahan University of Technology, Iran, in 1988 and 1992, respectively. From 1992 to 1993, he was with the faculty of the Department of Electrical Engineering, Urmia University, Iran. In 1993, he joined the faculty of Control Department, School of Electronic Technology, Shiraz University, Iran. He has completed all the requirements for the award of the PhD degree of IIT Madras, India. His research interests include filter banks, high data rate wireless communication, DSL and OFDM systems, space-time coding, and channel estimation.

R. Aravind received the B.Tech. degree from IIT Madras in 1982, the MS and PhD degrees from University of California, Santa Barbara, in 1984 and 1988, respectively, all in electrical engineering. From 1988 to 1995 he was a Member of Technical Staff at AT&T Bell Laboratories where his work was in image and video compression algorithms. He was an active participant in the MPEG-1 and MPEG-2 video codec standardization activities. Since 1995 he has been on the faculty of the Department of Electrical Engineering, IIT Madras. His activities have included speech and video coding as well as modem simulation studies. His interests are in the areas of voice and video coding and telecom applications of signal processing.

K.M.M. Prabhu obtained the BS (Engg.) degree in electronics and communication engineering, the MS (Engg.) degree in applied electronics engineering and the PhD degree in digital signal processing in 1971, 1973 and 1981. He joined the Department of Electrical Engineering, IIT Madras, in 1981, where he is currently a full Professor. While at IIT Madras, he has been a Principal Investigator of many projects funded by the Defence Research and Development Organization. He has published 100 papers in the International Journals and the International Conference Proceedings. Under his supervision, 17 MS (by research) and PhD students have completed their degrees. He has conducted 10 short-term courses in the area of DSP for the benefit of Engineers and Scientists from various organizations. He spent his sabbatical, from August 1999 to August 2000, with the Signal Processing Systems Group, Eindhoven University of Technology, The Netherlands. He has been a reviewer for a number of IEEE, IEE and other International Journals. His current research interests include DSP agorithms and their implementations, digital filter structures, time-frequency signal analysis, and wavelet-based signal processing. He is a Senior Member of the IEEE.