Can non-minimal coupling restore the consistency condition in bouncing universes?

Debottam Nandi[*] and L. Sriramkumar[†]
Department of Physics, Indian Institute of Technology Madras, Chennai 600036, India

An important property of the three-point functions generated in the early universe is the so-called consistency condition. According to the condition, in the squeezed limit wherein the wavenumber of one of the three modes (constituting the triangular configuration of wavevectors) is much smaller than the other two, the three-point functions can be completely expressed in terms of the two-point functions. It is found that, while the consistency condition is mostly satisfied by the primordial perturbations generated in the inflationary scenario, it is often violated in the bouncing models. The validity of the consistency condition in the context of inflation can be attributed to the fact that the amplitude of the scalar and tensor perturbations freeze on super-Hubble scales. Whereas, in the bouncing scenarios, the amplitude of the scalar and tensor perturbations often grow rapidly as one approaches the bounce, leading to a violation of the condition. In this work, with the help of a specific example involving the tensor perturbations, we explicitly show that suitable non-minimal couplings can restore the consistency condition even in the bouncing models. We briefly discuss the implications of the result.

I. INTRODUCTION

Arguably, the inflationary scenario is the most efficient and compelling paradigm to describe the origin of perturbations in the early universe [1–7]. Inflation corresponds to a brief phase of accelerated expansion in the early stages of the universe, and it is often invoked to resolve the horizon and flatness problems associated with the conventional hot big bang model. Its success can be attributed primarily to the fact that the simplest of inflationary models lead to nearly scale invariant primordial spectra which prove to be remarkably consistent with the cosmological data [8, 9]. However, the impressive efficiency of the inflationary paradigm also seems to harbor a possible pitfall. Despite the ever-tightening observational constraints, there seems to exist many inflationary models that continue remain consistent with the data [10–13], even leading to the concern whether inflation can be falsified at all [14].

A popular alternative to the inflationary paradigm are the classical bouncing scenarios [15–20]. In these scenarios, the universe undergoes a phase of contraction until the scale factor reaches a minimum value, before it enters the expanding phase. It is straightforward to establish that such scenarios can aid in overcoming the horizon problem. Moreover, it can be shown that a *non-accelerating* early phase of contraction will permit the standard Bunch-Davies initial conditions to be imposed on the perturbations, in the same manner as in inflation. However, in complete contrast to inflation, there arises many challenges in constructing viable bouncing models. The basic reason behind the difficulty is the fact that the null energy condition needs to be violated around the bounce. Nevertheless, the bouncing models continue to attract constant attention in the literature.

In this work, we shall focus on a specific bouncing scenario known as the matter bounce [21–23]. In such scenarios, during the early stages of contraction, the scale factor behaves as in a matter dominated universe. Also, matter bounces are guaranteed to lead to scale invariant spectra, as they are known to be 'dual' to de Sitter inflation (in this context, see Ref. [24]). Though both de Sitter inflation and matter bounces result in similar spectra, the amplitude and the shape of the three-point functions generated in these scenarios are expected to be considerably different. This is because of the difference in the behavior of the evolution of the perturbations in these alternative scenarios. In the context of inflation, it is well known that the amplitude of the perturbations freeze on super-Hubble scales. Due to this reason, the three-point functions generated during inflation exhibit an interesting property. One finds that, in the so-called squeezed limit wherein one of the three wavenumbers is much smaller than the other two, the inflationary three-point functions can be completely expressed in terms of the two-point functions, a property that is referred to as the consistency condition (see, for example, Refs. [25–27]). On the other hand, in most of the bouncing models, the amplitude of the perturbations (corresponding to scales of cosmological interest) grow rapidly as one approaches the bounce. Such a behavior results in a violation of the above-mentioned consistency condition (in this context, see, for instance, Ref. [28]).

Our goal in this work is to examine whether non-minimal coupling can restore the consistency condition in a matter bounce scenario. Since studying the case of scalars requires considerable modeling, for simplicity, we shall focus on the tensor perturbations. We shall consider a fairly generic scalar-tensor theory [29–32] and work with coupling functions that lead to a scale invariant tensor perturbation spectrum in a matter bounce (for discussions in the context of inflation, see Refs. [33, 34]). Moreover, we shall work with parameters that result in a tensor amplitude that is consistent with the current upper bounds on the tensor-to-scalar ratio r from the ob-

* E-mail: debottam@physics.iitm.ac.in
† E-mail: sriram@physics.iitm.ac.in

servations of the anisotropies in the Cosmic Microwave Background (CMB) [8]. As we shall show explicitly, it is possible to construct non-minimal couplings wherein the amplitude of the tensor perturbations freeze soon after they leave Hubble radius during the contracting phase and are hardly affected by the bounce. We shall illustrate that the consistency condition is indeed satisfied in such cases.

This paper is organized as follows. In the following section, we shall briefly describe the non-minimally coupled model of our interest and arrive at the actions describing the tensor perturbations at the second and the third orders. In Sec. III, we shall consider a specific form of the non-miminal coupling and show that the form leads to scale invariant tensor power spectrum. In Sec. IV, we shall evaluate the corresponding tensor bispectrum and, in Sec. V, we shall calculate the tensor non-Gaussianity parameter $h_{\rm NL}$, which is a dimensionless ratio involving the tensor bispectrum and power spectrum. We shall also explicitly show that the consistency condition is satisfied in the model. We shall conclude in Sec. VI with a brief discussion.

A few words on our conventions and notations are in order at this stage of our discussion. We shall work with natural units such that $\hbar = c = 1$, and we shall define the Planck mass to be $M_{\rm Pl} = (8\,\pi\,G)^{-1/2}$. We shall adopt the metric signature of $(-,+,+,+)$. While Greek indices shall denote the spacetime coordinates, Latin indices shall denote the spatial coordinates, with the exception of k which shall be reserved for representing the wavenumber of the perturbations. The overdots and overprimes, as usual, shall denote derivatives with respect to the cosmic time t and the conformal time η associated with the Friedmann-Lemaître-Robertson-Walker (FLRW) line-element, respectively. Lastly, a shall denote the scale factor and H shall denote the Hubble parameter defined as $H = \dot{a}/a$.

II. MODEL AND SCENARIO OF INTEREST

We shall consider a theory of gravitation which involves non-minimal coupling to a scalar field, say, ϕ. In four spacetime dimensions, such a theory, in general, can be described by the action [29–32]

$$\mathcal{S}[g_{\mu\nu},\phi] = \int {\rm d}^4x\,\sqrt{-g}\,\biggl[\mathcal{L}_2(g_{\mu\nu},X,\phi) + \mathcal{L}_3(g_{\mu\nu},X,\phi) + \mathcal{L}_4(g_{\mu\nu},X,\phi) + \mathcal{L}_5(g_{\mu\nu},X,\phi)\biggr], \tag{1}$$

where $X = -(\nabla_\mu\phi)^2/2$ denotes the standard kinetic term. The Lagrangian densities $\mathcal{L}$'s are defined as

$$\mathcal{L}_2 = K(X,\phi), \tag{2a}$$
$$\mathcal{L}_3 = -G_3(X,\phi)\,\Box\phi, \tag{2b}$$
$$\mathcal{L}_4 = G_4(X,\phi)\,R + G_{4X}(X,\phi)\,\left[(\Box\phi)^2 - (\nabla_{\mu\nu}\phi)^2\right], \tag{2c}$$
$$\mathcal{L}_5 = G_5(X,\phi)\,G_{\mu\nu}\,\nabla^{\mu\nu}\phi - \frac{1}{6}\,G_{5X}(X,\phi) \times \left[(\Box\phi)^3 - 3\,\Box\phi\,(\nabla_{\mu\nu}\phi)^2 + 2\,(\nabla_{\mu\nu}\phi)^3\right], \tag{2d}$$

where the quantities $K(X,\phi)$ and $G(X,\phi)$ are general functions of X and ϕ, while the subscript X denotes derivative of the function with respect to X. Note that the action (1) contains second time derivatives of the field ϕ. Hence, one may naively expect that the corresponding equations of motion may involve higher time derivatives. However, the complete action has been structured in a fashion such that the governing equations do not contain any higher time derivatives than the second. Therefore, the model is free of the so-called Ostrogradsky instabilities [35].

As we had mentioned, in this work, we shall be focusing on the three-point function describing the tensor perturbations. When the tensor perturbations, say, $\gamma_{ij}(\eta,\boldsymbol{x})$, are taken into account, the spatially flat, FLRW line-element can be written as

$${\rm d}s^2 = a(\eta)^2\,\left[-{\rm d}\eta^2 + \left\{\exp\gamma(\eta,\boldsymbol{x})\right\}_{ij}\,{\rm d}x^i\,{\rm d}x^j\right], \tag{3}$$

where, evidently, $a(\eta)$ denotes the scale factor, with η being the conformal time coordinate. The quantity $\{\exp\gamma(\eta,\boldsymbol{x})\}_{ij}$ contains the tensor perturbations and is defined as

$$\{{\rm e}^{\gamma}\}_{ij} = \delta_{ij} + \gamma_{ij} + \frac{1}{2}\,\gamma_i^l\,\gamma_{lj} + \frac{1}{6}\,\gamma_i^l\,\gamma_l^m\,\gamma_{mj} + \cdots, \tag{4}$$

where the spatial indices are to be raised and lowered with the aid of Kronecker δ_{ij}.

Since we shall be focusing on the tensor perturbations, we shall be interested in only the following part of the action (1):

$$\mathcal{S}[g_{\mu\nu}] = \int {\rm d}^4x\,\sqrt{-g}\,\left[\mathcal{L}_4(g_{\mu\nu},X,\phi) + \mathcal{L}_5(g_{\mu\nu},X,\phi)\right]. \tag{5}$$

Upon substituting the line-element (3) in this action, one can arrive at the following actions that describe the tensor perturbations γ_{ij} at the second and the third order in the perturbations (in this context, see Refs. [33, 34]):

$$\delta^3\mathcal{S}[\gamma_{ij}] = \frac{1}{8}\int \mathrm{d}\eta \int \mathrm{d}^3\boldsymbol{x}\, a^2(\eta)\, \Big[\mathcal{G}(\eta)\, {\gamma'_{ij}}^2 - \mathcal{F}(\eta)\, \left(\partial_l\gamma_{ij}\right)^2\Big], \tag{6a}$$

$$\delta^3\mathcal{S}[\gamma_{ij}] = \frac{1}{4}\int \mathrm{d}\eta \int \mathrm{d}^3\boldsymbol{x}\, \Big[a^2(\eta)\, \mathcal{F}(\eta)\, \partial_{lm}\gamma_{ij} \times \left(\gamma^{lj}\, \gamma^{im} - \frac{1}{2}\, \gamma^{ij}\, \gamma^{lm}\right) + \frac{1}{3}\, X\, \phi'\, G_{5X}(\eta)\, \gamma_j^{i\,\prime}\, \gamma_l^{j\,\prime}\, \gamma_i^{l\,\prime}\Big]. \tag{6b}$$

The functions $\mathcal{G}$ and $\mathcal{F}$ are background quantities and are given by

$$\mathcal{F} = 2\left[G_4 - X\left(\ddot{\phi}\, G_{5X} + G_{5\phi}\right)\right], \tag{7a}$$

$$\mathcal{G} = 2\left[G_4 - 2\, X\, G_{4X} - X\left(H\, \dot{\phi}\, G_{5X} - G_{5\phi}\right)\right], \tag{7b}$$

where the subscript ϕ denotes differentiation with respect to the scalar field.

Note that the standard Einstein's general theory of relativity corresponds to choosing

$$G_4(X,\phi) = \frac{M_{_{\mathrm{Pl}}}^2}{2}, \tag{8a}$$

$$G_5(X,\phi) = 0. \tag{8b}$$

In such a case, the general second and third order actions (6) that describe the tensor perturbations reduce to the following forms (see, for instance, Refs. [25, 27, 28]):

$$\delta^2\mathcal{S}[\gamma_{ij}] = \frac{M_{_{\mathrm{Pl}}}^2}{8}\int \mathrm{d}\eta \int \mathrm{d}^3\boldsymbol{x}\, a^2(\eta)\, \left[{\gamma'_{ij}}^2 - \left(\partial_l\gamma_{ij}\right)^2\right], \tag{9a}$$

$$\delta^3\mathcal{S}[\gamma_{ij}] = \frac{M_{_{\mathrm{Pl}}}^2}{4}\int \mathrm{d}\eta \int \mathrm{d}^3\boldsymbol{x}\, a^2(\eta)\, \partial_{lm}\gamma_{ij} \times \left(\gamma^{lj}\, \gamma^{im} - \frac{1}{2}\, \gamma^{ij}\, \gamma^{lm}\right). \tag{9b}$$

In an earlier work, assuming the matter bounce to be described by the scale factor

$$a(\eta) = a_0\, (1 + k_0^2\, \eta^2) = a_0\, \left(1 + \frac{\eta^2}{\eta_0^2}\right), \tag{10}$$

the tensor power and bispectra were evaluated analytically in Einstein's theory using the above second and third order actions (in this context, see Ref. [28]). Clearly, in the above scale factor, $\eta_0 = 1/k_0$, and the duration of the bounce (in terms of cosmic time) is of the order of $a_0\, \eta_0$. Moreover, the wavenumbers k of cosmological interest correspond to $k \ll k_0$. The tensor modes for such wavenumbers were obtained under a certain approximation and they were evolved across the bounce (at $\eta = 0$) to arrive at the power and bispectra after the bounce. Since the matter bounce is dual to de Sitter inflation, as expected, the tensor power spectrum proved to be strictly scale invariant for modes of cosmological interest. Interestingly, the power spectrum had depended only on the dimensionless ratio $k_0/(a_0\, M_{_{\mathrm{Pl}}})$ and, for $k_0/(a_0\, M_{_{\mathrm{Pl}}}) \lesssim 10^{-5}$, the tensor amplitude had proved to be consistent with the current constraints from the CMB data [8]. Having arrived at the modes, it was also possible to evaluate the tensor bispectrum and the corresponding non-Gaussianity parameter $h_{_{\mathrm{NL}}}$. The amplitude and shape of the tensor bispectrum and the non-Gaussianity parameter had turned out to be considerably different from what arises in de Sitter inflation. For instance, while the tensor non-Gaussianity parameter $h_{_{\mathrm{NL}}}$ is found to be strictly scale invariant in the equilateral and the squeezed limits in de Sitter inflation, the parameter had a strong dependence on the wavenumber (it had behaved as k^2) in the matter bounce scenario. The strong dependence of the non-Gaussianity parameter on the wavenumber also led to considerably smaller values for the parameter (when compared to the de Sitter case) over scales of cosmological interest. Moreover, it was found that the consistency condition is violated in the squeezed limit [28]. As we have already emphasized, such differences in the three-point functions are expected to help us discriminate between the alternative scenarios for the generation of perturbations in the early universe.

The differences between the tensor bispectra in de Sitter inflation and the matter bounce scenarios arise due to the behavior of the modes in these two cases [25, 28]. While in de Sitter, as we have already discussed, the amplitude of the tensor modes freeze once they leave the Hubble radius, in the matter bounce scenario, the amplitude of the modes grow rapidly as they approach the bounce after leaving the Hubble radius during the contracting phase. It is then interesting to inquire if, in non-minimally coupled theories of gravitation, the tensor perturbations can behave in a manner akin to de Sitter and thereby restore the consistency condition in the bouncing models.

With such a motivation in mind, we make the following choices for the functions $G_4(X,\phi)$ and $G_5(X,\phi)$ that appear in the non-minimal action (5):

$$G_4(X,\phi) = \frac{M_{_{\mathrm{Pl}}}^2}{2}\, G(\phi), \tag{11a}$$

$$G_5(X,\phi) = \mathrm{constant}. \tag{11b}$$

In such a case, since the derivatives G_{4X}, G_{5X} and $G_{5\phi}$ vanish, we find that [cf. Eqs. (7)]

$$\mathcal{G} = \mathcal{F} = 2\, G_4(\phi) = M_{_{\mathrm{Pl}}}^2\, G(\phi). \tag{12}$$

Then, the second and third order actions (6) describing the tensor perturbations reduce to

$$\delta^2\mathcal{S}[\gamma_{ij}] = \frac{M_{\rm Pl}^2}{8}\int {\rm d}\eta \int {\rm d}^3\boldsymbol{x}\, z^2(\eta)\, \left[{\gamma_{ij}'}^2 - (\partial_l\gamma_{ij})^2\right], \tag{13a}$$

$$\delta^3\mathcal{S}[\gamma_{ij}] = \frac{M_{\rm Pl}^2}{4}\int {\rm d}\eta \int {\rm d}^3\boldsymbol{x}\, z^2(\eta)\, \partial_{lm}\gamma_{ij} \times \left(\gamma^{lj}\,\gamma^{im} - \frac{1}{2}\,\gamma^{ij}\,\gamma^{lm}\right), \tag{13b}$$

where we have set

$$z^2 = a^2\, G. \tag{14}$$

Evidently, the structure of the above set of actions have the same form as in Einstein's theory with the overall factor $a(\eta)$ being replaced by the function $z(\eta)$, which is determined by $G(\phi)$. We shall choose to work with

$$G(\eta) = \frac{C^2}{a^3(\eta)}, \tag{15}$$

where C is a dimensionless constant. Such a choice leads to

$$z(\eta) = \frac{C}{\sqrt{a(\eta)}}, \tag{16}$$

and we shall discuss the reason for this choice in the final section. In the following two sections, we shall evaluate the tensor power and bispectra spectra for the above choice of $z(\eta)$ assuming that $a(\eta)$ describes a matter bounce as in Eq. (10).

III. EVOLUTION OF THE TENSOR MODES AND THE POWER SPECTRUM

In this section, we shall arrive at the solutions to the Fourier modes describing the tensor perturbations and evaluate the tensor perturbation spectrum *after* the bounce.

Let h_k denote the Fourier modes corresponding to the tensor perturbations γ_{ij}. Upon varying the action (13a) with respect to γ_{ij}, one finds that the corresponding Fourier modes h_k satisfy the differential equation

$$h_k'' + 2\,\frac{z''}{z}\,h_k' + k^2\,h_k = 0. \tag{17}$$

If we introduce the Mukhanov-Sasaki variable $u_k = (M_{\rm Pl}/\sqrt{2})\,h_k\,z$, this equation takes the form

$$u_k'' + \left(k^2 - \frac{z''}{z}\right)u_k = 0. \tag{18}$$

For $z(\eta) = C/\sqrt{a(\eta)}$ and the scale factor (10), one finds that

$$\frac{1}{k_0^2}\,\frac{z''}{z} = \frac{-1 + 2\,k_0^2\,\eta^2}{(1 + k_0^2\,\eta^2)^2}. \tag{19}$$

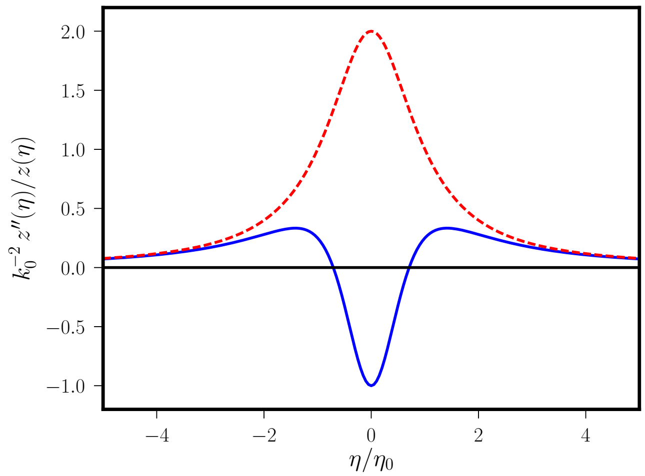

FIG. 1. The dimensionless quantity $k_0^{-2}\,(z''/z)$ has been plotted (in blue) as a function of η/η_0. We should point out that none of the parameters involved (*viz.* C, k_0 or a_0) need to be specified in plotting this figure. Note that the quantity $k_0^{-2}\,(z''/z)$ grows as one approaches the bounce, but exhibits a minimum at the bounce. The minimum is absent when $z = a$ (plotted in red), which is the case in Einsteinian gravity [28].

We have plotted the behavior of z''/z in Fig. 1. It turns out to be difficult to obtain an exact analytical solution for u_k (or, equivalently, h_k) for such a z''/z. However, one can obtain remarkably accurate solutions for the tensor modes of cosmological interest under certain approximations.

Note that $z''/z \to 0$ as $\eta \to -\infty$. Therefore, the standard Bunch-Davies initial conditions can be imposed on the modes at such early times. Our aim is to evolve the modes from such initial conditions across the bounce and evaluate the tensor perturbations after the bounce at, say, $\eta = \beta\,\eta_0$, where β is suitably large positive number. In order to arrive at analytical solutions for the modes, we shall divide the period of our interest, *viz.* $-\infty < \eta < \beta\,\eta_0$, into two domains, say, $-\infty < \eta < -\alpha\,\eta_0$ and $-\alpha\,\eta_0 < \eta < \beta\,\eta_0$, where we shall again choose α to be a large positive number. We shall indicate possible values for α and β in due course.

In the first domain, *i.e.* over $-\infty < \eta < -\alpha\,\eta_0$, the scale factor can be approximated as $a(\eta) \simeq a_0\,k_0^2\,\eta^2$. In such a case, we have $z(\eta) \simeq -C/(\sqrt{a_0}\,k_0\,\eta)$ and, hence, $z''/z \simeq 2/\eta^2$, which is exactly the behavior in de Sitter. Since u_k corresponding to the Bunch-Davies initial conditions is known in this case, the solution to the tensor modes h_k can be immediately written down to be [36]

$$h_k^{\rm I}(\eta) = \frac{\sqrt{2}}{M_{\rm Pl}}\,\frac{u_k(\eta)}{z(\eta)} \simeq -\frac{\sqrt{2}}{M_{\rm Pl}}\,\frac{\sqrt{a_0}\,k_0\,\eta}{C}\,\frac{1}{\sqrt{2\,k}}\left(1 - \frac{i}{k\,\eta}\right){\rm e}^{-i\,k\,\eta}. \tag{20}$$

As we had mentioned earlier, scales of cosmological interest correspond to $k \ll k_0$. Note that, during the contracting phase, for such wavenumbers, the solution $h_k^{\rm I}$ above oscillates over the domain wherein $k^2 > z''/z \simeq 2/\eta^2$. Needless to add, it is this oscillating behavior that permits one to impose the required initial conditions on the modes. The amplitude of the modes begin to freeze around the time when $k^2 = z''/z \simeq 2/\eta^2$, *i.e.* at $\eta_{\rm e} \simeq -\sqrt{2}/k$, which closely corresponds to the time when the modes exit the Hubble radius. Thereafter, the amplitude of the modes remain constant until one approaches close to the bounce (in this context, see Fig. 2).

Note that the quantity z''/z, though it grows to a maximum value [of $\mathcal{O}(k_0^2)$] as one approaches the bounce, it also contains a minimum [of $\mathcal{O}(-k_0^2)$] at the bounce. In the minimally coupled model wherein the tensor modes are governed only by the behavior of the scale factor, such a minimum is absent, and hence it is justified to assume that $k^2 \ll a''/a$ around the bounce. However, in the situation of our interest, we find that $k^2 = z''/z$ at $\eta_{\rm c} = \mp\eta_0/\sqrt{2}$. Close to these points, evidently, the conditions $k^2 \ll z''/z$ will not be satisfied. We shall nevertheless assume this condition is indeed satisfied and evaluate the modes across the bounce. We shall compare our analytical solutions with the numerical results to justify our assumption. We shall find that, since the period over which the condition is violated is rather brief (only very near $\eta = \eta_{\rm c}$), the corresponding effects on the modes prove to be completely negligible. If we now assume that $k^2 \ll z''/z$ around the bounce, we can obtain the solution in the second domain (*i.e.* over $-\alpha\,\eta_0 < \eta < \beta\,\eta_0$) to be

$$h_k^{\rm II}(\eta) = A_k + B_k\,\left(k_0\,\eta + \frac{k_0^3\,\eta^3}{3}\right), \qquad (21)$$

where the constants A_k and B_k can be expressed in terms of $h_k(\eta)$ in the first domain as follows:

$$B_k = \frac{h_k^{\rm I}{}'(\eta_*)}{k_0\,\left[a(\eta_*)/a_0\right]}, \qquad (22a)$$

$$A_k = h_k^{\rm I}(\eta_*) - B_k\,\left(k_0\,\eta_* + \frac{k_0^3\,\eta_*^3}{3}\right). \qquad (22b)$$

We can now choose $\eta_* = -\alpha\,\eta_0$ to be the time at which we match the solution and its time derivative in the two domains. It is then easy to determine A_k and B_k to be

$$B_k = -\frac{\sqrt{2}}{M_{_{\rm Pl}}}\,\frac{\sqrt{a_0}}{C}\,\frac{1}{\sqrt{2\,k}}\,\left(\frac{i\,k}{\alpha\,k_0}\right)\,{\rm e}^{i\,\alpha\,k/k_0}, \qquad (23a)$$

$$A_k = \frac{\sqrt{2}}{M_{_{\rm Pl}}}\,\frac{\sqrt{a_0}\,\alpha}{C}\,\frac{1}{\sqrt{2\,k}}\,\left(1 + \frac{i\,k_0}{\alpha\,k}\right)\,{\rm e}^{i\,\alpha\,k/k_0} + B_k\,\left(\alpha + \frac{\alpha^3}{3}\right). \qquad (23b)$$

For cosmological scales, we expect that $\eta_{\rm e} \simeq -\sqrt{2}/k \ll -\alpha\,\eta_0 = -\alpha/k_0$, which translates to the condition $k \ll k_0/\alpha$. It is for such wavenumbers that our approximation works well. In Fig. 2, we have plotted the analytical and the numerical solution for the tensor modes corresponding to a wavenumber that satisfies the above condition. It should be obvious from the figure that the analytical solution matches the numerical solution quite well, indicating the extent of accuracy of the analytical approximation. Also, note that the amplitude of the tensor mode freezes soon after it exits the Hubble radius during the contracting phase. Moreover, it is clear that the bounce does not affect its amplitude. In other words, the tensor modes broadly behave in a fashion similar to the way they do in de Sitter inflation.

Let us now turn to the evaluation of the power spectrum after the bounce. Recall that the tensor power spectrum $\mathcal{P}_{_{\rm T}}(k)$, evaluated at a given time, is defined as

$$\mathcal{P}_{_{\rm T}}(k) = 4\,\frac{k^3}{2\,\pi^2}\,|h_k(\eta)|^2. \qquad (24)$$

Upon using the solution (21) in the second domain, the tensor power spectrum, evaluated at $\eta_{\rm f} = \beta\,\eta_0$ can be expressed as

$$\mathcal{P}_{_{\rm T}}(k) = 4\,\frac{k^3}{2\,\pi^2}\,\left|A_k + B_k\,\left(\beta + \frac{\beta^3}{3}\right)\right|^2. \qquad (25)$$

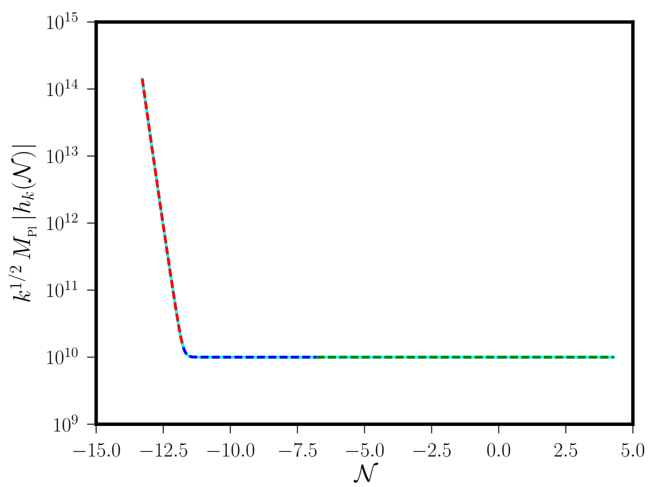

FIG. 2. The evolution of the amplitude of the tensor mode h_k evaluated analytically (in red until $\eta_{\rm e} \simeq -\sqrt{2}/k$, in blue over $\eta_{\rm e} < \eta < \eta_* = -\alpha\,\eta_0$ and in green over $\eta_* < \eta < \eta_{\rm f} = \beta\,\eta_0$) and numerically (in cyan) has been plotted as a function of e-N-folds $\mathcal{N}$, which is defined as $a(\mathcal{N}) = a_0\,\exp\left(\mathcal{N}^2/2\right)$ (in this context, see Refs. [28, 37]. Note that the bounce occurs at $\mathcal{N} = 0$, with negative and positive values of $\mathcal{N}$ corresponding to the contracting and expanding phases, respectively. We have set $k_0/(a_0\,M_{_{\rm Pl}}) = 10$, $\alpha = 10^5$ and $\beta = 10^2$ in plotting the figure. The mode of interest corresponds to the wavenumber $k = 10^{-11}\,(k_0/\alpha)$, which satisfies the required condition $k/(k_0/\alpha) \ll 1$ for the analytical approximations to be valid. It is clear that the analytical results match the exact numerical results very well. Also, evidently, the amplitude of the tensor modes freeze soon after they cross the Hubble radius (to be precise, when $k^2 \simeq z''/z$ at $\eta_{\rm e}$) during the contracting phase, and remain largely unaffected by the bounce.

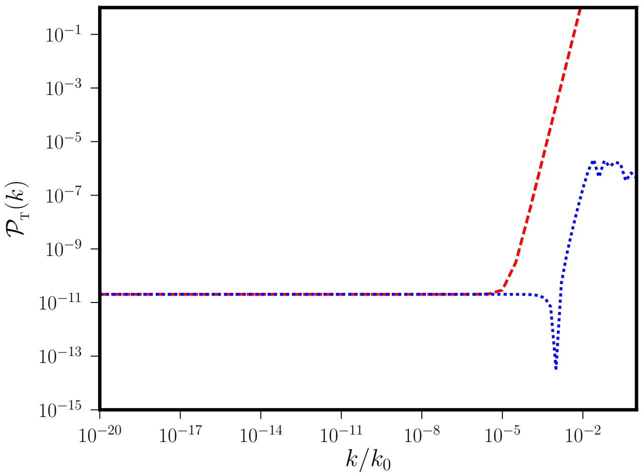

FIG. 3. The tensor power spectrum evaluated analytically [using Eq. (25)] (in red) and numerically (in blue) has been plotted as a function of k/k_0. We have worked with the same set of values of $k_0/(a_0\, M_{_{\rm Pl}})$, α and β as in the previous figure. Note that the analytical approximations are expected to be valid for $k/k_0 \ll 1/\alpha$, a domain over which the tensor power spectrum is strictly scale invariant. These values correspond to the analytical estimate of [cf. Eq. (26)] $\mathcal{P}_{_{\rm T}}(k) \simeq 2.026 \times 10^{-11}$ over the scale invariant domain, which is exactly what we obtain numerically. Needless to add, the analytical estimates are in very good agreement with the numerical results, indicating the validity of the approximations involved.

Clearly, it would desirable that the non-miminal action (5) reduces to the Einstein-Hilbert form at $\eta_{\rm f}$. This implies that we can set $C = a^{3/2}(\eta_{\rm f})$. Also, we expect z to behave as $z = a$ thereafter. In such a case, we find that, for $k \ll k_0/\alpha$, the power spectrum proves to be strictly scale invariant and has the following amplitude (if we assume that $\alpha \gg \beta$):

$$\mathcal{P}_{_{\rm T}}(k) = \frac{2}{\pi^2\, \beta^6}\, \left(\frac{k_0}{a_0\, M_{_{\rm Pl}}}\right)^2. \tag{26}$$

If we now choose that $k_0/(a_0\, M_{_{\rm Pl}}) < (\pi\, \beta^3/\sqrt{2})\, 10^{-5}$, one finds that $\mathcal{P}_{_{\rm T}}(k) < 10^{-10}$, which will be consistent with the current upper bound on the tensor-to-scalar ratio of $r \lesssim 0.07$ from the CMB [8]. In Fig. 3, we have plotted the analytical result (25) for the tensor power spectrum as well as the corresponding numerical results. Evidently, the analytical and numerical results match well for $k/k_0 \ll 1/\alpha$.

IV. THE TENSOR BI-SPECTRUM

Given the third order action (13b), the corresponding interaction Hamiltonian can be easily arrived at (see Ref. [38]; also see Refs. [39, 40]). With the interaction Hamiltonian at hand, the tensor bispectrum can be obtained by using the conventional rules of perturbative quantum field theory. The tensor bispectrum $G^{m_1 n_1 m_2 n_2 m_3 n_3}_{\gamma\gamma\gamma}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)$, calculated in the perturbative vacuum, can be expressed in terms of the modes h_k as follows [25, 27, 33, 34, 41, 42]:

$$G^{m_1 n_1 m_2 n_2 m_3 n_3}_{\gamma\gamma\gamma}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = M_{_{\rm Pl}}^2\, \Biggl[\biggl(\Pi^{\boldsymbol{k}_1}_{m_1 n_1, ij}\, \Pi^{\boldsymbol{k}_2}_{m_2 n_2, im}\, \Pi^{\boldsymbol{k}_3}_{m_3 n_3, lj} - \frac{1}{2}\, \Pi^{\boldsymbol{k}_1}_{m_1 n_1, ij}\, \Pi^{\boldsymbol{k}_2}_{m_2 n_2, ml}\, \Pi^{\boldsymbol{k}_3}_{m_3 n_3, ij}\biggr)\, k_{1m}\, k_{1l} + \text{five permutations}\Biggr]\, \Bigl[h_{k_1}(\eta_{\rm e})\, h_{k_2}(\eta_{\rm e})\, h_{k_3}(\eta_{\rm e})\, \mathcal{G}_{\gamma\gamma\gamma}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) + \text{complex conjugate}\Bigr], \tag{27}$$

where $\Pi^{\boldsymbol{k}}_{ij,mn} = \sum_{s=1,2} \varepsilon^s_{ij}(\boldsymbol{k})\, \varepsilon^{s*}_{mn}(\boldsymbol{k})$, with $\varepsilon^s_{ij}(\boldsymbol{k})$ being the polarization tensor characterizing the perturbations corresponding to helicity s. The quantity $\mathcal{G}_{\gamma\gamma\gamma}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)$ is described by the integral

$$\mathcal{G}_{\gamma\gamma\gamma}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = -\frac{i}{4} \int_{\eta_{\rm i}}^{\eta_{\rm f}} {\rm d}\eta\; z^2\, h^*_{k_1}\, h^*_{k_2}\, h^*_{k_3}, \tag{28}$$

with $\eta_{\rm i}$ being the time when the initial conditions are imposed on the perturbations and $\eta_{\rm f}$ denoting the final time when the bispectrum is to be evaluated. Also, we should clarify that (k_{1i}, k_{2i}, k_{3i}) denote the components of the three wavevectors $(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)$ along the i-spatial direction.

Our primary aim in this work is to calculate the magnitude and shape of the tensor bispectrum and the corresponding non-Gaussianity parameter in the model of our interest and compare them with, say, the results in de Sitter inflation. Therefore, for convenience, we shall set the polarization tensor $\varepsilon^s_{ij}(\boldsymbol{k})$ to unity. In such a case, the expression (27) for the tensor bi-spectrum above reduces to the following simpler form:

$$G_{\gamma\gamma\gamma}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = M_{_{\rm Pl}}^2 \biggl[h_{k_1}(\eta_{\rm f})\, h_{k_2}(\eta_{\rm f})\, h_{k_3}(\eta_{\rm f}) \times \left(k_1^2+k_2^2+k_3^2\right)\, \mathcal{G}_{\gamma\gamma\gamma}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) + \text{complex conjugate}\biggr]. \tag{29}$$

We shall choose $\eta_{\rm i} = -\infty$, *i.e.* the earliest time during the contracting phase when the initial conditions are imposed on the modes, and we shall set $\eta_{\rm f} = \beta\,\eta_0$, *viz.* the time after the bounce at which we had evaluated the power spectrum.

Since we had divided the domain of interest into two to arrive at the tensor modes, we can evaluate the quantity $\mathcal{G}_{\gamma\gamma\gamma}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)$ [cf. Eq. (28)] over these two domains as well. Recall that, in the first domain wherein $-\infty < \eta < -\alpha\,\eta_0$, we have $z \simeq -C/(\sqrt{a_0}\,k_0\,\eta)$. Upon using the modes (20) in the integral (28), we obtain the quantity $\mathcal{G}_{\gamma\gamma\gamma}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)$ in the first domain to be

$$\mathcal{G}^{\rm I}_{\gamma\gamma\gamma}(\mathbf{k_1},\mathbf{k_2},\mathbf{k_3}) = \frac{k_0}{4\,a_0\,(1+\beta^2)^{3/2}\,M_{_{\rm Pl}}^3}\,\frac{{\rm e}^{-i\,\alpha\,k_{_{\rm T}}/k_0}}{(k_1\,k_2\,k_3)^{3/2}}\,\left[\frac{k_0}{\alpha} - \frac{\alpha\,k_1\,k_2\,k_3}{k_0\,k_{_{\rm T}}} + \frac{i\,k_1\,k_2\,k_3}{k_{_{\rm T}}}\,\left(\frac{1}{k_1}+\frac{1}{k_2}+\frac{1}{k_3}+\frac{1}{k_{_{\rm T}}}\right)\right], \tag{30}$$

where $k_{_{\rm T}} = k_1+k_2+k_3$. Similarly, in the second domain, we can make use of the solution (21) for h_k. Also, over this domain, we have $z(\eta) = C/\sqrt{a(\eta)}$. Upon using these expressions and carrying out the integral (28) over $-\alpha\,\eta_0 < \eta < \beta\,\eta_0$, we find that $\mathcal{G}_{\gamma\gamma\gamma}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)$ in the second domain can be written as

$$\begin{aligned}\mathcal{G}^{\rm II}_{\gamma\gamma\gamma}(\mathbf{k_1},\mathbf{k_2},\mathbf{k_3}) = \frac{-i}{4\,k_0}\,a_0^2\,(1+\beta^2)^3\,\biggl\{&A_{k_1}^\ast\,A_{k_2}^\ast\,A_{k_3}^\ast\,\left(\tan^{-1}\beta+\tan^{-1}\alpha\right)\\
&+\frac{1}{6}\,\left(A_{k_1}^\ast\,A_{k_2}^\ast\,B_{k_3}^\ast + A_{k_1}^\ast\,B_{k_2}^\ast\,A_{k_3}^\ast + B_{k_1}^\ast\,A_{k_2}^\ast\,A_{k_3}^\ast\right)\,\left[\left(\beta^2-\alpha^2\right)+2\,\ln\,\left(\frac{1+\beta^2}{1+\alpha^2}\right)\right]\\
&+\frac{1}{135}\,\left(A_{k_1}^\ast\,B_{k_2}^\ast\,B_{k_3}^\ast + B_{k_1}^\ast\,A_{k_2}^\ast\,B_{k_3}^\ast + B_{k_1}^\ast\,B_{k_2}^\ast\,A_{k_3}^\ast\right)\\
&\times\left[3\,\left(\alpha^5+\beta^5\right)+25\,\left(\beta^3+\alpha^3\right)+60\,(\beta+\alpha)-60\,\left(\tan^{-1}\beta+\tan^{-1}\alpha\right)\right]\\
&+\frac{1}{648}\,B_{k_1}^\ast\,B_{k_2}^\ast\,B_{k_3}^\ast\\
&\times\left[3\,\left(\beta^8-\alpha^8\right)+32\,\left(\beta^6-\alpha^6\right)+114\,\left(\beta^4-\alpha^4\right)+96\,\left(\beta^2-\alpha^2\right)-96\,\ln\,\left(\frac{1+\beta^2}{1+\alpha^2}\right)\right]\biggr\}.\end{aligned} \tag{31}$$

With the aid of the quantities $\mathcal{G}^{\rm I}_{\gamma\gamma\gamma}(\mathbf{k_1},\mathbf{k_2},\mathbf{k_3})$ and $\mathcal{G}^{\rm II}_{\gamma\gamma\gamma}(\mathbf{k_1},\mathbf{k_2},\mathbf{k_3})$ we have evaluated above and, using the solution (21) to determine $h_k(\eta_{\rm f})$, we can arrive at the tensor bispectrum by substituting them in the expression (29). As the resulting expression proves to be rather lengthy, we do not explicitly write down the complete bispectrum here. In the following section, we shall plot the corresponding tensor non-Gaussianity parameter $h_{_{\rm NL}}$ in the equilateral and the squeezed limits.

V. THE TENSOR NON-GAUSSIANITY PARAMETER

The dimensionless non-Gaussianity parameter that characterizes the amplitude of the tensor bi-spectrum can be defined to be (in this context, see, for instance, Refs. [28, 42])

$$\begin{aligned}h_{_{\rm NL}}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = -\left(\frac{4}{2\,\pi^2}\right)^2\,&\left[k_1^3\,k_2^3\,k_3^3\,G_{\gamma\gamma\gamma}^{m_1n_1m_2n_2m_3n_3}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)\right]\\
&\times\left[\Pi^{\boldsymbol{k}_1}_{m_1n_1,m_3n_3}\,\Pi^{\boldsymbol{k}_2}_{m_2n_2,\bar{m}\bar{n}}\,k_3^3\,\mathcal{P}_{_{\rm T}}(k_1)\,\mathcal{P}_{_{\rm T}}(k_2)+\text{five permutations}\right]^{-1},\end{aligned} \tag{32}$$

where the overbars on the indices imply that they need to be summed over all allowed values. If we ignore the factors involving the polarization tensor, the non-Gaussianity parameter $h_{_{\rm NL}}$ simplifies to

$$h_{_{\rm NL}}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = -\left(\frac{4}{2\,\pi^2}\right)^2\,\left[k_1^3\,k_2^3\,k_3^3\,G_{\gamma\gamma\gamma}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)\right]\,\left[2\,k_3^3\,\mathcal{P}_{_{\rm T}}(k_1)\,\mathcal{P}_{_{\rm T}}(k_2)+\text{two permutations}\right]^{-1}. \tag{33}$$

The non-Gaussianity parameter $h_{_{\rm NL}}$ in the model of our interest can be arrived at using the power and bispectra arrived at in the previous two sections. In Fig. 4, we have plotted the parameter $h_{_{\rm NL}}$ over a wide range of wavenumbers in the equilateral and the squeezed limits. If the consistency condition is satisfied, in the squeezed limit (say, when $k_1 \to 0$ and $k_2 = k_3 = k$) one expects that $h_{_{\rm NL}} = (3 - n_{_{\rm T}})/8$, where $n_{_{\rm T}}$ is the tensor spectral index (see, for instance, Ref. [27]). For $n_{_{\rm T}} = 0$, $h_{_{\rm NL}} = 3/8 = 0.375$ when the consistency condition is satisfied in the squeezed limit, which is exactly what we obtain. Moreover, in the equilateral limit (say, when $k_1 = k_2 = k_3 = k$), we find that $h_{_{\rm NL}} = 0.472$, just as in de Sitter inflation (see, for example, Ref. [42]). In fact, we find that that the contribution to $h_{_{\rm NL}}$ due to the second domain is completely negligible, while the contribution due to the first domain closely resembles the $h_{_{\rm NL}}$ that arises in de Sitter for modes such that $k \ll k_0/\alpha$. These should be contrasted with the results in a matter bounce in Einsteinian gravity, wherein $h_{_{\rm NL}}$ behaves as k^2 in the equilateral as well as the squeezed limits. This behavior leads to rather small values for $h_{_{\rm NL}}$ over scales of cosmological interest. Also, it leads to a violation of the consistency condition (in this context, see Ref. [28]).

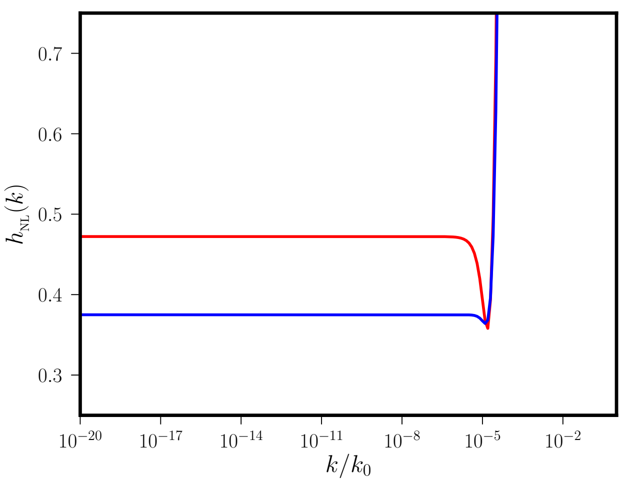

FIG. 4. The tensor non-Gaussianity parameter $h_{_{\rm NL}}$ has been plotted in the equilateral (in red) and in the squeezed (in blue) limits. We have worked with the same values for the parameters as in the previous two figures wherein we had plotted the evolution of the tensor modes and the corresponding power spectrum. As in the case of the tensor power spectrum, the analytical results are valid for $k/k_0 < 1/\alpha$, where $h_{_{\rm NL}}$ is strictly scale invariant. In the scale invariant regime, we find that $h_{_{\rm NL}} = 0.472$ and $h_{_{\rm NL}} = 0.375$ in the equilateral and squeezed limits, respectively, which exactly correspond to the values under these limits in de Sitter inflation. In particular, we find that the consistency condition in the squeezed limit is indeed satisfied.

VI. DISCUSSION

The duality principle suggests that bouncing scenarios such as the matter bounce can lead to strictly scale invariant spectra as de Sitter inflation does [24]. It is expected that the three-point functions can help us discriminate between alternative scenarios such as the matter bounce and de Sitter inflation, which otherwise lead to identical two-point functions. In inflation, the three-point functions satisfy the consistency condition in the squeezed limit, according to which the three-point functions can be completely expressed in terms of the two-points functions. This condition arises due to the fact that the amplitude of the long wavelength modes freeze on super-Hubble scales during inflation. However, in the bouncing models, often, the amplitudes of the modes grow strongly as one approaches the bounce, a behavior that results in the violation of the consistency condition in such scenarios. It then becomes interesting to examine whether it is possible at all to restore the consistency condition in the bouncing scenarios.

In this work, we have considered a fairly generic non-minimally coupled model of gravitation and examined the behavior of the tensor bispectrum in the matter bounce scenario. We had focused on the case of tensors since it requires limited modeling and also the modes are easier to determine analytically. We had explicitly evaluated the tensor bispectrum for a specific form of the non-minimal coupling function G [cf. Eq. (15)]. We had found that, for such a coupling function, the tensor modes behave in exactly the same manner as in de Sitter inflation on sub-Hubble as well as super-Hubble scales during the contracting phase. While there is some difference in the behavior of the tensor modes (when compared to their behavior at late times in de Sitter inflation) as they approach and cross the bounce, the difference is not substantial enough to alter the shape of the tensor power and bispectra.

In retrospect, our choice of the coupling function (15) is not difficult to understand. In the Jordan frame wherein gravity is coupled non-minimally to the matter fields, it is the function $z(\eta)$ that determines the behavior of the tensor modes and the resulting correlation functions. But, in the Einstein frame wherein gravity is coupled minimally, the $z(\eta)$ we have worked with has to be treated as the scale factor. Note that, in the matter bounce scenario of our interest, at early times during the contracting phase, we have $z(\eta) = C/\sqrt{a(\eta)} \propto 1/\eta$ [cf. Eq. (16)], which is exactly the behavior of the scale factor in de Sitter inflation. As we had discussed, it is due to this reason that the modes in the first domain $h_k^{\rm I}$ have exactly the same form as in de Sitter [cf. Eq. (20)]. However, it should be emphasized that the equivalence to de Sitter inflation is not exact and the equivalence breaks down as one approaches the bounce. In fact, while the amplitude of the tensor modes freeze after leaving the Hubble radius during the contracting phase, one finds that there is a weak growth in their amplitude soon after the bounce [this should be clear from the solution (21)]. Despite this behavior, we find that the domain around the bounce does not contribute to the tensor bispectrum significantly. Therefore, the tensor power and bispectra largely retain their forms as in de Sitter inflation. Specifically, we find that our choice of the non-minimal coupling indeed restores the consistency condition governing the tensor bispectrum in the squeezed limit.

While we have been able to restore the consistency condition, it is possible that we have achieved it at some cost. Note that the non-minimal coupling function behaves as $G \propto 1/\eta^6$ [cf. Eq. (15)] at early stages of the contracting phase. This implies that the gravitational coupling to matter is rather strong during early times, which can turn out to be undesirable. We are currently working towards circumventing such difficulties and examining the corresponding results for the correlation functions involving the scalar perturbations in stable contracting phases [43, 44].

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